

8 Quadrilaterals

- 8.1 Find Angle Measures in Polygons
- 8.2 Use Properties of Parallelograms
- 8.3 Show that a Quadrilateral is a Parallelogram
- 8.4 Properties of Rhombuses, Rectangles, and Squares
- 8.5 Use Properties of Trapezoids and Kites
- 8.6 Identify Special Quadrilaterals

Before

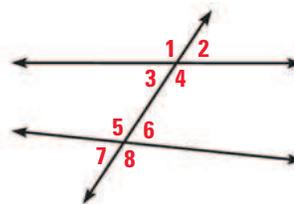
In previous chapters, you learned the following skills, which you'll use in Chapter 8: identifying angle pairs, using the Triangle Sum Theorem, and using parallel lines.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

- $\angle 1$ and $\underline{\quad ? \quad}$ are vertical angles.
- $\angle 3$ and $\underline{\quad ? \quad}$ are consecutive interior angles.
- $\angle 7$ and $\underline{\quad ? \quad}$ are corresponding angles.
- $\angle 5$ and $\underline{\quad ? \quad}$ are alternate interior angles.

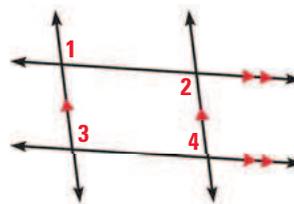


SKILLS AND ALGEBRA CHECK

- In $\triangle ABC$, $m\angle A = x^\circ$, $m\angle B = 3x^\circ$, and $m\angle C = (4x - 12)^\circ$. Find the measures of the three angles. (Review p. 217 for 8.1.)

Find the measure of the indicated angle. (Review p. 154 for 8.2–8.5.)

- If $m\angle 3 = 105^\circ$, then $m\angle 2 = \underline{\quad ? \quad}$.
- If $m\angle 1 = 98^\circ$, then $m\angle 3 = \underline{\quad ? \quad}$.
- If $m\angle 4 = 82^\circ$, then $m\angle 1 = \underline{\quad ? \quad}$.
- If $m\angle 2 = 102^\circ$, then $m\angle 4 = \underline{\quad ? \quad}$.



@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 8, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 559. You will also use the key vocabulary listed below.

Big Ideas

- 1 Using angle relationships in polygons
- 2 Using properties of parallelograms
- 3 Classifying quadrilaterals by their properties

KEY VOCABULARY

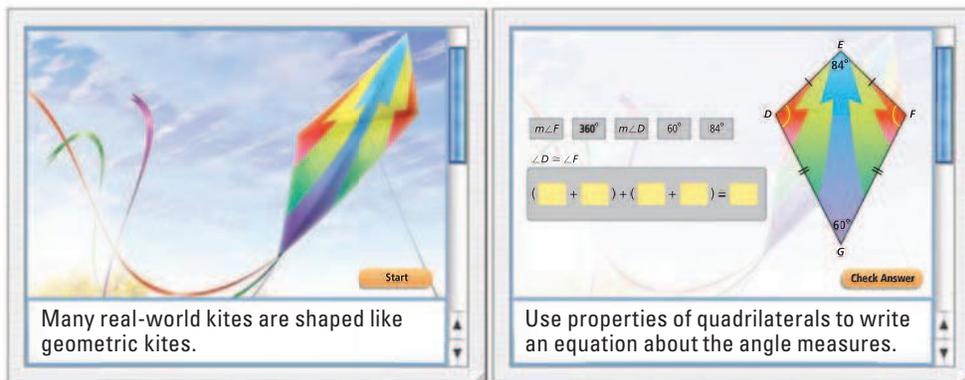
- diagonal, p. 507
- parallelogram, p. 515
- rhombus, p. 533
- rectangle, p. 533
- square, p. 533
- trapezoid, p. 542
bases, base angles, legs
- isosceles trapezoid, p. 543
- midsegment of a trapezoid, p. 544
- kite, p. 545

Why?

You can use properties of quadrilaterals and other polygons to find side lengths and angle measures.

Animated Geometry

The animation illustrated below for Example 4 on page 545 helps you answer this question: How can classifying a quadrilateral help you draw conclusions about its sides and angles?



Many real-world kites are shaped like geometric kites.

Use properties of quadrilaterals to write an equation about the angle measures.

Animated Geometry at classzone.com

Other animations for Chapter 8: pages 509, 519, 527, 535, 551, and 553

8.1 Investigate Angle Sums in Polygons

MATERIALS • straightedge • ruler

QUESTION What is the sum of the measures of the interior angles of a convex n -gon?

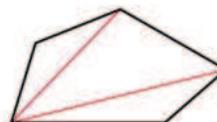
Recall from page 43 that an n -gon is a polygon with n sides and n vertices.

EXPLORE Find sums of interior angle measures

STEP 1 *Draw polygons* Use a straightedge to draw convex polygons with three sides, four sides, five sides, and six sides. An example is shown.



STEP 2 *Draw diagonals* In each polygon, draw all the diagonals from one vertex. A *diagonal* is a segment that joins two nonconsecutive vertices. Notice that the diagonals divide the polygon into triangles.



STEP 3 *Make a table* Copy the table below. By the Triangle Sum Theorem, the sum of the measures of the interior angles of a triangle is 180° . Use this theorem to complete the table.

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Triangle	3	1	$1 \cdot 180^\circ = 180^\circ$
Quadrilateral	?	?	$2 \cdot 180^\circ = 360^\circ$
Pentagon	?	?	?
Hexagon	?	?	?

DRAW CONCLUSIONS Use your observations to complete these exercises

- Look for a pattern in the last column of the table. What is the sum of the measures of the interior angles of a convex heptagon? a convex octagon? *Explain* your reasoning.
- Write an expression for the sum of the measures of the interior angles of a convex n -gon.
- Measure the side lengths in the hexagon you drew. Compare the lengths with those in hexagons drawn by other students. Do the side lengths affect the sum of the interior angle measures of a hexagon? *Explain*.

8.1 Find Angle Measures in Polygons

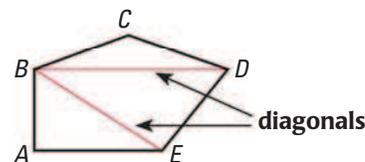


- Before** You classified polygons.
- Now** You will find angle measures in polygons.
- Why?** So you can describe a baseball park, as in Exs. 28–29.

Key Vocabulary

- **diagonal**
- **interior angle**, p. 218
- **exterior angle**, p. 218

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two *nonconsecutive vertices*. Polygon $ABCDE$ has two diagonals from vertex B , \overline{BD} and \overline{BE} .



As you can see, the diagonals from one vertex form triangles. In the Activity on page 506, you used these triangles to find the sum of the interior angle measures of a polygon. Your results support the following theorem and corollary.

THEOREMS

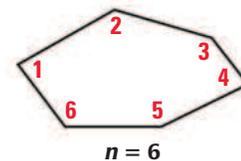
For Your Notebook

THEOREM 8.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n -gon is $(n - 2) \cdot 180^\circ$.

$$m\angle 1 + m\angle 2 + \dots + m\angle n = (n - 2) \cdot 180^\circ$$

Proof: Ex. 33, p. 512 (for pentagons)



COROLLARY TO THEOREM 8.1 Interior Angles of a Quadrilateral

The sum of the measures of the interior angles of a quadrilateral is 360° .

Proof: Ex. 34, p. 512

EXAMPLE 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex octagon.



Solution

An octagon has 8 sides. Use the Polygon Interior Angles Theorem.

$$\begin{aligned}
 (n - 2) \cdot 180^\circ &= (8 - 2) \cdot 180^\circ && \text{Substitute 8 for } n. \\
 &= 6 \cdot 180^\circ && \text{Subtract.} \\
 &= 1080^\circ && \text{Multiply.}
 \end{aligned}$$

▶ The sum of the measures of the interior angles of an octagon is 1080° .

EXAMPLE 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 900° . Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n . Then solve the equation to find the number of sides.

$$(n - 2) \cdot 180^\circ = 900^\circ \quad \text{Polygon Interior Angles Theorem}$$

$$n - 2 = 5 \quad \text{Divide each side by } 180^\circ.$$

$$n = 7 \quad \text{Add 2 to each side.}$$

► The polygon has 7 sides. It is a heptagon.



GUIDED PRACTICE for Examples 1 and 2

1. The coin shown is in the shape of a regular 11-gon. Find the sum of the measures of the interior angles.
2. The sum of the measures of the interior angles of a convex polygon is 1440° . Classify the polygon by the number of sides.



EXAMPLE 3 Find an unknown interior angle measure

xy ALGEBRA Find the value of x in the diagram shown.



Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x . Then solve the equation.

$$x^\circ + 108^\circ + 121^\circ + 59^\circ = 360^\circ \quad \text{Corollary to Theorem 8.1}$$

$$x + 288 = 360 \quad \text{Combine like terms.}$$

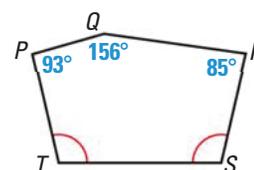
$$x = 72 \quad \text{Subtract 288 from each side.}$$

► The value of x is 72.



GUIDED PRACTICE for Example 3

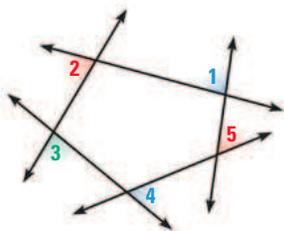
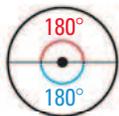
3. Use the diagram at the right. Find $m\angle S$ and $m\angle T$.
4. The measures of three of the interior angles of a quadrilateral are 89° , 110° , and 46° . Find the measure of the fourth interior angle.



EXTERIOR ANGLES Unlike the sum of the interior angle measures of a convex polygon, the sum of the exterior angle measures does *not* depend on the number of sides of the polygon. The diagrams below suggest that the sum of the measures of the exterior angles, one at each vertex, of a pentagon is 360° . In general, this sum is 360° for any convex polygon.

VISUALIZE IT

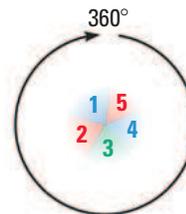
A circle contains two straight angles. So, there are $180^\circ + 180^\circ$, or 360° , in a circle.



STEP 1 Shade one exterior angle at each vertex.



STEP 2 Cut out the exterior angles.



STEP 3 Arrange the exterior angles to form 360° .

Animated Geometry at classzone.com

THEOREM

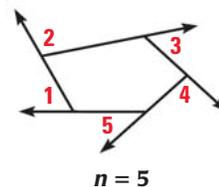
For Your Notebook

THEOREM 8.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360° .

$$m\angle 1 + m\angle 2 + \dots + m\angle n = 360^\circ$$

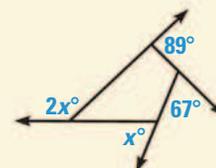
Proof: Ex. 35, p. 512



EXAMPLE 4 Standardized Test Practice

What is the value of x in the diagram shown?

- (A) 67
- (B) 68
- (C) 91
- (D) 136



ELIMINATE CHOICES

You can quickly eliminate choice *D*. If x were equal to 136, then the sum of only two of the angle measures (x° and $2x^\circ$) would be greater than 360° .

Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$$x^\circ + 2x^\circ + 89^\circ + 67^\circ = 360^\circ \quad \text{Polygon Exterior Angles Theorem}$$

$$3x + 156 = 360 \quad \text{Combine like terms.}$$

$$x = 68 \quad \text{Solve for } x.$$

▶ The correct answer is B. (A) (B) (C) (D)



GUIDED PRACTICE for Example 4

- 5. A convex hexagon has exterior angles with measures 34° , 49° , 58° , 67° , and 75° . What is the measure of an exterior angle at the sixth vertex?

EXAMPLE 5 Find angle measures in regular polygons

READ VOCABULARY

Recall that a *dodecagon* is a polygon with 12 sides and 12 vertices.

TRAMPOLINE The trampoline shown is shaped like a regular dodecagon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.



Solution

- a. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$$(n - 2) \cdot 180^\circ = (12 - 2) \cdot 180^\circ = 1800^\circ$$

Then find the measure of one interior angle. A regular dodecagon has 12 congruent interior angles. Divide 1800° by 12: $1800^\circ \div 12 = 150^\circ$.

▶ The measure of each interior angle in the dodecagon is 150° .

- b. By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is 360° . Divide 360° by 12 to find the measure of one of the 12 congruent exterior angles: $360^\circ \div 12 = 30^\circ$.

▶ The measure of each exterior angle in the dodecagon is 30° .



GUIDED PRACTICE for Example 5

6. An interior angle and an adjacent exterior angle of a polygon form a linear pair. How can you use this fact as another method to find the exterior angle measure in Example 5?

8.1 EXERCISES

HOMEWORK KEY

- = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 9, 11, and 29
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 18, 23, and 37
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 36

SKILL PRACTICE

1. **VOCABULARY** Sketch a convex hexagon. Draw all of its diagonals.
2. ★ **WRITING** How many exterior angles are there in an n -gon? Are all the exterior angles considered when you use the Polygon Exterior Angles Theorem? *Explain.*

EXAMPLES 1 and 2

on pp. 507–508
for Exs. 3–10

INTERIOR ANGLE SUMS Find the sum of the measures of the interior angles of the indicated convex polygon.

3. Nonagon 4. 14-gon 5. 16-gon 6. 20-gon

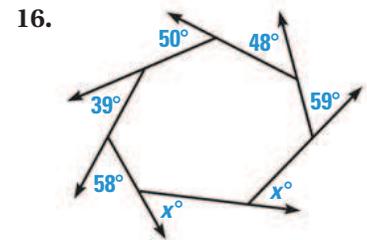
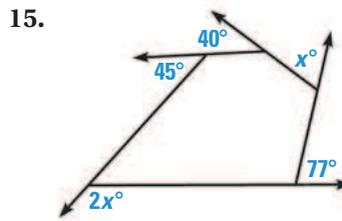
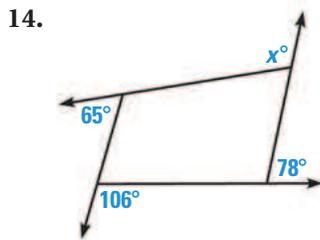
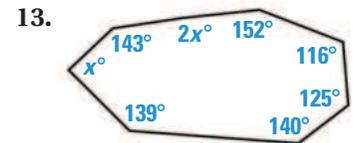
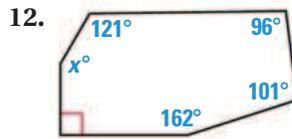
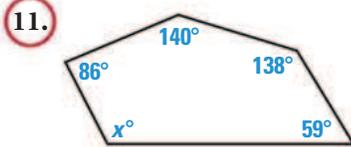
FINDING NUMBER OF SIDES The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.

7. 360° 8. 720° 9. 1980° 10. 2340°

EXAMPLES 3 and 4

on pp. 508–509
for Exs. 11–18

xy ALGEBRA Find the value of x .



17. **ERROR ANALYSIS** A student claims that the sum of the measures of the exterior angles of an octagon is greater than the sum of the measures of the exterior angles of a hexagon. The student justifies this claim by saying that an octagon has two more sides than a hexagon. *Describe* and correct the error the student is making.

18. **★ MULTIPLE CHOICE** The measures of the interior angles of a quadrilateral are x° , $2x^\circ$, $3x^\circ$, and $4x^\circ$. What is the measure of the largest interior angle?

- (A) 120° (B) 144° (C) 160° (D) 360°

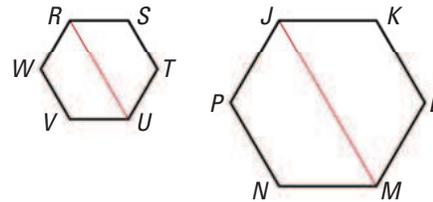
EXAMPLE 5

on p. 510
for Exs. 19–21

REGULAR POLYGONS Find the measures of an interior angle and an exterior angle of the indicated regular polygon.

19. Regular pentagon 20. Regular 18-gon 21. Regular 90-gon

22. **DIAGONALS OF SIMILAR FIGURES**
Hexagons $RSTUVW$ and $JKLMNP$ are similar. \overline{RU} and \overline{JM} are diagonals. Given $ST = 6$, $KL = 10$, and $RU = 12$, find JM .



23. **★ SHORT RESPONSE** *Explain* why any two regular pentagons are similar.

REGULAR POLYGONS Find the value of n for each regular n -gon described.

24. Each interior angle of the regular n -gon has a measure of 156° .
25. Each exterior angle of the regular n -gon has a measure of 9° .
26. **POSSIBLE POLYGONS** Determine if it is possible for a regular polygon to have an interior angle with the given angle measure. *Explain* your reasoning.
a. 165° b. 171° c. 75° d. 40°
27. **CHALLENGE** Sides are added to a convex polygon so that the sum of its interior angle measures is increased by 540° . How many sides are added to the polygon? *Explain* your reasoning.

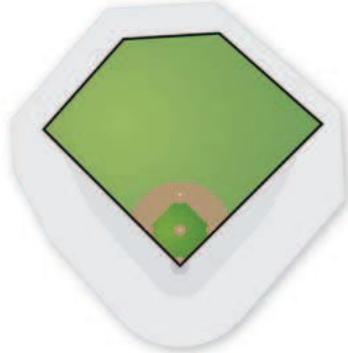
PROBLEM SOLVING

EXAMPLE 1

on p. 507
for Exs. 28–29

BASEBALL The outline of the playing field at a baseball park is a polygon, as shown. Find the sum of the measures of the interior angles of the polygon.

28.



29.



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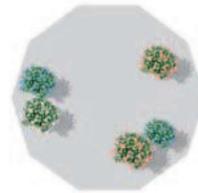
EXAMPLE 5

on p. 510
for Exs. 30–31

30. **JEWELRY BOX** The base of a jewelry box is shaped like a regular hexagon. What is the measure of each interior angle of the hexagon?

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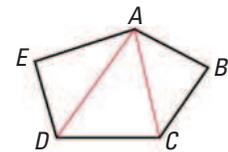
31. **GREENHOUSE** The floor of the greenhouse shown is shaped like a regular decagon. Find the measure of an interior angle of the regular decagon. Then find the measure of an exterior angle.



32. **MULTI-STEP PROBLEM** In pentagon $PQRST$, $\angle P$, $\angle Q$, and $\angle S$ are right angles, and $\angle R \cong \angle T$.

- Draw a Diagram** Sketch pentagon $PQRST$. Mark the right angles and the congruent angles.
- Calculate** Find the sum of the interior angle measures of $PQRST$.
- Calculate** Find $m\angle R$ and $m\angle T$.

33. **PROVING THEOREM 8.1 FOR PENTAGONS** The Polygon Interior Angles Theorem states that the sum of the measures of the interior angles of an n -gon is $(n - 2) \cdot 180^\circ$. Write a paragraph proof of this theorem for the case when $n = 5$.

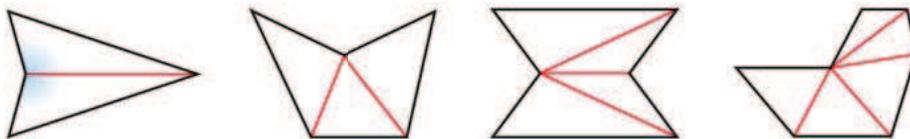


34. **PROVING A COROLLARY** Write a paragraph proof of the Corollary to the Polygon Interior Angles Theorem.

35. **PROVING THEOREM 8.2** Use the plan below to write a paragraph proof of the Polygon Exterior Angles Theorem.

Plan for Proof In a convex n -gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180° . Multiply by n to get the sum of all such sums at each vertex. Then subtract the sum of the interior angles derived by using the Polygon Interior Angles Theorem.

36. **MULTIPLE REPRESENTATIONS** The formula for the measure of each interior angle in a regular polygon can be written in function notation.
- Writing a Function** Write a function $h(n)$, where n is the number of sides in a regular polygon and $h(n)$ is the measure of any interior angle in the regular polygon.
 - Using a Function** Use the function from part (a) to find $h(9)$. Then use the function to find n if $h(n) = 150^\circ$.
 - Graphing a Function** Graph the function from part (a) for $n = 3, 4, 5, 6, 7,$ and 8 . Based on your graph, *describe* what happens to the value of $h(n)$ as n increases. *Explain* your reasoning.
37. **★ EXTENDED RESPONSE** In a concave polygon, at least one interior angle measure is greater than 180° . For example, the measure of the shaded angle in the concave quadrilateral below is 210° .



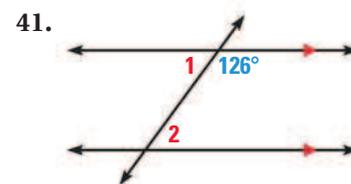
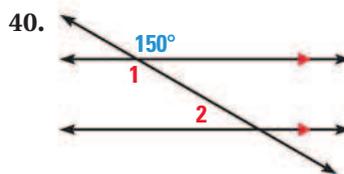
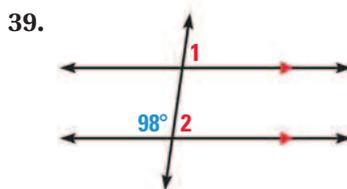
- In the diagrams above, the interiors of a concave quadrilateral, pentagon, hexagon, and heptagon are divided into triangles. Make a table like the one in the Activity on page 506. For each of the polygons shown above, record the number of sides, the number of triangles, and the sum of the measures of the interior angles.
 - Write an algebraic expression that you can use to find the sum of the measures of the interior angles of a concave polygon. *Explain*.
38. **CHALLENGE** Polygon $ABCDEFGH$ is a regular octagon. Suppose sides \overline{AB} and \overline{CD} are extended to meet at a point P . Find $m\angle BPC$. *Explain* your reasoning. Include a diagram with your answer.

MIXED REVIEW

PREVIEW

Prepare for
Lesson 8.2
in Exs. 39–41.

Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning. (p. 154)



42. Quadrilaterals $JKLM$ and $PQRS$ are similar. If $JK = 3.6$ centimeters and $PQ = 1.2$ centimeters, find the scale factor of $JKLM$ to $PQRS$. (p. 372)
43. Quadrilaterals $ABCD$ and $EFGH$ are similar. The scale factor of $ABCD$ to $EFGH$ is $8:5$, and the perimeter of $ABCD$ is 90 feet. Find the perimeter of $EFGH$. (p. 372)

Let $\angle A$ be an acute angle in a right triangle. Approximate the measure of $\angle A$ to the nearest tenth of a degree. (p. 483)

44. $\sin A = 0.77$

45. $\sin A = 0.35$

46. $\cos A = 0.81$

47. $\cos A = 0.23$