

12.4 Volume of Prisms and Cylinders



- Before**
- Now**
- Why**

You found surface areas of prisms and cylinders.
 You will find volumes of prisms and cylinders.
 So you can determine volume of water in an aquarium, as in Ex. 33.

Key Vocabulary
 • volume

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3).

POSTULATES
For Your Notebook

POSTULATE 27 Volume of a Cube Postulate
 The volume of a cube is the cube of the length of its side.

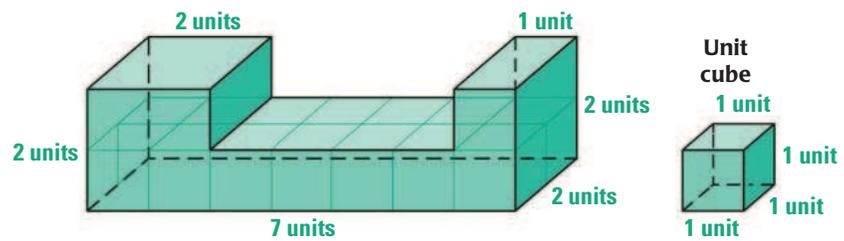
POSTULATE 28 Volume Congruence Postulate
 If two polyhedra are congruent, then they have the same volume.

POSTULATE 29 Volume Addition Postulate
 The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

$V = s^3$

EXAMPLE 1 Find the number of unit cubes

3-D PUZZLE Find the volume of the puzzle piece in cubic units.



Solution

To find the volume, find the number of unit cubes it contains. Separate the piece into three rectangular boxes as follows:

- The *base* is 7 units by 2 units. So, it contains $7 \cdot 2$, or 14 unit cubes.
- The *upper left box* is 2 units by 2 units. So, it contains $2 \cdot 2$, or 4 unit cubes.
- The *upper right box* is 1 unit by 2 units. So, it contains $1 \cdot 2$, or 2 unit cubes.

► By the Volume Addition Postulate, the total volume of the puzzle piece is $14 + 4 + 2 = 20$ cubic units.

VOLUME FORMULAS The volume of any right prism or right cylinder can be found by multiplying the area of its base by its height.

THEOREMS

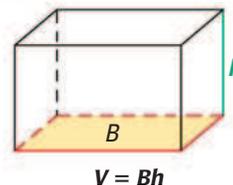
For Your Notebook

THEOREM 12.6 Volume of a Prism

The volume V of a prism is

$$V = Bh,$$

where B is the area of a base and h is the height.

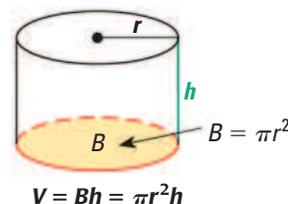


THEOREM 12.7 Volume of a Cylinder

The volume V of a cylinder is

$$V = Bh = \pi r^2 h,$$

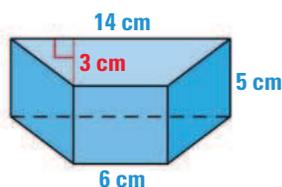
where B is the area of a base, h is the height, and r is the radius of a base.



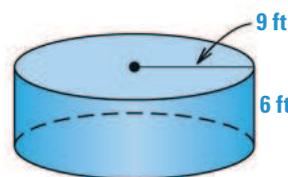
EXAMPLE 2 Find volumes of prisms and cylinders

Find the volume of the solid.

a. Right trapezoidal prism



b. Right cylinder



Solution

a. The area of a base is $\frac{1}{2}(3)(6 + 14) = 30 \text{ cm}^2$ and $h = 5 \text{ cm}$.

$$V = Bh = 30(5) = 150 \text{ cm}^3$$

b. The area of the base is $\pi \cdot 9^2$, or $81\pi \text{ ft}^2$. Use $h = 6 \text{ ft}$ to find the volume.

$$V = Bh = 81\pi(6) = 486\pi \approx 1526.81 \text{ ft}^3$$

REVIEW AREA

For help with finding the area of a trapezoid, see p. 730.

EXAMPLE 3 Use volume of a prism

xy ALGEBRA The volume of the cube is 90 cubic inches. Find the value of x .

Solution

A side length of the cube is x inches.

$$V = x^3 \quad \text{Formula for volume of a cube}$$

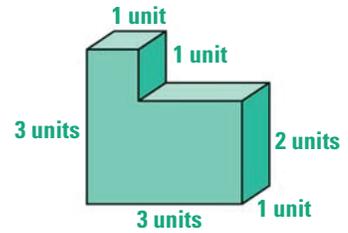
$$90 \text{ in.}^3 = x^3 \quad \text{Substitute for } V.$$

$$4.48 \text{ in.} \approx x \quad \text{Find the cube root.}$$

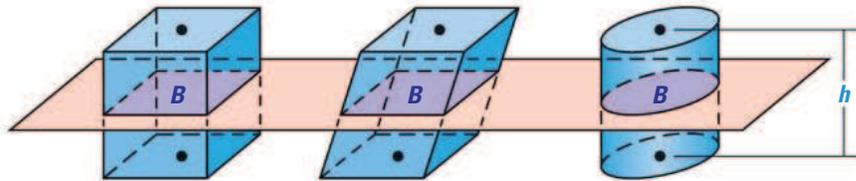


**GUIDED PRACTICE** for Examples 1, 2, and 3

1. Find the volume of the puzzle piece shown in cubic units.
2. Find the volume of a square prism that has a base edge length of 5 feet and a height of 12 feet.
3. The volume of a right cylinder is 684π cubic inches and the height is 18 inches. Find the radius.



USING CAVALIERI'S PRINCIPLE Consider the solids below. All three have equal heights h and equal cross-sectional areas B . Mathematician Bonaventura Cavalieri (1598–1647) claimed that all three of the solids have the same volume. This principle is stated below.



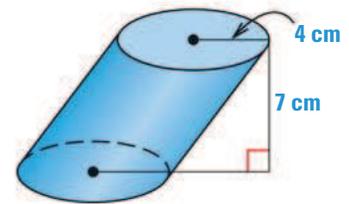
Animated Geometry at classzone.com

THEOREM*For Your Notebook***THEOREM 12.8 Cavalieri's Principle**

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

EXAMPLE 4 Find the volume of an oblique cylinder

Find the volume of the oblique cylinder.

**APPLY THEOREMS**

Cavalieri's Principle tells you that the volume formulas on page 820 work for oblique prisms and cylinders.

Solution

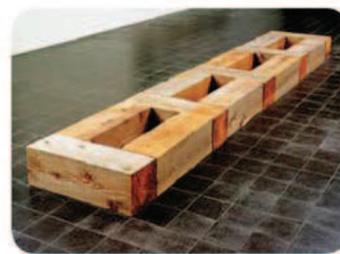
Cavalieri's Principle allows you to use Theorem 12.7 to find the volume of the oblique cylinder.

$$\begin{aligned}
 V &= \pi r^2 h && \text{Formula for volume of a cylinder} \\
 &= \pi(4^2)(7) && \text{Substitute known values.} \\
 &= 112\pi && \text{Simplify.} \\
 &\approx 351.86 && \text{Use a calculator.}
 \end{aligned}$$

▶ The volume of the oblique cylinder is about 351.86 cm^3 .

EXAMPLE 5 Solve a real-world problem

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.



Romartyr Hamburg, 1989 © Carl Andre/
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Solution

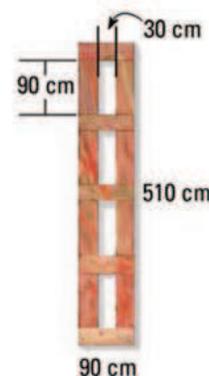
The area of the base B can be found by subtracting the area of the small rectangles from the area of the large rectangle.

$$\begin{aligned} B &= \text{Area of large rectangle} - 4 \cdot \text{Area of small rectangle} \\ &= 90 \cdot 510 - 4(30 \cdot 90) \\ &= 35,100 \text{ cm}^2 \end{aligned}$$

Use the formula for the volume of a prism.

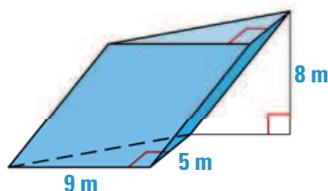
$$\begin{aligned} V &= Bh && \text{Formula for volume of a prism} \\ &= 35,100(90) && \text{Substitute.} \\ &= 1,053,000 \text{ cm}^3 && \text{Simplify.} \end{aligned}$$

▶ The volume of the sculpture is $1,053,000 \text{ cm}^3$, or 1.053 m^3 .

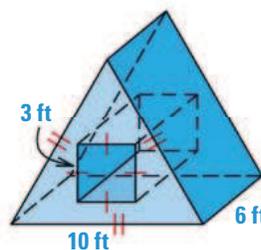


GUIDED PRACTICE for Examples 4 and 5

4. Find the volume of the oblique prism shown below.



5. Find the volume of the solid shown below.



12.4 EXERCISES

HOMWORK KEY

○ = WORKED-OUT SOLUTIONS
on p. WS1 for Exs. 7, 11, and 29

★ = STANDARDIZED TEST PRACTICE
Exs. 2, 3, 21, and 33

SKILL PRACTICE

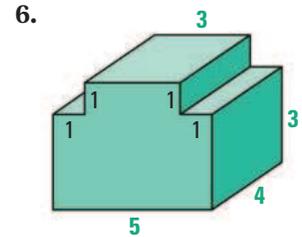
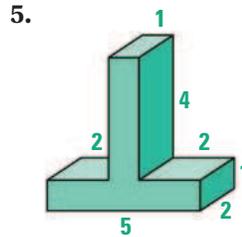
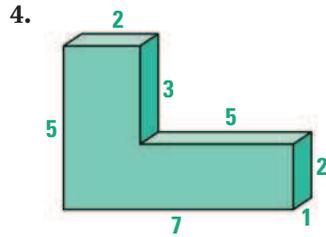
- VOCABULARY** In what type of units is the volume of a solid measured?
- ★ **WRITING** Two solids have the same surface area. Do they have the same volume? *Explain* your reasoning.
- ★ **MULTIPLE CHOICE** How many 3 inch cubes can fit completely in a box that is 15 inches long, 9 inches wide, and 3 inches tall?

- (A) 15 (B) 45 (C) 135 (D) 405

EXAMPLE 1

on p. 819
for Exs. 3–6

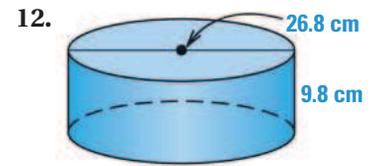
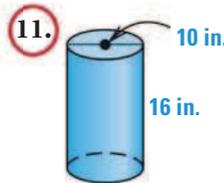
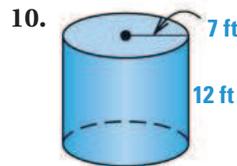
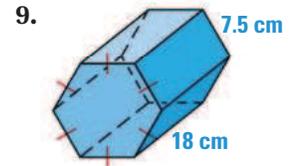
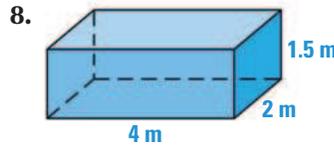
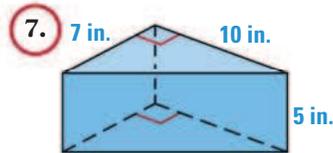
USING UNIT CUBES Find the volume of the solid by determining how many unit cubes are contained in the solid.



EXAMPLE 2

on p. 820
for Exs. 7–13

FINDING VOLUME Find the volume of the right prism or right cylinder. Round your answer to two decimal places.



13. **ERROR ANALYSIS** Describe and correct the error in finding the volume of a right cylinder with radius 4 feet and height 3 feet.

$$\begin{aligned} V &= 2\pi rh \\ &= 2\pi(4)(3) \\ &= 24\pi \text{ ft}^3 \end{aligned}$$

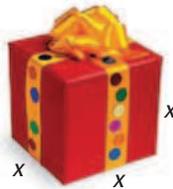
14. **FINDING VOLUME** Sketch a rectangular prism with height 3 feet, width 11 inches, and length 7 feet. Find its volume.

EXAMPLE 3

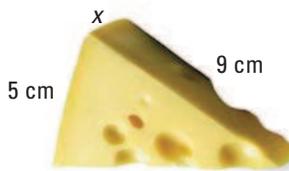
on p. 820
for Exs. 15–17

ALGEBRA Find the length x using the given volume V .

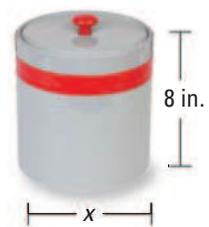
15. $V = 1000 \text{ in.}^3$



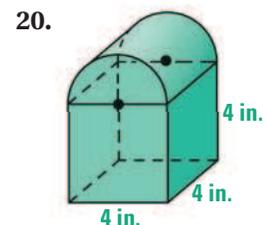
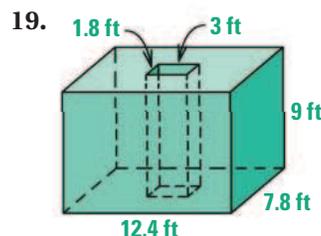
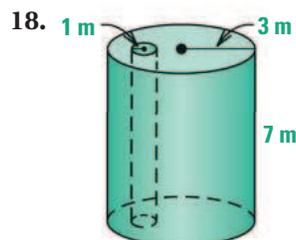
16. $V = 45 \text{ cm}^3$



17. $V = 128\pi \text{ in.}^3$



COMPOSITE SOLIDS Find the volume of the solid. The prisms and cylinders are right. Round your answer to two decimal places, if necessary.



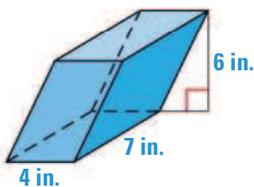
21. ★ **MULTIPLE CHOICE** What is the height of a cylinder with radius 4 feet and volume 64π cubic feet?
- (A) 4 feet (B) 8 feet (C) 16 feet (D) 256 feet
22. **FINDING HEIGHT** The bases of a right prism are right triangles with side lengths of 3 inches, 4 inches, and 5 inches. The volume of the prism is 96 cubic inches. What is the height of the prism?
23. **FINDING DIAMETER** A cylinder has height 8 centimeters and volume 1005.5 cubic centimeters. What is the diameter of the cylinder?

EXAMPLE 4

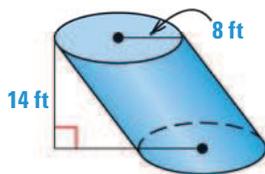
on p. 821
for Exs. 24–26

VOLUME OF AN OBLIQUE SOLID Use Cavalieri's Principle to find the volume of the oblique prism or cylinder. Round your answer to two decimal places.

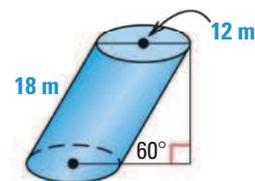
24.



25.



26.



27. **CHALLENGE** The bases of a right prism are rhombuses with diagonals 12 meters and 16 meters long. The height of the prism is 8 meters. Find the lateral area, surface area, and volume of the prism.

PROBLEM SOLVING

EXAMPLE 5

on p. 822
for Exs. 28–30

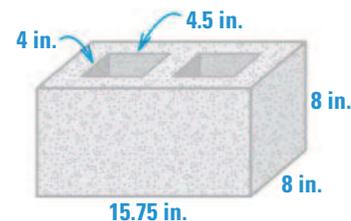
28. **JEWELRY** The bead at the right is a rectangular prism of length 17 millimeters, width 9 millimeters, and height 5 millimeters. A 3 millimeter wide hole is drilled through the smallest face. Find the volume of the bead.



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29. **MULTI-STEP PROBLEM** In the concrete block shown, the holes are 8 inches deep.

- a. Find the volume of the block using the Volume Addition Postulate.
- b. Find the volume of the block using the formula in Theorem 12.6.
- c. Compare your answers in parts (a) and (b).



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30. **OCEANOGRAPHY** The Blue Hole is a cylindrical trench located on Lighthouse Reef Atoll, an island off the coast of Central America. It is approximately 1000 feet wide and 400 feet deep.
- a. Find the volume of the Blue Hole.
- b. About how many gallons of water does the Blue Hole contain? ($1 \text{ ft}^3 = 7.48$ gallons)

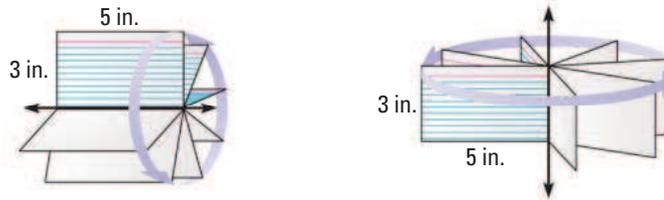


31. **ARCHITECTURE** A cylindrical column in the building shown has circumference 10 feet and height 20 feet. Find its volume. Round your answer to two decimal places.

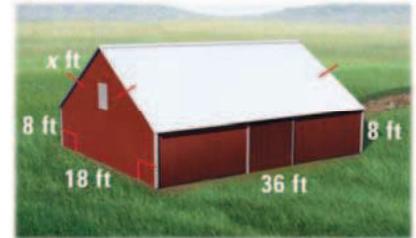
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32. **ROTATIONS** A 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? *Explain* your reasoning.



33. **★ EXTENDED RESPONSE** An aquarium shaped like a rectangular prism has length 30 inches, width 10 inches, and height 20 inches.
- Calculate** You fill the aquarium $\frac{3}{4}$ full with water. What is the volume of the water?
 - Interpret** When you submerge a rock in the aquarium, the water level rises 0.25 inch. Find the volume of the rock.
 - Interpret** How many rocks of the same size as the rock in part (b) can you place in the aquarium before water spills out?
34. **CHALLENGE** A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.

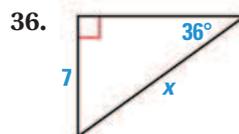
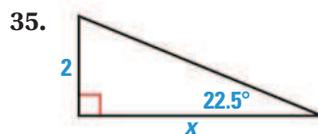


MIXED REVIEW

PREVIEW

Prepare for
Lesson 12.5 in
Exs. 35–40.

Find the value of x . Round your answer to two decimal places. (pp. 466, 473)



Find the area of the figure described. Round your answer to two decimal places. (pp. 755, 762)

- A circle with radius 9.5 inches
- An equilateral triangle with perimeter 78 meters and apothem 7.5 meters
- A regular pentagon with radius 10.6 inches

Another Way to Solve Example 5, page 822



MULTIPLE REPRESENTATIONS In Lesson 12.4, you used volume postulates and theorems to find volumes of prisms and cylinders. Now, you will learn two different ways to solve Example 5 on page 822.

PROBLEM

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.



METHOD 1

Finding Volume by Subtracting Empty Spaces One alternative approach is to compute the volume of the prism formed if the holes in the sculpture were filled. Then, to get the correct volume, you must subtract the volume of the four holes.

STEP 1 Read the problem. In centimeters, each beam measures 30 by 30 by 90.

The dimensions of the entire sculpture are 30 by 90 by $(4 \cdot 90 + 5 \cdot 30)$, or 30 by 90 by 510.

The dimensions of each hole are equal to the dimensions of one beam.

STEP 2 Apply the Volume Addition Postulate. The volume of the sculpture is equal to the volume of the larger prism minus 4 times the volume of a hole.

$$\begin{aligned} \text{Volume } V \text{ of sculpture} &= \text{Volume of larger prism} - \text{Volume of 4 holes} \\ &= 30 \cdot 90 \cdot 510 - 4(30 \cdot 30 \cdot 90) \\ &= 1,377,000 - 4 \cdot 81,000 \\ &= 1,377,000 - 324,000 \\ &= 1,053,000 \end{aligned}$$

► The volume of the sculpture is 1,053,000 cubic centimeters, or 1.053 cubic meters.

STEP 3 Check page 822 to verify your new answer, and confirm that it is the same.

METHOD 2

Finding Volume of Pieces Another alternative approach is to use the dimensions of each beam.

STEP 1 Look at the sculpture. Notice that the sculpture consists of 13 beams, each with the same dimensions. Therefore, the volume of the sculpture will be 13 times the volume of one beam.

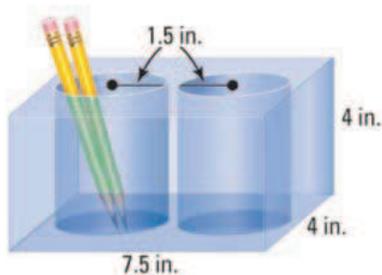
STEP 2 Write an expression for the volume of the sculpture and find the volume.

$$\begin{aligned} \text{Volume of sculpture} &= 13(\text{Volume of one beam}) \\ &= 13(30 \cdot 30 \cdot 90) \\ &= 13 \cdot 81,000 \\ &= 1,053,000 \end{aligned}$$

► The volume of the sculpture is $1,053,000 \text{ cm}^3$, or 1.053 m^3 .

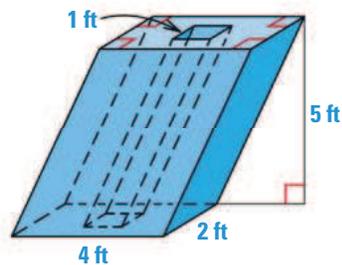
PRACTICE

1. **PENCIL HOLDER** The pencil holder has the dimensions shown.

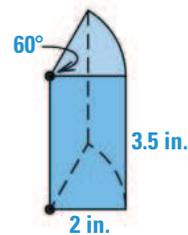


- Find its volume using the Volume Addition Postulate.
 - Use its base area to find its volume.
2. **ERROR ANALYSIS** A student solving Exercise 1 claims that the surface area is found by subtracting four times the base area of the cylinders from the surface area of the rectangular prism. *Describe* and correct the student's error.
3. **REASONING** You drill a circular hole of radius r through the base of a cylinder of radius R . Assume the hole is drilled completely through to the other base. You want the volume of the hole to be half the volume of the cylinder. Express r as a function of R .

4. **FINDING VOLUME** Find the volume of the solid shown below. Assume the hole has square cross sections.



5. **FINDING VOLUME** Find the volume of the solid shown to the right.



6. **SURFACE AREA** Refer to the diagram of the sculpture on page 826.
- Describe* a method to find the surface area of the sculpture.
 - Explain* why adding the individual surface areas of the beams will give an incorrect result for the total surface area.