

# 10.2 Find Arc Measures



**Before** You found angle measures.

**Now** You will use angle measures to find arc measures.

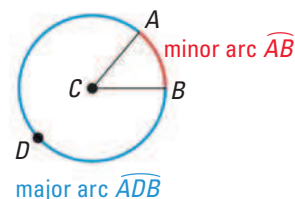
**Why?** So you can describe the arc made by a bridge, as in Ex. 22.

## Key Vocabulary

- central angle
- minor arc
- major arc
- semicircle
- measure  
minor arc, major arc
- congruent circles
- congruent arcs

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram,  $\angle ACB$  is a central angle of  $\odot C$ .

If  $m\angle ACB$  is less than  $180^\circ$ , then the points on  $\odot C$  that lie in the interior of  $\angle ACB$  form a **minor arc** with endpoints  $A$  and  $B$ . The points on  $\odot C$  that do not lie on minor arc  $\widehat{AB}$  form a **major arc** with endpoints  $A$  and  $B$ . A **semicircle** is an arc with endpoints that are the endpoints of a diameter.



**NAMING ARCS** Minor arcs are named by their endpoints. The minor arc associated with  $\angle ACB$  is named  $\widehat{AB}$ . Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with  $\angle ACB$  can be named  $\widehat{ADB}$ .

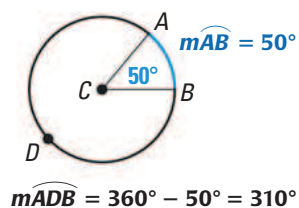
## KEY CONCEPT

## For Your Notebook

### Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression  $m\widehat{AB}$  is read as “the measure of arc  $AB$ .”

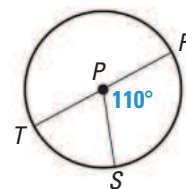
The measure of the entire circle is  $360^\circ$ . The **measure of a major arc** is the difference between  $360^\circ$  and the measure of the related minor arc. The measure of a semicircle is  $180^\circ$ .



## EXAMPLE 1 Find measures of arcs

Find the measure of each arc of  $\odot P$ , where  $\overline{RT}$  is a diameter.

- a.  $\widehat{RS}$                       b.  $\widehat{RTS}$                       c.  $\widehat{RST}$



### Solution

- a.  $\widehat{RS}$  is a minor arc, so  $m\widehat{RS} = m\angle RPS = 110^\circ$ .
- b.  $\widehat{RTS}$  is a major arc, so  $m\widehat{RTS} = 360^\circ - 110^\circ = 250^\circ$ .
- c.  $\overline{RT}$  is a diameter, so  $\widehat{RST}$  is a semicircle, and  $m\widehat{RST} = 180^\circ$ .

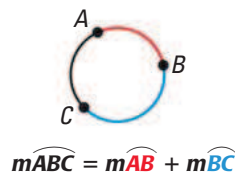
**ADJACENT ARCS** Two arcs of the same circle are *adjacent* if they have a common endpoint. You can add the measures of two adjacent arcs.

**POSTULATE**

*For Your Notebook*

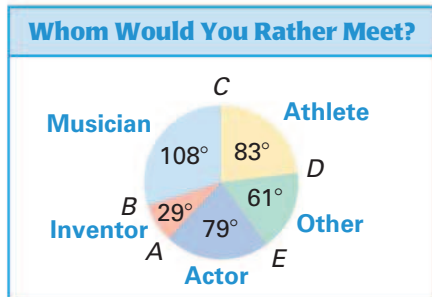
**POSTULATE 23 Arc Addition Postulate**

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



**EXAMPLE 2 Find measures of arcs**

**SURVEY** A recent survey asked teenagers if they would rather meet a famous musician, athlete, actor, inventor, or other person. The results are shown in the circle graph. Find the indicated arc measures.



- a.  $m\widehat{AC}$
- b.  $m\widehat{ACD}$
- c.  $m\widehat{ADC}$
- d.  $m\widehat{EBD}$

**Solution**

$$\begin{aligned} \text{a. } m\widehat{AC} &= m\widehat{AB} + m\widehat{BC} \\ &= 29^\circ + 108^\circ \\ &= 137^\circ \end{aligned}$$

$$\begin{aligned} \text{b. } m\widehat{ACD} &= m\widehat{AC} + m\widehat{CD} \\ &= 137^\circ + 83^\circ \\ &= 220^\circ \end{aligned}$$

$$\begin{aligned} \text{c. } m\widehat{ADC} &= 360^\circ - m\widehat{AC} \\ &= 360^\circ - 137^\circ \\ &= 223^\circ \end{aligned}$$

$$\begin{aligned} \text{d. } m\widehat{EBD} &= 360^\circ - m\widehat{ED} \\ &= 360^\circ - 61^\circ \\ &= 299^\circ \end{aligned}$$

**ARC MEASURES**

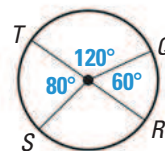
The measure of a minor arc is less than  $180^\circ$ .

The measure of a major arc is greater than  $180^\circ$ .

**GUIDED PRACTICE** for Examples 1 and 2

Identify the given arc as a *major arc*, *minor arc*, or *semicircle*, and find the measure of the arc.

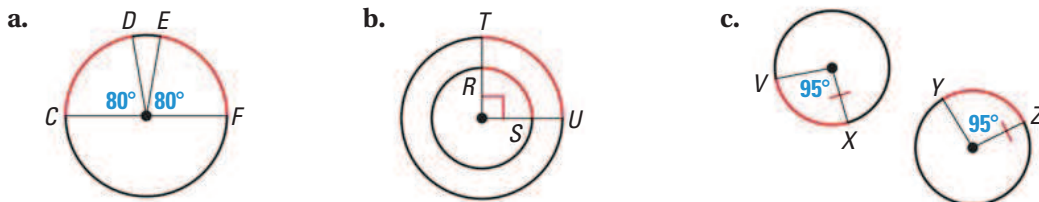
- 1.  $\widehat{TQ}$
- 2.  $\widehat{QRT}$
- 3.  $\widehat{TQR}$
- 4.  $\widehat{QS}$
- 5.  $\widehat{TS}$
- 6.  $\widehat{RST}$



**CONGRUENT CIRCLES AND ARCS** Two circles are **congruent circles** if they have the same radius. Two arcs are **congruent arcs** if they have the same measure and they are arcs of the same circle or of congruent circles. If  $\odot C$  is congruent to  $\odot D$ , then you can write  $\odot C \cong \odot D$ .

### EXAMPLE 3 Identify congruent arcs

Tell whether the red arcs are congruent. Explain why or why not.



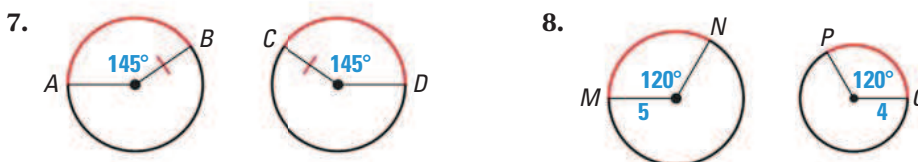
#### Solution

- $\widehat{CD} \cong \widehat{EF}$  because they are in the same circle and  $m\widehat{CD} = m\widehat{EF}$ .
- $\widehat{RS}$  and  $\widehat{TU}$  have the same measure, but are not congruent because they are arcs of circles that are not congruent.
- $\widehat{VX} \cong \widehat{YZ}$  because they are in congruent circles and  $m\widehat{VX} = m\widehat{YZ}$ .

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### GUIDED PRACTICE for Example 3

Tell whether the red arcs are congruent. Explain why or why not.



## 10.2 EXERCISES

### HOMEWORK KEY

= WORKED-OUT SOLUTIONS on p. WS1 for Exs. 5, 13, and 23

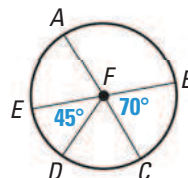
= STANDARDIZED TEST PRACTICE Exs. 2, 11, 17, 18, and 24

### SKILL PRACTICE

- VOCABULARY** Copy and complete: If  $\angle ACB$  and  $\angle DCE$  are congruent central angles of  $\odot C$ , then  $\widehat{AB}$  and  $\widehat{DE}$  are     .
- ★ WRITING** What do you need to know about two circles to show that they are congruent? *Explain.*

**MEASURING ARCS**  $\overline{AC}$  and  $\overline{BE}$  are diameters of  $\odot F$ . Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of  $\odot F$ . Then find the measure of the arc.

- $\widehat{BC}$
- $\widehat{DB}$
- $\widehat{AD}$
- $\widehat{ACD}$
- $\widehat{DC}$
- $\widehat{AE}$
- $\widehat{ABC}$
- $\widehat{EAC}$

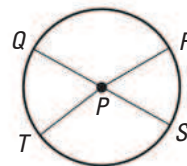


#### EXAMPLES 1 and 2

on pp. 659–660 for Exs. 3–11

11. ★ **MULTIPLE CHOICE** In the diagram,  $\overline{QS}$  is a diameter of  $\odot P$ . Which arc represents a semicircle?

- (A)  $\overline{QR}$                       (B)  $\overline{RQT}$   
 (C)  $\overline{QRS}$                       (D)  $\overline{QRT}$

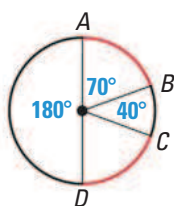


**EXAMPLE 3**

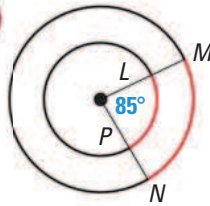
on p. 661  
 for Exs. 12–14

- CONGRUENT ARCS** Tell whether the red arcs are congruent. *Explain why or why not.*

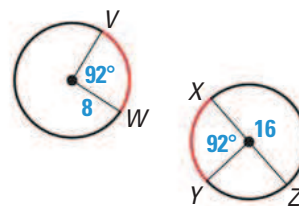
12.



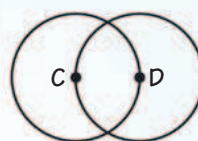
13.



14.



15. **ERROR ANALYSIS** *Explain* what is wrong with the statement.



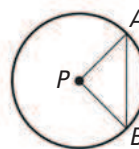
You cannot tell if  $\odot C \cong \odot D$  because the radii are not given.



16. **ARCS** Two diameters of  $\odot P$  are  $\overline{AB}$  and  $\overline{CD}$ . If  $m\widehat{AD} = 20^\circ$ , find  $m\widehat{ACD}$  and  $m\widehat{AC}$ .

17. ★ **MULTIPLE CHOICE**  $\odot P$  has a radius of 3 and  $\widehat{AB}$  has a measure of  $90^\circ$ . What is the length of  $\overline{AB}$ ?

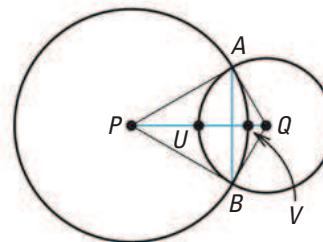
- (A)  $3\sqrt{2}$                       (B)  $3\sqrt{3}$   
 (C) 6                              (D) 9



18. ★ **SHORT RESPONSE** On  $\odot C$ ,  $m\widehat{EF} = 100^\circ$ ,  $m\widehat{FG} = 120^\circ$ , and  $m\widehat{EFG} = 220^\circ$ . If  $H$  is on  $\odot C$  so that  $m\widehat{GH} = 150^\circ$ , *explain* why  $H$  must be on  $\widehat{EF}$ .

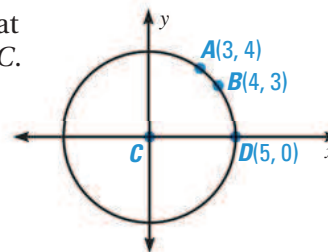
19. **REASONING** In  $\odot R$ ,  $m\widehat{AB} = 60^\circ$ ,  $m\widehat{BC} = 25^\circ$ ,  $m\widehat{CD} = 70^\circ$ , and  $m\widehat{DE} = 20^\circ$ . Find two possible values for  $m\widehat{AE}$ .

20. **CHALLENGE** In the diagram shown,  $\overline{PQ} \perp \overline{AB}$ ,  $\overline{QA}$  is tangent to  $\odot P$ , and  $m\widehat{AVB} = 60^\circ$ . What is  $m\widehat{AUB}$ ?



21. **CHALLENGE** In the coordinate plane shown,  $C$  is at the origin. Find the following arc measures on  $\odot C$ .

- a.  $m\widehat{BD}$   
 b.  $m\widehat{AD}$   
 c.  $m\widehat{AB}$




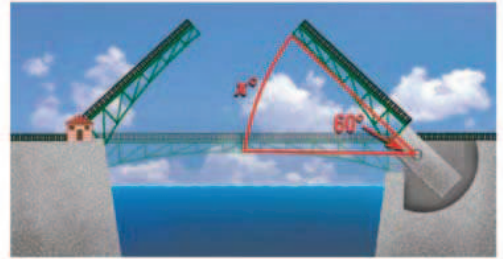
## PROBLEM SOLVING

### EXAMPLE 1


on p. 659  
for Ex. 22

22. **BRIDGES** The deck of a bascule bridge creates an arc when it is moved from the closed position to the open position. Find the measure of the arc.

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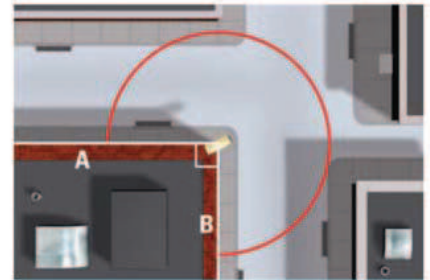
23. **DARTS** On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?

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24. **★ EXTENDED RESPONSE** A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of  $10^\circ$  per minute.

- What is the measure of the arc surveyed by the camera?
- How long does it take the camera to survey the entire area once?
- If the camera is at an angle of  $85^\circ$  from Wall B while rotating counterclockwise, how long will it take for the camera to return to that same position?
- The camera is rotating counterclockwise and is  $50^\circ$  from Wall A. Find the location of the camera after 15 minutes.



25. **CHALLENGE** A clock with hour and minute hands is set to 1:00 P.M.
- After 20 minutes, what will be the measure of the minor arc formed by the hour and minute hands?
  - At what time before 2:00 P.M., to the nearest minute, will the hour and minute hands form a diameter?

## MIXED REVIEW

### PREVIEW

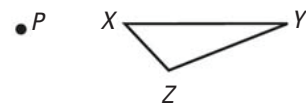
Prepare for  
Lesson 10.3  
in Exs. 26–27.

Determine if the lines with the given equations are parallel. (p. 180)

26.  $y = 5x + 2$ ,  $y = 5(1 - x)$

27.  $2y + 2x = 5$ ,  $y = 4 - x$

28. Trace  $\triangle XYZ$  and point  $P$ . Draw a counterclockwise rotation of  $\triangle XYZ$   $145^\circ$  about  $P$ . (p. 598)



Find the product. (p. 641)

29.  $(x + 2)(x + 3)$

30.  $(2y - 5)(y + 7)$

31.  $(x + 6)(x - 6)$

32.  $(z - 3)^2$

33.  $(3x + 7)(5x + 4)$

34.  $(z - 1)(z - 4)$