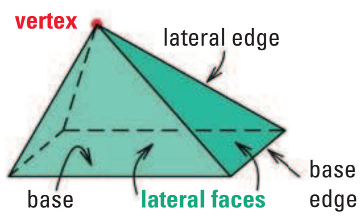
12.3 Surface Area of Pyramids and Cones

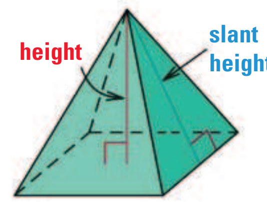
Before	You found surface areas of prisms and cylinders.
Now	You will find surface areas of pyramids and cones.
Why?	So you can find the surface area of a volcano, as in Ex. 33.

Key Vocabulary
- **pyramid**
- **vertex of a pyramid**
- **regular pyramid**
- **slant height**
- **cone**
- **vertex of a cone**
- **right cone**
- **lateral surface**

A **pyramid** is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex, called the **vertex of the pyramid**. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The height of the pyramid is the perpendicular distance between the base and the vertex.



Pyramid



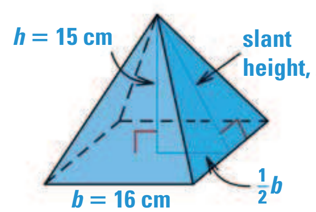
Regular pyramid

NAME PYRAMIDS
Pyramids are classified by the shapes of their bases.

A **regular pyramid** has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces of a regular pyramid are congruent isosceles triangles. The **slant height** of a regular pyramid is the height of a lateral face of the regular pyramid. A nonregular pyramid does not have a slant height.

EXAMPLE 1 Find the area of a lateral face of a pyramid

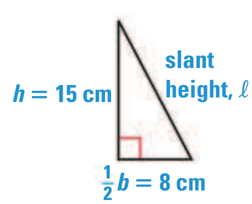
A regular square pyramid has a height of 15 centimeters and a base edge length of 16 centimeters. Find the area of each lateral face of the pyramid.



Solution

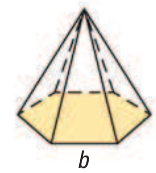
Use the Pythagorean Theorem to find the slant height ℓ.

$\ell^2 = h^2 + \left(\frac{1}{2}b\right)^2$	**Write formula.**
$\ell^2 = 15^2 + 8^2$	**Substitute for h and $\frac{1}{2}b$.**
$\ell^2 = 289$	**Simplify.**
$\ell = 17$	**Find the positive square root.**



▶ The area of each triangular face is $A = \frac{1}{2}b\ell = \frac{1}{2}(16)(17) = 136$ square centimeters.

SURFACE AREA A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.



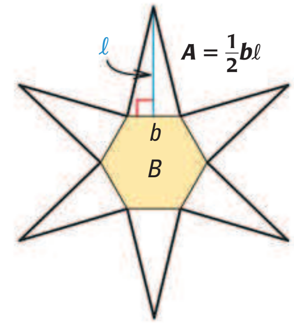
The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area S is as follows.

$S =$ (Area of base) $+ 6$(Area of lateral face)

$S = B + 6\left(\frac{1}{2}b\ell\right)$ **Substitute.**

$S = B + \frac{1}{2}(6b)\ell$ **Rewrite $6\left(\frac{1}{2}b\ell\right)$ as $\frac{1}{2}(6b)\ell$.**

$S = B + \frac{1}{2}P\ell$ **Substitute P for $6b$.**



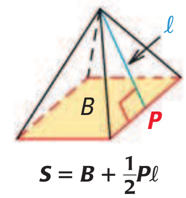
THEOREM — *For Your Notebook*

THEOREM 12.4 Surface Area of a Regular Pyramid

The surface area S of a regular pyramid is

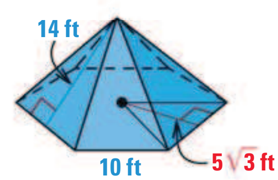
$$S = B + \frac{1}{2}P\ell,$$

where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



EXAMPLE 2 Find the surface area of a pyramid

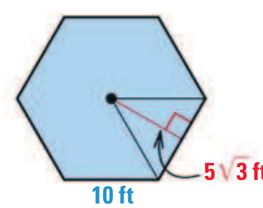
Find the surface area of the regular hexagonal pyramid.



Solution

REVIEW AREA
For help with finding the area of regular polygons, see p. 762.

First, find the area of the base using the formula for the area of a regular polygon, $\frac{1}{2}aP$. The apothem a of the hexagon is $5\sqrt{3}$ feet and the perimeter P is $6 \cdot 10 = 60$ feet. So, the area of the base B is $\frac{1}{2}(5\sqrt{3})(60) = 150\sqrt{3}$ square feet. Then, find the surface area.



$S = B + \frac{1}{2}P\ell$ **Formula for surface area of regular pyramid**

$= 150\sqrt{3} + \frac{1}{2}(60)(14)$ **Substitute known values.**

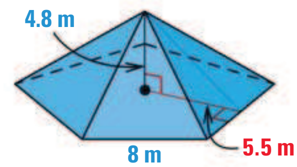
$= 150\sqrt{3} + 420$ **Simplify.**

≈ 679.81 **Use a calculator.**

▶ The surface area of the regular hexagonal pyramid is about $679.81\ \text{ft}^2$.

✓ GUIDED PRACTICE for Examples 1 and 2

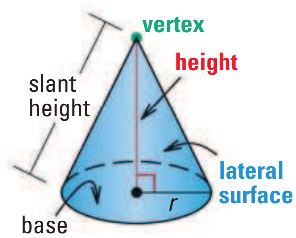
1. Find the area of each lateral face of the regular pentagonal pyramid shown.
2. Find the surface area of the regular pentagonal pyramid shown.



CONES A **cone** has a circular base and a **vertex** that is not in the same plane as the base. The radius of the base is the *radius* of the cone. The height is the perpendicular distance between the vertex and the base.

In a **right cone**, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

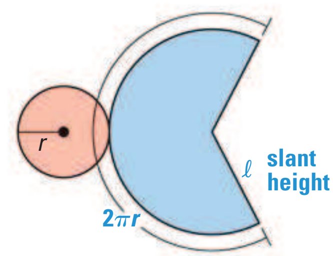
The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge.



Right cone

SURFACE AREA When you cut along the slant height and base edge and lay a right cone flat, you get the net shown at the right.

The circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can use a proportion to find the area of the sector, as shown below.



$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ **Set up proportion.**

$$\frac{\text{Area of sector}}{\pi\ell^2} = \frac{2\pi r}{2\pi\ell}$$ **Substitute.**

$$\text{Area of sector} = \pi\ell^2 \cdot \frac{2\pi r}{2\pi\ell}$$ **Multiply each side by $\pi\ell^2$.**

$$\text{Area of sector} = \pi r\ell$$ **Simplify.**

The surface area of a cone is the sum of the base area, πr^2, and the lateral area, $\pi r\ell$. Notice that the quantity $\pi r\ell$ can be written as $\frac{1}{2}(2\pi r)\ell$, or $\frac{1}{2}C\ell$.

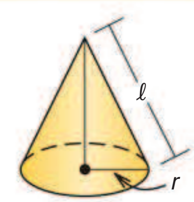
THEOREM — *For Your Notebook*

THEOREM 12.5 Surface Area of a Right Cone

The surface area S of a right cone is

$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell,$$

where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.



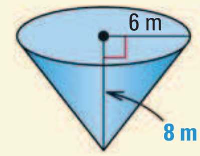
$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$

EXAMPLE 3 Standardized Test Practice

What is the surface area of the right cone?

(A) $72\pi\ m^2$ (B) $96\pi\ m^2$

(C) $132\pi\ m^2$ (D) $136\pi\ m^2$



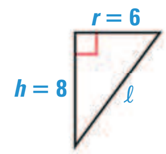
Solution

To find the slant height ℓ of the right cone, use the Pythagorean Theorem.

$\ell^2 = h^2 + r^2$ **Write formula.**

$\ell^2 = 8^2 + 6^2$ **Substitute.**

$\ell = 10$ **Find positive square root.**



ANOTHER WAY

You can use a Pythagorean triple to find ℓ.
$6 = 2 \cdot 3$ and $8 = 2 \cdot 4$, so $\ell = 2 \cdot 5 = 10$.

Use the formula for the surface area of a right cone.

$S = \pi r^2 + \pi r\ell$ **Formula for surface area of a right cone**

$= \pi(6^2) + \pi(6)(10)$ **Substitute.**

$= 96\pi$ **Simplify.**

▶ The correct answer is B. (A) (B) (C) (D)

EXAMPLE 4 Find the lateral area of a cone

TRAFFIC CONE The traffic cone can be approximated by a right cone with radius 5.7 inches and height 18 inches. Find the approximate lateral area of the traffic cone.

Solution

To find the slant height ℓ, use the Pythagorean Theorem.

$\ell^2 = 18^2 + (5.7)^2$, so $\ell \approx 18.9$ inches.

Find the lateral area.

Lateral area $= \pi r\ell$ **Write formula.**

$= \pi(5.7)(18.9)$ **Substitute known values.**

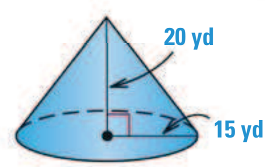
≈ 338.4 **Simplify and use a calculator.**

▶ The lateral area of the traffic cone is about 338.4 square inches.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the lateral area of the right cone shown.

4. Find the surface area of the right cone shown.



12.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 29

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, and 31

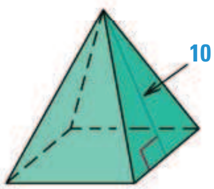
SKILL PRACTICE

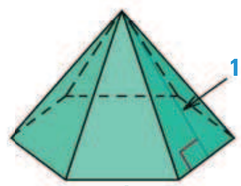
1. **VOCABULARY** Draw a regular square pyramid. Label its *height*, *slant height*, and *base*.

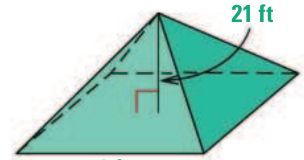
2. ★ **WRITING** *Compare* the height and slant height of a right cone.

EXAMPLE 1 on p. 810 for Exs. 3–5

AREA OF A LATERAL FACE **Find the area of each lateral face of the regular pyramid.**

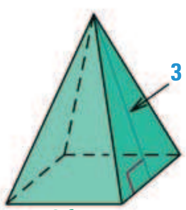
3. 

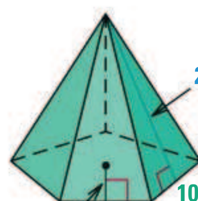
4. 

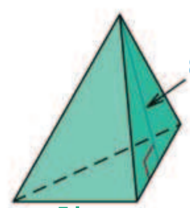
5. 

EXAMPLE 2 on p. 811 for Exs. 6–9

SURFACE AREA OF A PYRAMID **Find the surface area of the regular pyramid. Round your answer to two decimal places.**

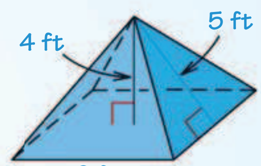
6. 

7. 

8. 

9. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the regular pyramid.

$$S = B + \frac{1}{2}P\ell$$
$$= 6^2 + \frac{1}{2}(24)(4)$$
$$= 84 \text{ ft}^2$$



EXAMPLES 3 and 4 on p. 813 for Exs. 10–17

LATERAL AREA OF A CONE **Find the lateral area of the right cone. Round your answer to two decimal places.**

10. 

$r = 7.5$ cm
$h = 25$ cm

11. 

$r = 1$ in.
$h = 4$ in.

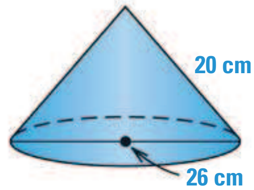
12. 

$d = 7$ in.
$h = 1$ ft

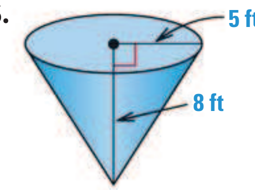
SURFACE AREA OF A CONE **Find the surface area of the right cone. Round your answer to two decimal places.**

13.

14.

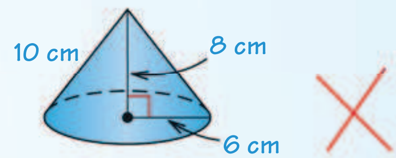


15.



16. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the right cone.

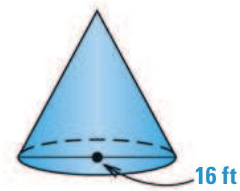
$$S = \pi(r^2) + \pi r^2 \ell$$
$$= \pi(36) + \pi(36)(10)$$
$$= 396\pi \text{ cm}^2$$



17. ★ **MULTIPLE CHOICE** The surface area of the right cone is 200π square feet. What is the slant height of the cone?

Ⓐ 10.5 ft Ⓑ 17 ft

Ⓒ 23 ft Ⓓ 24 ft



VISUAL REASONING **In Exercises 18–21, sketch the described solid and find its surface area. Round your answer to two decimal places.**

18. A right cone has a radius of 15 feet and a slant height of 20 feet.

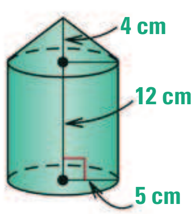
19. A right cone has a diameter of 16 meters and a height of 30 meters.

20. A regular pyramid has a slant height of 24 inches. Its base is an equilateral triangle with a base edge length of 10 inches.

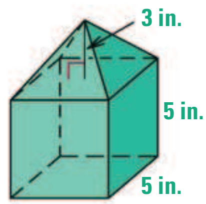
21. A regular pyramid has a hexagonal base with a base edge length of 6 centimeters and a slant height of 9 centimeters.

COMPOSITE SOLIDS **Find the surface area of the solid. The pyramids are regular and the cones are right. Round your answers to two decimal places, if necessary.**

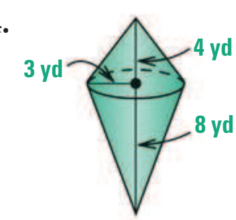
22.



23.



24.



25. **TETRAHEDRON** Find the surface area of a regular tetrahedron with edge length 4 centimeters.

26. **CHALLENGE** A right cone with a base of radius 4 inches and a regular pyramid with a square base both have a slant height of 5 inches. Both solids have the same surface area. Find the length of a base edge of the pyramid. Round your answer to the nearest hundredth of an inch.

PROBLEM SOLVING

EXAMPLE 2 on p. 811 for Ex. 27

27. **CANDLES** A candle is in the shape of a regular square pyramid with base edge length 6 inches. Its height is 4 inches. Find its surface area.

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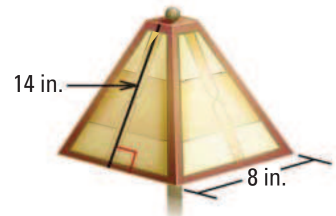
28. **LAMPSHADE** A glass lampshade is shaped like a regular square pyramid.



   a. Approximate the lateral area of the lampshade shown.

   b. *Explain* why your answer to part (a) is not the exact lateral area.

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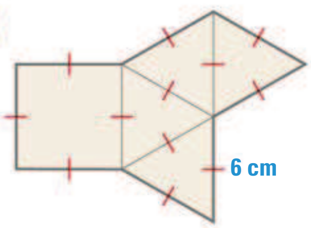
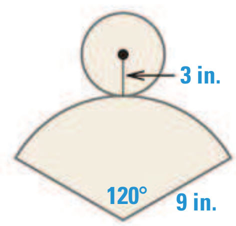
USING NETS **Name the figure that is represented by the net. Then find its surface area. Round your answer to two decimal places.**

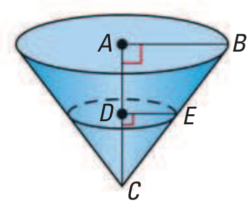
29.



30.



31. ★ **SHORT RESPONSE** In the figure, $AC = 4$, $AB = 3$, and $DC = 2$.

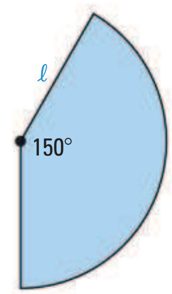


   a. Prove $\triangle ABC \sim \triangle DEC$.

   b. Find BC, DE, and EC.

   c. Find the surface areas of the larger cone and the smaller cone in terms of π. *Compare* the surface areas using a percent.

32. **MULTI-STEP PROBLEM** The sector shown can be rolled to form the lateral surface of a right cone. The lateral surface area of the cone is 20 square meters.



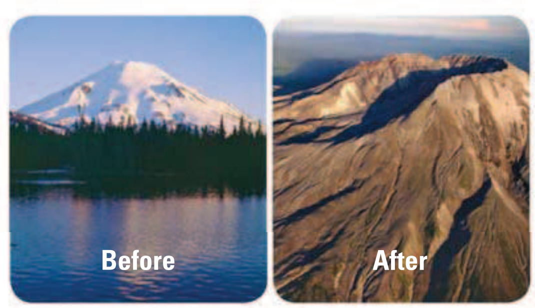
   a. Write the formula for the area of a sector.

   b. Use the formula in part (a) to find the slant height of the cone. *Explain* your reasoning.

   c. Find the radius and height of the cone.

33. **VOLCANOES** Before 1980, Mount St. Helens was a conic volcano with a height from its base of about 1.08 miles and a base radius of about 3 miles. In 1980, the volcano erupted, reducing its height to about 0.83 mile.

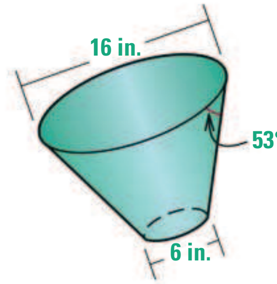
   Approximate the lateral area of the volcano after 1980. (*Hint:* The ratio of the radius of the destroyed cone-shaped top to its height is the same as the ratio of the radius of the original volcano to its height.)



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

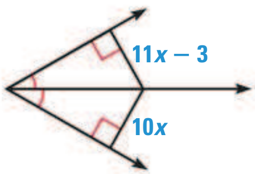
34. CHALLENGE An *Elizabethan collar* is used to prevent an animal from irritating a wound. The angle between the opening with a 16 inch diameter and the side of the collar is 53°. Find the surface area of the collar shown.



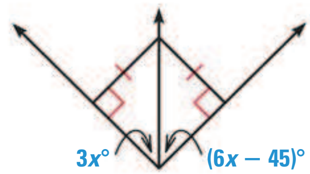
MIXED REVIEW

Find the value of *x*. *(p. 310)*

35.



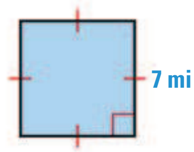
36.



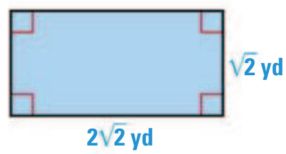
PREVIEW
Prepare for Lesson 12.4 in Exs. 37–39.

In Exercises 37–39, find the area of the polygon. *(pp. 720, 730)*

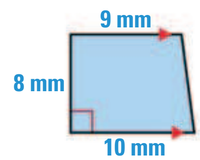
37.



38.



39.

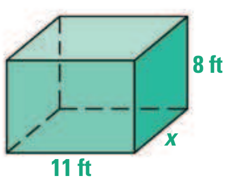


QUIZ *for Lessons 12.1–12.3*

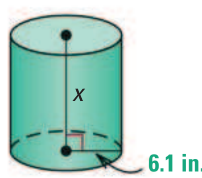
1. A polyhedron has 8 vertices and 12 edges. How many faces does the polyhedron have? *(p. 794)*

Solve for *x* given the surface area *S* of the right prism or right cylinder. Round your answer to two decimal places. *(p. 803)*

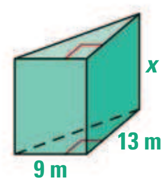
2. $S = 366 \text{ ft}^2$



3. $S = 717 \text{ in.}^2$

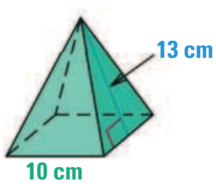


4. $S = 567 \text{ m}^2$



Find the surface area of the regular pyramid or right cone. Round your answer to two decimal places. *(p. 810)*

5.



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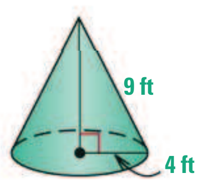

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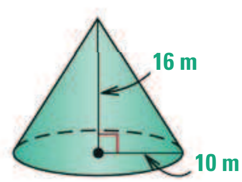

MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
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Lessons 12.1–12.3

1. **SHORT RESPONSE** Using Euler's Theorem, *explain* why it is not possible for a polyhedron to have 6 vertices and 7 edges.

2. **SHORT RESPONSE** *Describe* two methods of finding the surface area of a rectangular solid.

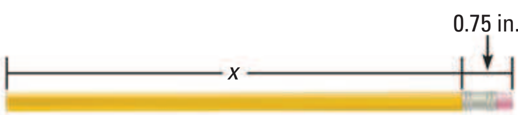
3. **EXTENDED RESPONSE** Some pencils are made from slats of wood that are machined into right regular hexagonal prisms.

   a. The formula for the surface area of a new unsharpened pencil without an eraser is

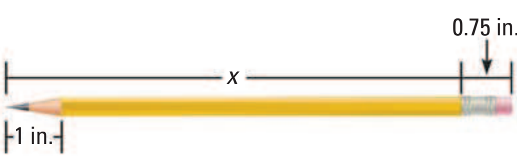
   $$S = 3\sqrt{3}r^2 + 6rh.$$

   Tell what each variable in this formula represents.

   b. After a pencil is painted, a metal band that holds an eraser is wrapped around one end. Write a formula for the surface area of the visible portion of the pencil, shown below.

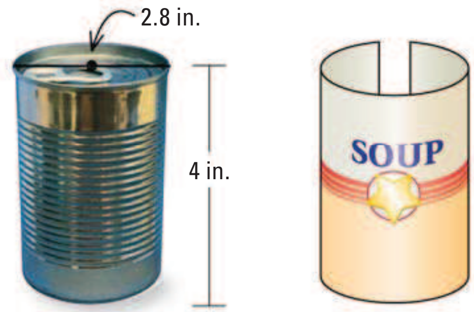


   c. After a pencil is sharpened, the end is shaped like a cone. Write a formula to find the surface area of the visible portion of the pencil, shown below.



   d. Use your formulas from parts (b) and (c) to write a formula for the difference of the surface areas of the two pencils. Define any variables in your formula.

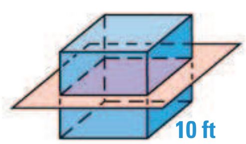
4. **GRIDDED ANSWER** The amount of paper needed for a soup can label is approximately equal to the lateral area of the can. Find the lateral area of the soup can in square inches. Round your answer to two decimal places.



5. **SHORT RESPONSE** If you know the diameter d and slant height ℓ of a right cone, how can you find the surface area of the cone?

6. **OPEN-ENDED** Identify an object in your school or home that is a rectangular prism. Measure its length, width, and height to the nearest quarter inch. Then approximate the surface area of the object.

7. **MULTI-STEP PROBLEM** The figure shows a plane intersecting a cube parallel to its base. The cube has a side length of 10 feet.



   a. Describe the shape formed by the cross section.

   b. Find the perimeter and area of the cross section.

   c. When the cross section is cut along its diagonal, what kind of triangles are formed?

   d. Find the area of one of the triangles formed in part (c).

8. **SHORT RESPONSE** A cone has a base radius of $3x$ units and a height of $4x$ units. The surface area of the cone is 1944π square units. Find the value of x. *Explain* your steps.