11.2 Areas of Trapezoids and Kites

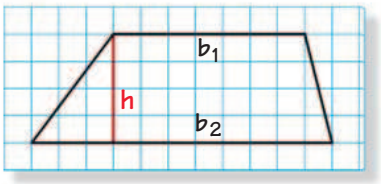
MATERIALS • graph paper • straightedge • scissors • tape

QUESTION How can you use a parallelogram to find other areas?

A trapezoid or a kite can be cut out and rearranged to form a parallelogram.

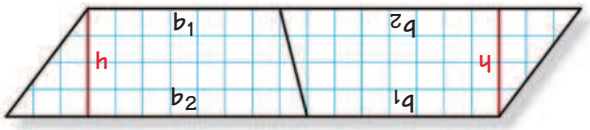
EXPLORE 1 Use two congruent trapezoids to form a parallelogram

STEP 1



Draw a trapezoid Fold graph paper in half and draw a trapezoid. Cut out two congruent trapezoids. Label as shown.

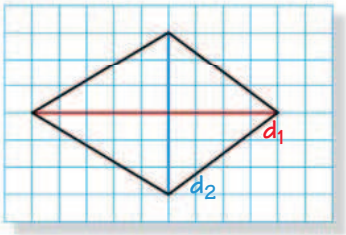
STEP 2



Create a parallelogram Arrange the two trapezoids from Step 1 to form a parallelogram. Then tape them together.

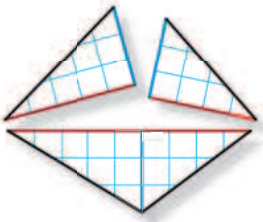
EXPLORE 2 Use one kite to form a rectangle

STEP 1



Draw a kite Draw a kite and its perpendicular diagonals. Label the diagonal that is a line of symmetry d_1. Label the other diagonal d_2.

STEP 2



Cut triangles Cut out the kite. Cut along d_1 to form two congruent triangles. Then cut one triangle along part of d_2 to form two right triangles.

STEP 3

Create a rectangle Turn over the right triangles. Place each with its hypotenuse along a side of the larger triangle to form a rectangle. Then tape the pieces together.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. In Explore 1, how does the area of one trapezoid compare to the area of the parallelogram formed from two trapezoids? Write expressions in terms of b_1, b_2, and h for the base, height, and area of the parallelogram. Then write a formula for the area of a trapezoid.
2. In Explore 2, how do the base and height of the rectangle compare to d_1 and d_2? Write an expression for the area of the rectangle in terms of d_1 and d_2. Then use that expression to write a formula for the area of a kite.

11.2 Areas of Trapezoids, Rhombuses, and Kites

Before You found areas of triangles and parallelograms.

Now You will find areas of other types of quadrilaterals.

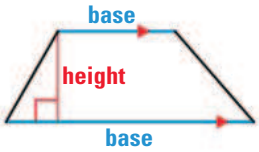
Why? So you can find the area of an irregular shape, as in Example 1.

Key Vocabulary
- **height of a trapezoid**
- **diagonal,** *p. 507*
- **bases of a trapezoid,** *p. 542*

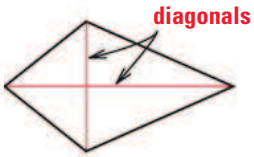
As you saw in the Activity on page 729, you can use the area formula for a parallelogram to develop area formulas for other special quadrilaterals. The areas of the figures below are related to the lengths of the marked segments.

The **height of a trapezoid** is the perpendicular distance between its bases.

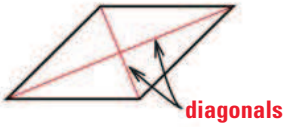
Trapezoid



Kite



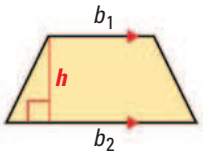
Rhombus



THEOREM *For Your Notebook*

THEOREM 11.4 Area of a Trapezoid

The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases.

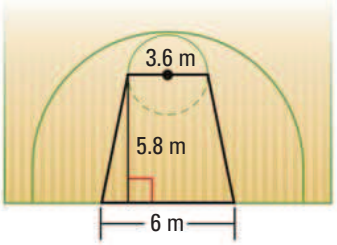


$$A = \frac{1}{2}h(b_1 + b_2)$$

Proof: Ex. 40, p. 736

EXAMPLE 1 Find the area of a trapezoid

BASKETBALL The free-throw lane on an international basketball court is shaped like a trapezoid. Find the area of the free-throw lane.



Solution

The height of the trapezoid is 5.8 meters. The lengths of the bases are 3.6 meters and 6 meters.

$A = \frac{1}{2}h(b_1 + b_2)$ **Formula for area of a trapezoid**

$= \frac{1}{2}(5.8)(3.6 + 6)$ **Substitute 5.8 for h, 3.6 for b_1, and 6 for b_2.**

$= 27.84$ **Simplify.**

▶ The area of the free-throw lane is about 27.8 square meters.

ANOTHER WAY
In a trapezoid, the average of the lengths of the bases is also the length of the midsegment. So, you can also find the area by multiplying the midsegment by the height.

ANOTHER WAY

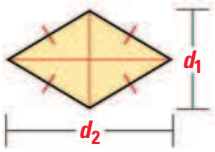
Remember that a rhombus is also a parallelogram, so you can also use the formula $A = bh$.

THEOREMS *For Your Notebook*

THEOREM 11.5 Area of a Rhombus

The area of a rhombus is one half the product of the lengths of its diagonals.

Justification: Ex. 39, p. 735

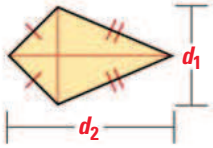


$A = \frac{1}{2}d_1d_2$

THEOREM 11.6 Area of a Kite

The area of a kite is one half the product of the lengths of its diagonals.

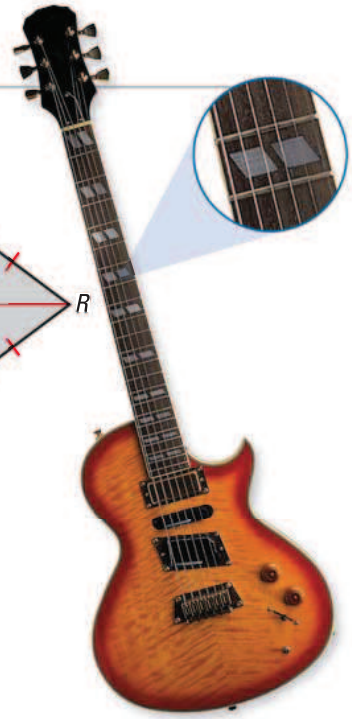
Proof: Ex. 41, p. 736

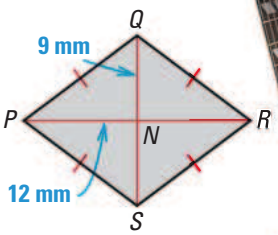


$A = \frac{1}{2}d_1d_2$

EXAMPLE 2 Find the area of a rhombus

MUSIC Rhombus $PQRS$ represents one of the inlays on the guitar in the photo. Find the area of the inlay.





READ DIAGRAMS

When you read a diagram, look for information you need to find. The diagram gives the lengths of $\overline{QN}$ and $\overline{PN}$, but not the lengths of $\overline{QS}$ and $\overline{PR}$.

Solution

STEP 1 **Find** the length of each diagonal. The diagonals of a rhombus bisect each other, so $QN = NS$ and $PN = NR$.

$QS = QN + NS = 9 + 9 = 18$ mm

$PR = PN + NR = 12 + 12 = 24$ mm

STEP 2 **Find** the area of the rhombus. Let d_1 represent QS and d_2 represent PR.

$A = \frac{1}{2}d_1d_2$ **Formula for area of a rhombus**

$= \frac{1}{2}(18)(24)$ **Substitute.**

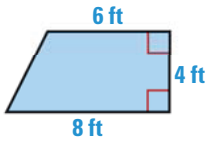
$= 216$ **Simplify.**

▶ The area of the inlay is 216 square millimeters.

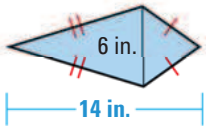
✓ GUIDED PRACTICE for Examples 1 and 2

Find the area of the figure.

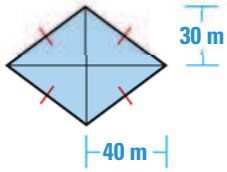
1.



2.



3.



EXAMPLE 3 Standardized Test Practice

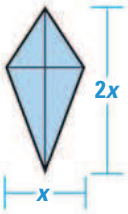
ELIMINATE CHOICES
In Example 3, you can eliminate choices A and B because in each case, one diagonal is not twice as long as the other diagonal.

One diagonal of a kite is twice as long as the other diagonal. The area of the kite is 72.25 square inches. What are the lengths of the diagonals?

Ⓐ 6 in., 6 in. Ⓑ 8.5 in., 8.5 in. Ⓒ 8.5 in., 17 in. Ⓓ 6 in., 12 in.

Solution

Draw and label a diagram. Let x be the length of one diagonal. The other diagonal is twice as long, so label it $2x$. Use the formula for the area of a kite to find the value of x.



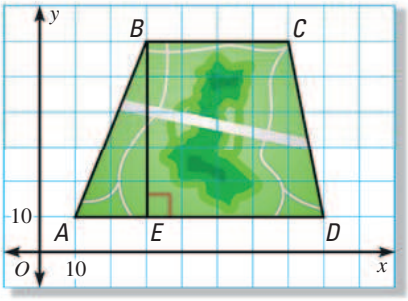
$A = \frac{1}{2}d_1d_2$	**Formula for area of a kite**
$72.25 = \frac{1}{2}(x)(2x)$	**Substitute 72.25 for A, x for d_1, and $2x$ for d_2.**
$72.25 = x^2$	**Simplify.**
$8.5 = x$	**Find the positive square root of each side.**

The lengths of the diagonals are 8.5 inches and $2(8.5) = 17$ inches.

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 4 Find an area in the coordinate plane

CITY PLANNING You have a map of a city park. Each grid square represents a 10 meter by 10 meter square. Find the area of the park.



Solution

STEP 1 **Find** the lengths of the bases and the height of trapezoid $ABCD$.

$b_1 = BC = |70 - 30| = 40$ m

$b_2 = AD = |80 - 10| = 70$ m

$h = BE = |60 - 10| = 50$ m

STEP 2 **Find** the area of $ABCD$.

$$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(50)(40 + 70) = 2750$$

▶ The area of the park is 2750 square meters.

✓ GUIDED PRACTICE for Examples 3 and 4

4. The area of a kite is 80 square feet. One diagonal is 4 times as long as the other. Find the diagonal lengths.

5. Find the area of a rhombus with vertices $M(1, 3)$, $N(5, 5)$, $P(9, 3)$, and $Q(5, 1)$.

11.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 17, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 30, 39, and 42

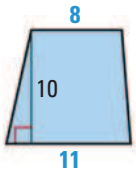
SKILL PRACTICE

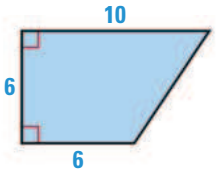
1. **VOCABULARY** Copy and complete: The perpendicular distance between the bases of a trapezoid is called the __?__ of the trapezoid.

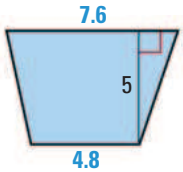
2. ★ **WRITING** Sketch a kite and its diagonals. *Describe* what you know about the segments and angles formed by the intersecting diagonals.

EXAMPLE 1 on p. 730 for Exs. 3–6

FINDING AREA **Find the area of the trapezoid.**

3. 

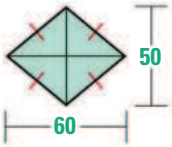
4. 

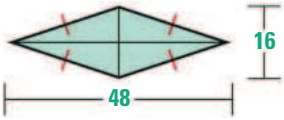
5. 

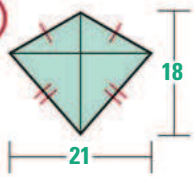
6. **DRAWING DIAGRAMS** The lengths of the bases of a trapezoid are 5.4 centimeters and 10.2 centimeters. The height is 8 centimeters. Draw and label a trapezoid that matches this description. Then find its area.

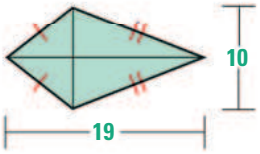
EXAMPLE 2 on p. 731 for Exs. 7–14

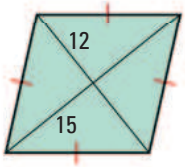
FINDING AREA **Find the area of the rhombus or kite.**

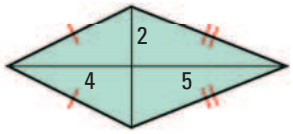
7. 

8. 

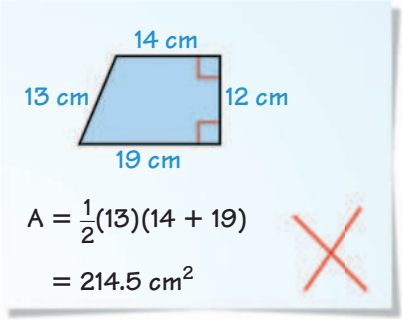
9. 

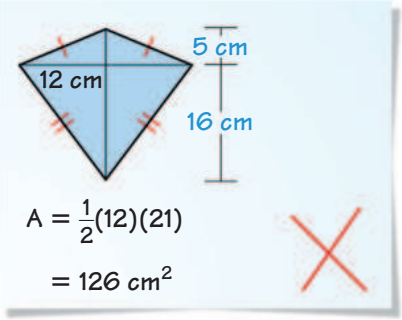
10. 

11. 

12. 

ERROR ANALYSIS ***Describe*** **and correct the error in finding the area.**

13. 

14. 

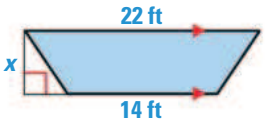
EXAMPLE 3 on p. 732 for Exs. 15–18

15. ★ **MULTIPLE CHOICE** One diagonal of a rhombus is three times as long as the other diagonal. The area of the rhombus is 24 square feet. What are the lengths of the diagonals?

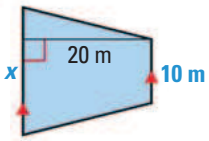
Ⓐ 8 ft, 11 ft Ⓑ 4 ft, 12 ft Ⓒ 2 ft, 6 ft Ⓓ 6 ft, 24 ft

ALGEBRA **Use the given information to find the value of x.**

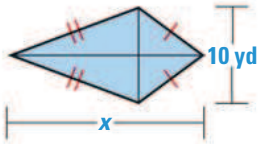
16. Area = 108 ft^2



17. Area = 300 m^2



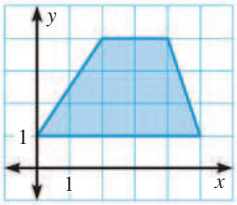
18. Area = 100 yd^2



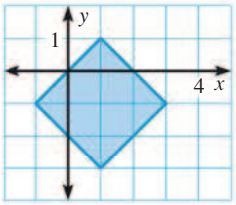
EXAMPLE 4 on p. 732 for Exs. 19–21

COORDINATE GEOMETRY **Find the area of the figure.**

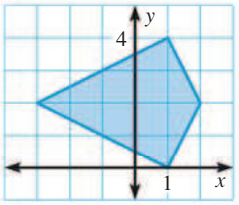
19.



20.



21.



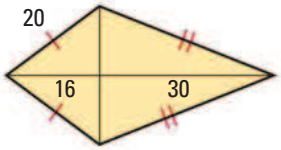
ALGEBRA **Find the lengths of the bases of the trapezoid described.**

22. The height is 3 feet. One base is twice as long as the other base. The area is 13.5 square feet.

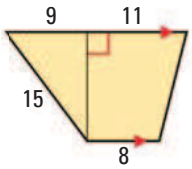
23. One base is 8 centimeters longer than the other base. The height is 6 centimeters and the area is 54 square centimeters.

FINDING AREA **Find the area of the shaded region.**

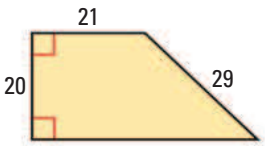
24.



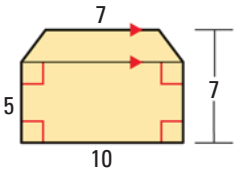
25.



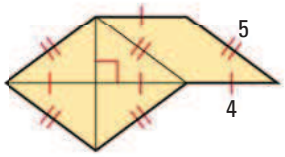
26.



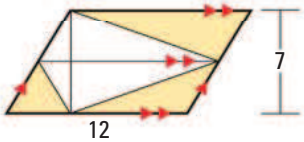
27.



28.



29.



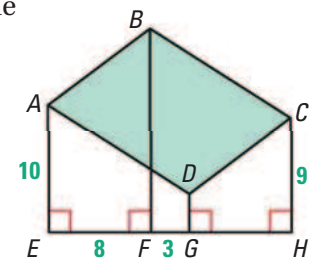
30. ★ **OPEN-ENDED MATH** Draw three examples of trapezoids that match this description: The height of the trapezoid is 3 units and its area is the same as the area of a parallelogram with height 3 units and base 8 units.

VISUALIZING **Sketch the figure. Then determine its perimeter and area.**

31. The figure is a trapezoid. It has two right angles. The lengths of its bases are 7 and 15. Its height is 6.

32. The figure is a rhombus. Its side length is 13. The length of one of its diagonals is 24.

33. **CHALLENGE** In the diagram shown at the right, $ABCD$ is a parallelogram and $BF = 16$. Find the area of ▱$ABCD$. *Explain* your reasoning. (*Hint:* Draw auxiliary lines through point A and through point D that are parallel to $\overline{EH}$.)



○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 1 on p. 730 for Ex. 34

34. TRUCKS The windshield in a truck is in the shape of a trapezoid. The lengths of the bases of the trapezoid are 70 inches and 79 inches. The height is 35 inches. Find the area of the glass in the windshield.

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EXAMPLE 2 on p. 731 for Ex. 35

35. INTERNET You are creating a kite-shaped logo for your school's website. The diagonals of the logo are 8 millimeters and 5 millimeters long. Find the area of the logo. Draw two different possible shapes for the logo.

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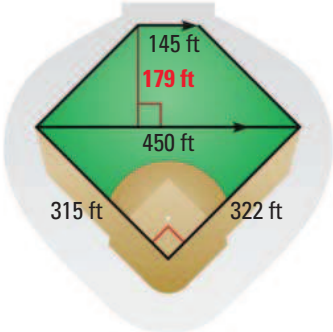
36. DESIGN You are designing a wall hanging that is in the shape of a rhombus. The area of the wall hanging is 432 square inches and the length of one diagonal is 36 inches. Find the length of the other diagonal.

37. MULTI-STEP PROBLEM As shown, a baseball stadium's playing field is shaped like a pentagon. To find the area of the playing field shown at the right, you can divide the field into two smaller polygons.

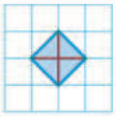
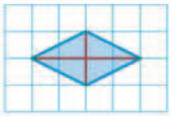
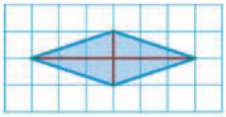
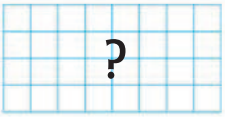


a. Classify the two polygons.

b. Find the area of the playing field in square feet. Then express your answer in square yards. Round to the nearest square foot.

38. VISUAL REASONING Follow the steps in parts (a)–(c).

a. Analyze Copy the table and extend it to include a column for $n = 5$. Complete the table for $n = 4$ and $n = 5$.

Rhombus number, *n*	1	2	3	4
Diagram				
Area, *A*	2	4	6	?

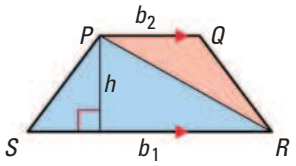
b. Use Algebra *Describe* the relationship between the rhombus number n and the area of the rhombus. Then write an algebraic rule for finding the area of the nth rhombus.

c. Compare In each rhombus, the length of one diagonal (d_1) is 2. What is the length of the other diagonal (d_2) for the nth rhombus? Use the formula for the area of a rhombus to write a rule for finding the area of the nth rhombus. *Compare* this rule with the one you wrote in part (b).

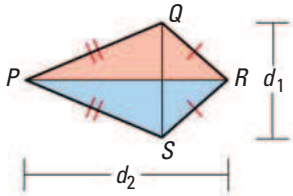
39. ★ SHORT RESPONSE Look back at the Activity on page 729. *Explain* how the results for kites in Explore 2 can be used to justify Theorem 11.5, the formula for the area of a rhombus.

PROVING THEOREMS 11.4 AND 11.6 **Use the triangle area formula and the triangles in the diagram to write a plan for the proof.**

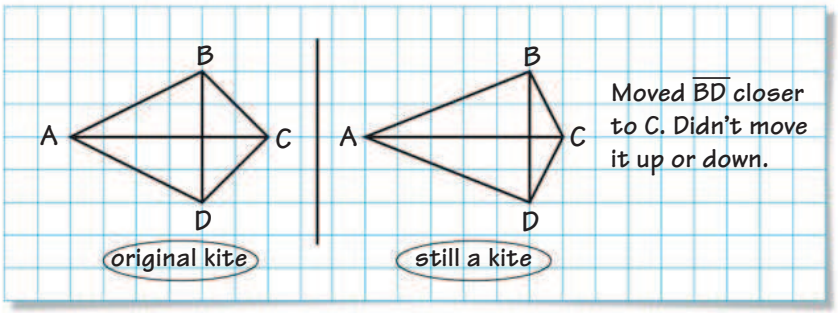
40. Show that the area A of the trapezoid shown is $\frac{1}{2}h(b_1 + b_2)$.



41. Show that the area A of the kite shown is $\frac{1}{2}d_1d_2$.



42. ★ **EXTENDED RESPONSE** You will explore the effect of moving a diagonal.

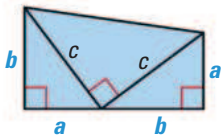


a. Investigate Draw a kite in which the longer diagonal is horizontal. Suppose this diagonal is fixed and you can slide the vertical diagonal left or right and up or down. You can keep sliding as long as the diagonals continue to intersect. Draw and identify each type of figure you can form.

b. Justify Is it possible to form any shapes that are not quadrilaterals? *Explain*.

c. Compare Compare the areas of the different shapes you found in part (b). What do you notice about the areas? *Explain*.

43. **CHALLENGE** James A. Garfield, the twentieth president of the United States, discovered a proof of the Pythagorean Theorem in 1876. His proof involved the fact that a trapezoid can be formed from two congruent right triangles and an isosceles right triangle. Use the diagram to show that $a^2 + b^2 = c^2$.



MIXED REVIEW

Solve for the indicated variable. Write a reason for each step. *(p. 105)*

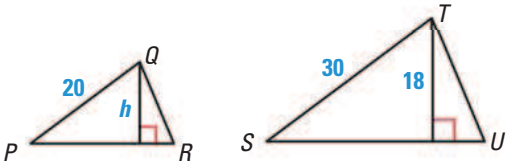
44. $d = rt$; solve for t

45. $A = \frac{1}{2}bh$; solve for h

46. $P = 2\ell + 2w$; solve for w

47. Find the angle measures of an isosceles triangle if the measure of a base angle is 4 times the measure of the vertex angle. *(p. 264)*

48. In the diagram at the right, $\triangle PQR \sim \triangle STU$. The perimeter of $\triangle STU$ is 81 inches. Find the height h and the perimeter of $\triangle PQR$. *(p. 372)*



PREVIEW
Prepare for Lesson 11.3 in Ex. 48.