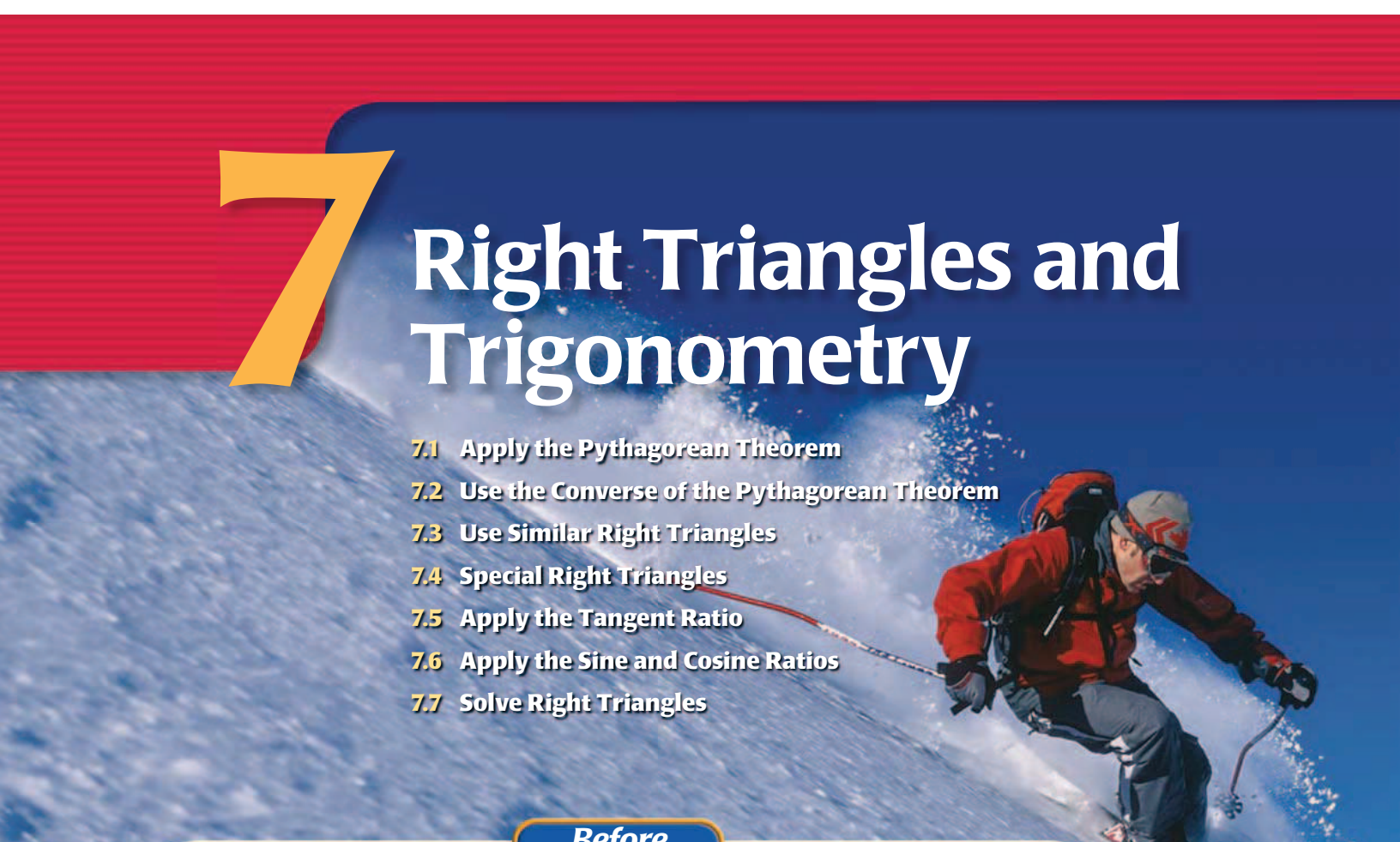
7 Right Triangles and Trigonometry



Before

In previous courses and in Chapters 1–6, you learned the following skills, which you'll use in Chapter 7: classifying triangles, simplifying radicals, and solving proportions.

Prerequisite Skills

VOCABULARY CHECK

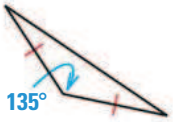
Name the triangle shown.

1.

2.

3.

4.



SKILLS AND ALGEBRA CHECK

Simplify the radical. *(Review p. 874 for 7.1, 7.2, 7.4.)*

5. $\sqrt{45}$ **6.** $(3\sqrt{7})^2$ **7.** $\sqrt{3} \cdot \sqrt{5}$ **8.** $\frac{7}{\sqrt{2}}$

Solve the proportion. *(Review p. 356 for 7.3, 7.5–7.7.)*

9. $\frac{3}{x} = \frac{12}{16}$ **10.** $\frac{2}{3} = \frac{x}{18}$ **11.** $\frac{x+5}{4} = \frac{1}{2}$ **12.** $\frac{x+4}{x-4} = \frac{6}{5}$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 7, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 493. You will also use the key vocabulary listed below.

Big Ideas

1. **Using the Pythagorean Theorem and its converse**
2. **Using special relationships in right triangles**
3. **Using trigonometric ratios to solve right triangles**

KEY VOCABULARY

- Pythagorean triple, *p. 435*
- trigonometric ratio, *p. 466*
- tangent, *p. 466*
- sine, *p. 473*
- cosine, *p. 473*
- angle of elevation, *p. 475*
- angle of depression, *p. 475*
- solve a right triangle, *p. 483*
- inverse tangent, *p. 483*
- inverse sine, *p. 483*
- inverse cosine, *p. 483*

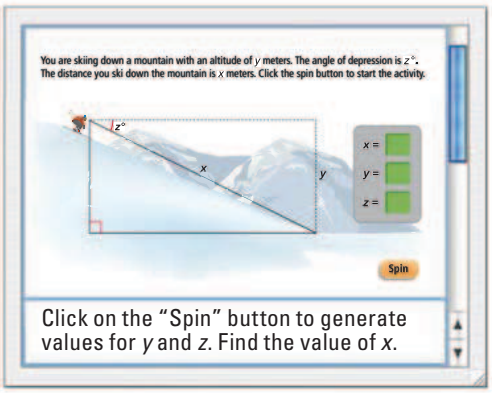
Why?

You can use trigonometric ratios to find unknown side lengths and angle measures in right triangles. For example, you can find the length of a ski slope.

Animated Geometry

The animation illustrated below for Example 4 on page 475 helps you answer this question: How far will you ski down the mountain?

You can use right triangles to find the distance you ski down a mountain.



Click on the "Spin" button to generate values for *y* and *z*. Find the value of *x*.

Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 7: pages 434, 442, 450, 460, and 462

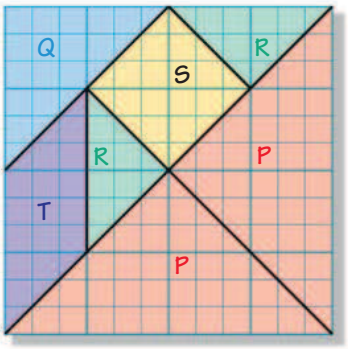
7.1 Pythagorean Theorem

MATERIALS • graph paper • ruler • pencil • scissors

QUESTION What relationship exists among the sides of a right triangle?

Recall that a square is a four sided figure with four right angles and four congruent sides.

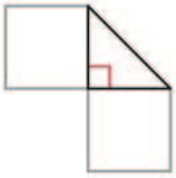
EXPLORE Make and use a tangram set



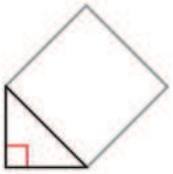
STEP 1 ***Make a tangram set*** On your graph paper, copy the tangram set as shown. Label each piece with the given letters. Cut along the solid black lines to make seven pieces.

STEP 2 ***Trace a triangle*** On another piece of paper, trace one of the large triangles P of the tangram set.

STEP 3 ***Assemble pieces along the legs*** Use all of the tangram pieces to form two squares along the legs of your triangle so that the length of each leg is equal to the side length of the square. Trace all of the pieces.



STEP 4 ***Assemble pieces along the hypotenuse*** Use all of the tangram pieces to form a square along the hypotenuse so that the side length of the square is equal to the length of the hypotenuse. Trace all of the pieces.



DRAW CONCLUSIONS Use your observations to complete these exercises

1. Find the sum of the areas of the two squares formed in Step 3. Let the letters labeling the figures represent the area of the figure. How are the side lengths of the squares related to Triangle P?
2. Find the area of the square formed in Step 4. How is the side length of the square related to Triangle P?
3. Compare your answers from Exercises 1 and 2. Make a conjecture about the relationship between the legs and hypotenuse of a right triangle.
4. The triangle you traced in Step 2 is an isosceles right triangle. Why? Do you think that your conjecture is true for all isosceles triangles? Do you think that your conjecture is true for all right triangles? *Justify* your answers.

7.1 Apply the Pythagorean Theorem

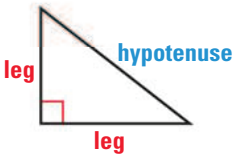
Before You learned about the relationships within triangles.

Now You will find side lengths in right triangles.

Why? So you can find the shortest distance to a campfire, as in Ex. 35.

Key Vocabulary
- **Pythagorean triple**
- **right triangle,** *p. 217*
- **leg of a right triangle,** *p. 241*
- **hypotenuse,** *p. 241*

One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras (around 500 B.C.). This theorem can be used to find information about the lengths of the sides of a right triangle.

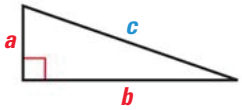


THEOREM *For Your Notebook*

THEOREM 7.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

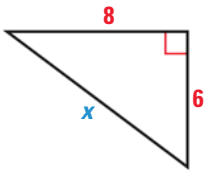
Proof: p. 434; Ex. 32, p. 455



$c^2 = a^2 + b^2$

EXAMPLE 1 Find the length of a hypotenuse

Find the length of the hypotenuse of the right triangle.



Solution

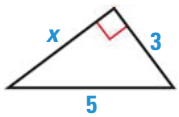
ABBREVIATE In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	Pythagorean Theorem
$x^2 = 6^2 + 8^2$	Substitute.
$x^2 = 36 + 64$	Multiply.
$x^2 = 100$	Add.
$x = 10$	Find the positive square root.

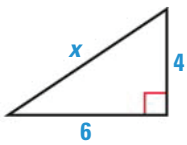
✓ GUIDED PRACTICE for Example 1

Identify the unknown side as a *leg* or *hypotenuse*. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.

1.

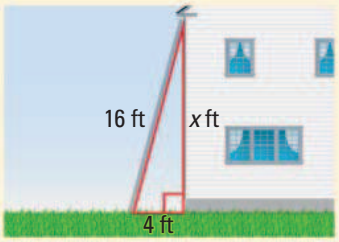


2.



EXAMPLE 2 Standardized Test Practice

A 16 foot ladder rests against the side of the house, and the base of the ladder is 4 feet away. Approximately how high above the ground is the top of the ladder?



Ⓐ 240 feet Ⓑ 20 feet

Ⓒ 16.5 feet Ⓓ 15.5 feet

Solution

$$\begin{pmatrix}\text{Length}\\ \text{of ladder}\end{pmatrix}^2 = \begin{pmatrix}\text{Distance}\\ \text{from house}\end{pmatrix}^2 + \begin{pmatrix}\text{Height}\\ \text{of ladder}\end{pmatrix}^2$$

$16^2 = 4^2 + x^2$	**Substitute.**
$256 = 16 + x^2$	**Multiply.**
$240 = x^2$	**Subtract 16 from each side.**
$\sqrt{240} = x$	**Find positive square root.**
$15.491 \approx x$	**Approximate with a calculator.**

The ladder is resting against the house at about 15.5 feet above the ground.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

APPROXIMATE
In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

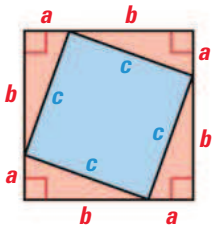
✓ GUIDED PRACTICE for Example 2

3. The top of a ladder rests against a wall, 23 feet above the ground. The base of the ladder is 6 feet away from the wall. What is the length of the ladder?

4. The Pythagorean Theorem is only true for what type of triangle?

PROVING THE PYTHAGOREAN THEOREM There are many proofs of the Pythagorean Theorem. An informal proof is shown below. You will write another proof in Exercise 32 on page 455.

In the figure at the right, the four right triangles are congruent, and they form a small square in the middle. The area of the large square is equal to the area of the four triangles plus the area of the smaller square.



$$\begin{pmatrix}\text{Area of}\\ \text{large square}\end{pmatrix} = \begin{pmatrix}\text{Area of}\\ \text{four triangles}\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{smaller square}\end{pmatrix}$$

$(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$	**Use area formulas.**
$a^2 + 2ab + b^2 = 2ab + c^2$	**Multiply.**
$a^2 + b^2 = c^2$	**Subtract 2*ab* from each side.**

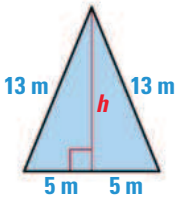
REVIEW AREA
Recall that the area of a square with side length s is $A = s^2$. The area of a triangle with base b and height h is $A = \frac{1}{2}bh$.

Animated Geometry at classzone.com

EXAMPLE 3 Find the area of an isosceles triangle

Find the area of the isosceles triangle with side lengths 10 meters, 13 meters, and 13 meters.

Solution



STEP 1 **Draw** a sketch. By definition, the length of an altitude is the height of a triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.

STEP 2 **Use** the Pythagorean Theorem to find the height of the triangle.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$13^2 = 5^2 + h^2$	**Substitute.**
$169 = 25 + h^2$	**Multiply.**
$144 = h^2$	**Subtract 25 from each side.**
$12 = h$	**Find the positive square root.**

STEP 3 **Find** the area.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(10)(12) = 60 \text{ m}^2$$

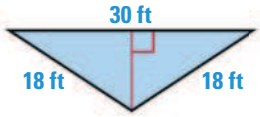
▶ The area of the triangle is 60 square meters.

READ TABLES
You may find it helpful to use the Table of Squares and Square Roots on p. 924.

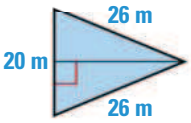
✓ GUIDED PRACTICE for Example 3

Find the area of the triangle.

5.



6.



PYTHAGOREAN TRIPLES A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

STANDARDIZED TESTS
You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

KEY CONCEPT *For Your Notebook*

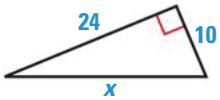
Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

EXAMPLE 4 Find the length of a hypotenuse using two methods

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is **5**, **12**, **13**. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: **5** • 2 = 10 and **12** • 2 = 24. So, the length of the hypotenuse is **13** • 2 = 26.

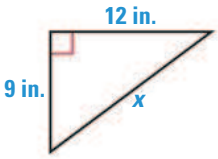
Method 2: Use the Pythagorean Theorem.

$x^2 = 10^2 + 24^2$	**Pythagorean Theorem**
$x^2 = 100 + 576$	**Multiply.**
$x^2 = 676$	**Add.**
$x = 26$	**Find the positive square root.**

✓ GUIDED PRACTICE for Example 4

Find the unknown side length of the right triangle using the Pythagorean Theorem. Then use a Pythagorean triple.

7.



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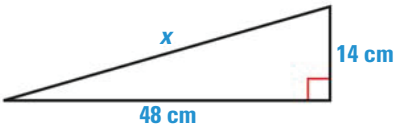


7.1 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 33
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 27, 33, and 36
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

SKILL PRACTICE

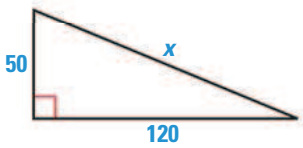
1. **VOCABULARY** Copy and complete: A set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$ is called a __?__.

2. ★ **WRITING** *Describe* the information you need to have in order to use the Pythagorean Theorem to find the length of a side of a triangle.

EXAMPLE 1 on p. 433 for Exs. 3–7

xy ALGEBRA **Find the length of the hypotenuse of the right triangle.**

3.



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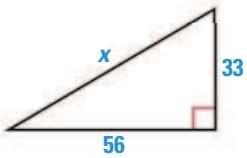

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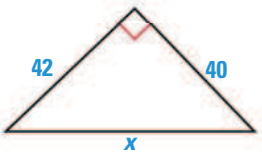
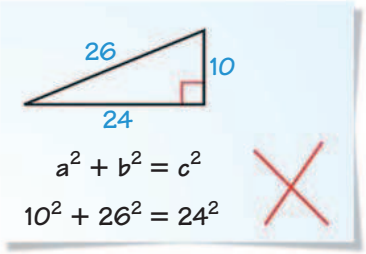


ERROR ANALYSIS ***Describe*** **and correct the error in using the Pythagorean Theorem.**

6.



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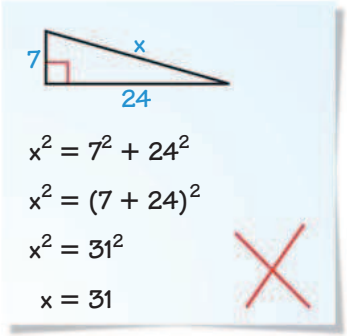
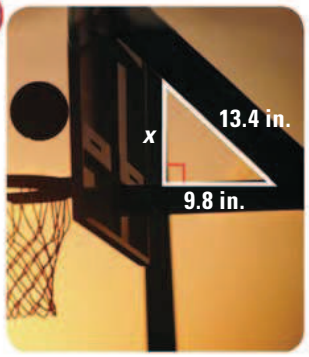


EXAMPLE 2 on p. 434 for Exs. 8–10

FINDING A LENGTH **Find the unknown leg length** ***x*****.**

8.

9.

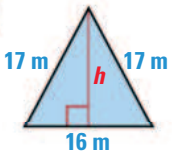


10.

EXAMPLE 3 on p. 435 for Exs. 11–13

FINDING THE AREA **Find the area of the isosceles triangle.**

11.



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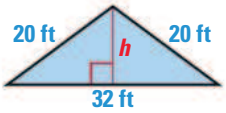

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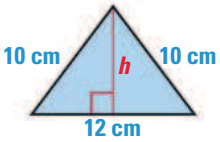
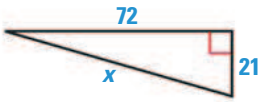


EXAMPLE 4 on p. 436 for Exs. 14–17

FINDING SIDE LENGTHS **Find the unknown side length of the right triangle using the Pythagorean Theorem or a Pythagorean triple.**

14.



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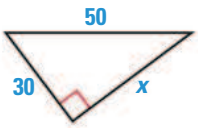

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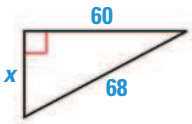


17. ★ **MULTIPLE CHOICE** What is the length of the hypotenuse of a right triangle with leg lengths of 8 inches and 15 inches?

Ⓐ 13 inches Ⓑ 17 inches Ⓒ 21 inches Ⓓ 25 inches

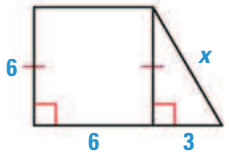
PYTHAGOREAN TRIPLES **The given lengths are two sides of a right triangle. All three side lengths of the triangle are integers and together form a Pythagorean triple. Find the length of the third side and tell whether it is a leg or the hypotenuse.**

18. 24 and 51 **19.** 20 and 25 **20.** 28 and 96

21. 20 and 48 **22.** 75 and 85 **23.** 72 and 75

FINDING SIDE LENGTHS **Find the unknown side length x. Write your answer in simplest radical form.**

24.



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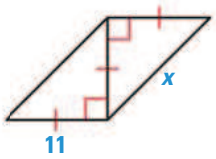
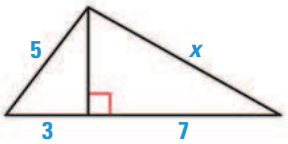


26.



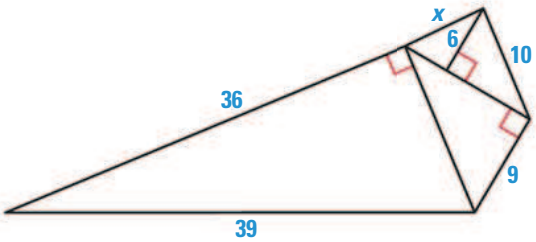
27. ★ **MULTIPLE CHOICE** What is the area of a right triangle with a leg length of 15 feet and a hypotenuse length of 39 feet?

Ⓐ 270 ft^2 Ⓑ 292.5 ft^2 Ⓒ 540 ft^2 Ⓓ 585 ft^2

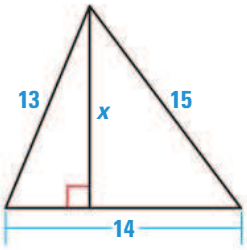
28. **ALGEBRA** Solve for x if the lengths of the two legs of a right triangle are $2x$ and $2x + 4$, and the length of the hypotenuse is $4x - 4$.

CHALLENGE **In Exercises 29 and 30, solve for x.**

29.



30.



PROBLEM SOLVING

EXAMPLE 2 on p. 434 for Exs. 31–32

31. **BASEBALL DIAMOND** In baseball, the distance of the paths between each pair of consecutive bases is 90 feet and the paths form right angles. How far does the ball need to travel if it is thrown from home plate directly to second base?

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32. **APPLE BALLOON** You tie an apple balloon to a stake in the ground. The rope is 10 feet long. As the wind picks up, you observe that the balloon is now 6 feet away from the stake. How far above the ground is the balloon now?

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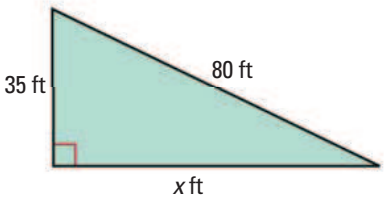
33. ★ **SHORT RESPONSE** Three side lengths of a right triangle are 25, 65, and 60. *Explain* how you know which side is the hypotenuse.

34. **MULTI-STEP PROBLEM** In your town, there is a field that is in the shape of a right triangle with the dimensions shown.

a. Find the perimeter of the field.

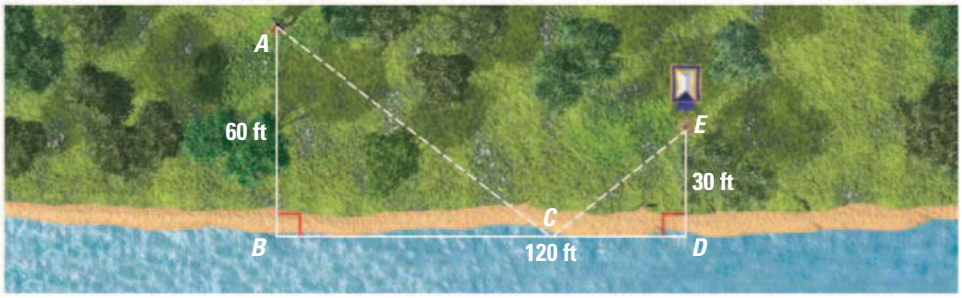
b. You are going to plant dogwood seedlings about every ten feet around the field's edge. How many trees do you need?

c. If each dogwood seedling sells for $12, how much will the trees cost?



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

35. **MULTIPLE REPRESENTATIONS** As you are gathering leaves for a science project, you look back at your campsite and see that the campfire is not completely out. You want to get water from a nearby river to put out the flames with the bucket you are using to collect leaves. Use the diagram and the steps below to determine the shortest distance you must travel.



a. Making a Table Make a table with columns labeled *BC*, *AC*, *CE*, and *AC* + *CE*. Enter values of *BC* from 10 to 120 in increments of 10.

b. Calculating Values Calculate *AC*, *CE*, and *AC* + *CE* for each value of *BC*, and record the results in the table. Then, use your table of values to determine the shortest distance you must travel.

c. Drawing a Picture Draw an accurate picture to scale of the shortest distance.

36. ★ **SHORT RESPONSE** *Justify* the Distance Formula using the Pythagorean Theorem.

37. **PROVING THEOREM 4.5** Find the Hypotenuse-Leg (HL) Congruence Theorem on page 241. Assign variables for the side lengths in the diagram. Use your variables to write GIVEN and PROVE statements. Use the Pythagorean Theorem and congruent triangles to prove Theorem 4.5.

38. **CHALLENGE** Trees grown for sale at nurseries should stand at least five feet from one another while growing. If the trees are grown in parallel rows, what is the smallest allowable distance between rows?

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.2 in Exs. 39–42.

Evaluate the expression. *(p. 874)*

39. $(\sqrt{7})^2$ **40.** $(4\sqrt{3})^2$ **41.** $(-6\sqrt{81})^2$ **42.** $(-8\sqrt{2})^2$

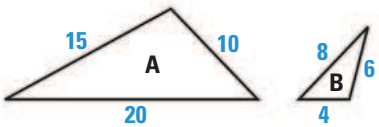
Describe **the possible lengths of the third side of the triangle given the lengths of the other two sides.** *(p. 328)*

43. 3 feet, 6 feet **44.** 5 inches, 11 inches **45.** 14 meters, 21 meters

46. 12 inches, 27 inches **47.** 18 yards, 18 yards **48.** 27 meters, 39 meters

Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A. *(p. 388)*

49.



50.