

3.4 Find and Use Slopes of Lines



- Before** You used properties of parallel lines to find angle measures.
- Now** You will find and compare slopes of lines.
- Why** So you can compare rates of speed, as in Example 4.

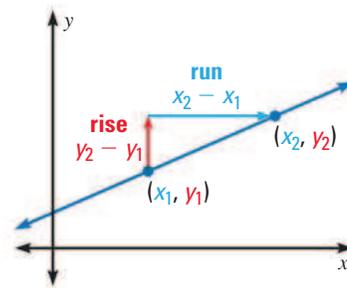
Key Vocabulary

- **slope**, p. 879
- **rise**, p. 879
- **run**, p. 879

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line.

If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2) then the slope m is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



KEY CONCEPT

For Your Notebook

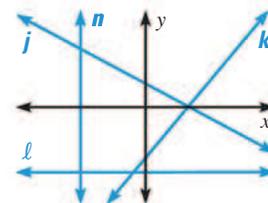
Slope of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line j

Positive slope: rises from left to right, as in line k

Zero slope (slope of 0): horizontal, as in line l

Undefined slope: vertical, as in line n



EXAMPLE 1 Find slopes of lines in a coordinate plane

REVIEW SLOPE

For more help with slope, see p. 879.

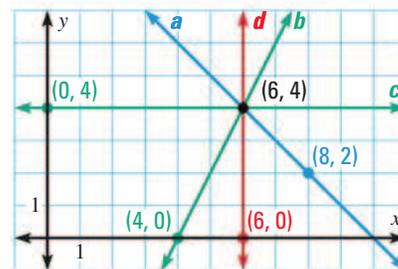
Find the slope of line a and line d .

Solution

Slope of line a : $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{6 - 8} = \frac{2}{-2} = -1$

Slope of line d : $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - 6} = \frac{4}{0}$,

which is undefined.



GUIDED PRACTICE for Example 1

Use the graph in Example 1. Find the slope of the line.

1. Line b
2. Line c

COMPARING SLOPES When two lines intersect in a coordinate plane, the steeper line has the slope with greater absolute value. You can also compare slopes to tell whether two lines are parallel or perpendicular.

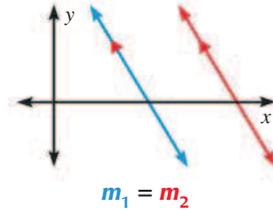
POSTULATES

For Your Notebook

POSTULATE 17 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

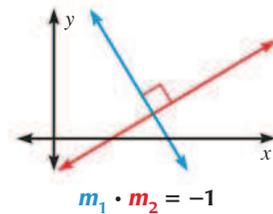
Any two vertical lines are parallel.



POSTULATE 18 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1 .

Horizontal lines are perpendicular to vertical lines.



READ VOCABULARY

If the product of two numbers is -1 , then the numbers are called *negative reciprocals*.

EXAMPLE 2 Identify parallel lines

Find the slope of each line. Which lines are parallel?

Solution

Find the slope of k_1 through $(-2, 4)$ and $(-3, 0)$.

$$m_1 = \frac{0 - 4}{-3 - (-2)} = \frac{-4}{-1} = 4$$

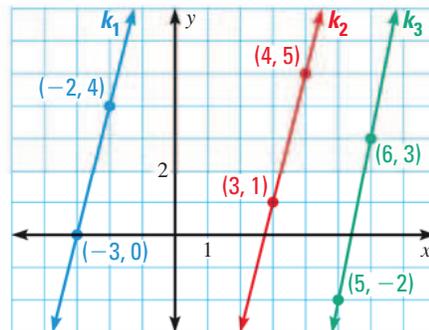
Find the slope of k_2 through $(4, 5)$ and $(3, 1)$.

$$m_2 = \frac{1 - 5}{3 - 4} = \frac{-4}{-1} = 4$$

Find the slope of k_3 through $(6, 3)$ and $(5, -2)$.

$$m_3 = \frac{-2 - 3}{5 - 6} = \frac{-5}{-1} = 5$$

► Compare the slopes. Because k_1 and k_2 have the same slope, they are parallel. The slope of k_3 is different, so k_3 is not parallel to the other lines.



GUIDED PRACTICE for Example 2

3. Line m passes through $(-1, 3)$ and $(4, 1)$. Line t passes through $(-2, -1)$ and $(3, -3)$. Are the two lines parallel? *Explain* how you know.

EXAMPLE 3 Draw a perpendicular line

Line h passes through $(3, 0)$ and $(7, 6)$. Graph the line perpendicular to h that passes through the point $(2, 5)$.

Solution

STEP 1 Find the slope m_1 of line h through $(3, 0)$ and $(7, 6)$.

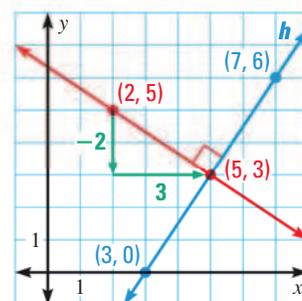
$$m_1 = \frac{6 - 0}{7 - 3} = \frac{6}{4} = \frac{3}{2}$$

STEP 2 Find the slope m_2 of a line perpendicular to h . Use the fact that the product of the slopes of two perpendicular lines is -1 .

$$\frac{3}{2} \cdot m_2 = -1 \quad \text{Slopes of perpendicular lines}$$

$$m_2 = \frac{-2}{3} \quad \text{Multiply each side by } \frac{2}{3}.$$

STEP 3 Use the rise and run to graph the line.



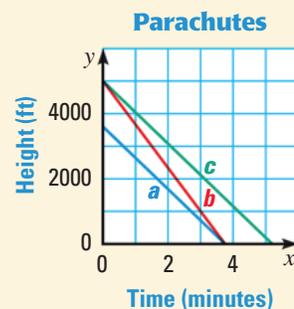
REVIEW GRAPHING

Given a point on a line and the line's slope, you can use the rise and run to find a second point and draw the line.



EXAMPLE 4 Standardized Test Practice

A skydiver made jumps with three parachutes. The graph shows the height of the skydiver from the time the parachute opened to the time of the landing for each jump. Which statement is true?



ELIMINATE CHOICES

The y-intercept represents the height when the parachute opened, so the heights in jumps a and b were not the same. So you can eliminate choice A.

- (A) The parachute opened at the same height in jumps a and b .
- (B) The parachute was open for the same amount of time in jumps b and c .
- (C) The skydiver descended at the same rate in jumps a and b .
- (D) The skydiver descended at the same rate in jumps a and c .

Solution

The rate at which the skydiver descended is represented by the slope of the segments. The segments that have the same slope are a and c .

▶ The correct answer is D. (A) (B) (C) (D)



GUIDED PRACTICE for Examples 3 and 4

- Line n passes through $(0, 2)$ and $(6, 5)$. Line m passes through $(2, 4)$ and $(4, 0)$. Is $n \perp m$? Explain.
- In Example 4, which parachute is in the air for the longest time? Explain.
- In Example 4, what do the x -intercepts represent in the situation? How can you use this to eliminate one of the choices?



EXAMPLE 5 Solve a real-world problem

ROLLER COASTERS During the climb on the Magnum XL-200 roller coaster, you move 41 feet upward for every 80 feet you move horizontally. At the crest of the hill, you have moved 400 feet forward.

- a. **Making a Table** Make a table showing the height of the Magnum at every 80 feet it moves horizontally. How high is the roller coaster at the top of its climb?
- b. **Calculating** Write a fraction that represents the height the Magnum climbs for each foot it moves horizontally. What does the numerator represent?
- c. **Using a Graph** Another roller coaster, the Millenium Force, climbs at a slope of 1. At its crest, the horizontal distance from the starting point is 310 feet. Compare this climb to that of the Magnum. Which climb is steeper?



Solution

a.

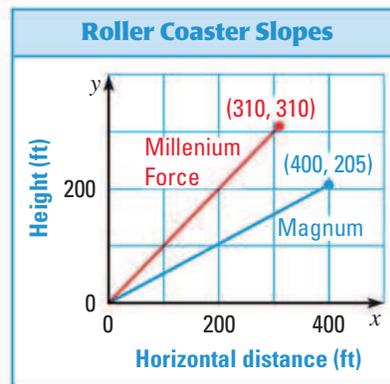
Horizontal distance (ft)	80	160	240	320	400
Height (ft)	41	82	123	164	205

The Magnum XL-200 is 205 feet high at the top of its climb.

b. Slope of the Magnum = $\frac{\text{rise}}{\text{run}} = \frac{41}{80} = \frac{41 \div 80}{80 \div 80} = \frac{0.5125}{1}$

The numerator, 0.5125, represents the slope in decimal form.

- c. Use a graph to compare the climbs. Let x be the horizontal distance and let y be the height. Because the slope of the Millenium Force is 1, the rise is equal to the run. So the highest point must be at (310, 310).
- ▶ The graph shows that the Millenium Force has a steeper climb, because the slope of its line is greater ($1 > 0.5125$).



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GUIDED PRACTICE for Example 5

- 7. Line q passes through the points (0, 0) and (−4, 5). Line t passes through the points (0, 0) and (−10, 7). Which line is steeper, q or t ?
- 8. **WHAT IF?** Suppose a roller coaster climbed 300 feet upward for every 350 feet it moved horizontally. Is it *more steep* or *less steep* than the Magnum? than the Millenium Force?

3.4 EXERCISES

HOMEWORK KEY

- = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 7, 13, and 35
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 34, 35, and 41
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 37

SKILL PRACTICE

1. **VOCABULARY** Describe what is meant by the slope of a nonvertical line.

2. **★ WRITING** What happens when you apply the slope formula to a horizontal line? What happens when you apply it to a vertical line?

EXAMPLE 1

on p. 171
for Exs. 3–12

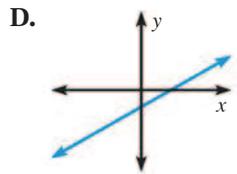
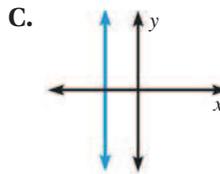
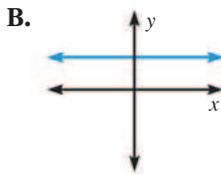
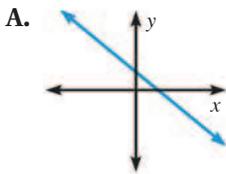
MATCHING Match the description of the slope of a line with its graph.

3. m is positive.

4. m is negative.

5. m is zero.

6. m is undefined.



FINDING SLOPE Find the slope of the line that passes through the points.

7. $(3, 5), (5, 6)$

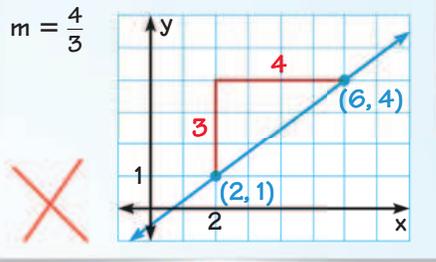
8. $(-2, 2), (2, -6)$

9. $(-5, -1), (3, -1)$

10. $(2, 1), (0, 6)$

ERROR ANALYSIS Describe and correct the error in finding the slope of the line.

11.



12.

Slope of the line through $(2, 7)$ and $(4, 5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{4 - 2} = \frac{2}{2} = 1$$

EXAMPLES 2 and 3

on pp. 172–173
for Exs. 13–18

TYPES OF LINES Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. Justify your answer.

13. Line 1: $(1, 0), (7, 4)$
Line 2: $(7, 0), (3, 6)$

14. Line 1: $(-3, 1), (-7, -2)$
Line 2: $(2, -1), (8, 4)$

15. Line 1: $(-9, 3), (-5, 7)$
Line 2: $(-11, 6), (-7, 2)$

GRAPHING Graph the line through the given point with the given slope.

16. $P(3, -2)$, slope $-\frac{1}{6}$

17. $P(-4, 0)$, slope $\frac{5}{2}$

18. $P(0, 5)$, slope $\frac{2}{3}$

EXAMPLES 4 and 5

on pp. 173–174
for Exs. 19–22

STEEPNESS OF A LINE Tell which line through the given points is steeper.

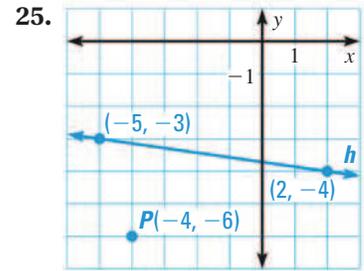
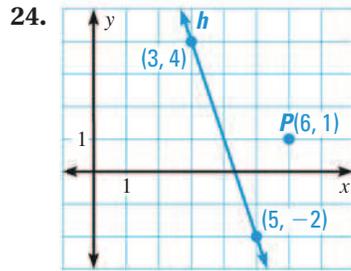
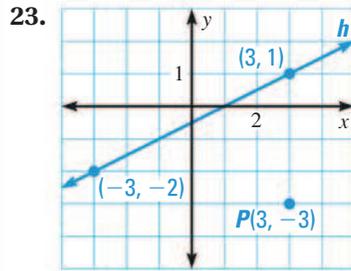
19. Line 1: $(-2, 3), (3, 5)$
Line 2: $(3, 1), (6, 5)$

20. Line 1: $(-2, -1), (1, -2)$
Line 2: $(-5, -3), (-1, -4)$

21. Line 1: $(-4, 2), (-3, 6)$
Line 2: $(1, 6), (3, 8)$

22. **REASONING** Use your results from Exercises 19–21. Describe a way to determine which of two lines is steeper without graphing them.

PERPENDICULAR LINES Find the slope of line n perpendicular to line h and passing through point P . Then copy the graph and graph line n .



26. **REASONING** Use the concept of slope to decide whether the points $(-3, 3)$, $(1, -2)$, and $(4, 0)$ lie on the same line. *Explain* your reasoning and include a diagram.

GRAPHING Graph a line with the given description.

27. Through $(0, 2)$ and parallel to the line through $(-2, 4)$ and $(-5, 1)$

28. Through $(1, 3)$ and perpendicular to the line through $(-1, -1)$ and $(2, 0)$

29. Through $(-2, 1)$ and parallel to the line through $(3, 1)$ and $(4, -\frac{1}{2})$

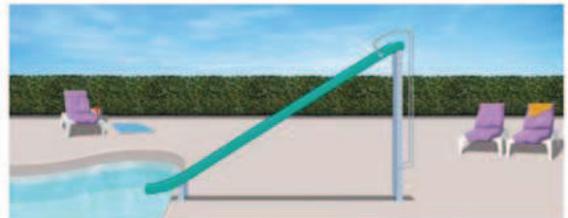
CHALLENGE Find the unknown coordinate so the line through the points has the given slope.

30. $(-3, 2)$, $(0, y)$; slope -2 31. $(-7, -4)$, $(x, 0)$; slope $\frac{1}{3}$ 32. $(4, -3)$, $(x, 1)$; slope -4

PROBLEM SOLVING

33. **WATER SLIDE** The water slide is 6 feet tall, and the end of the slide is 9 feet from the base of the ladder. About what slope does the slide have?

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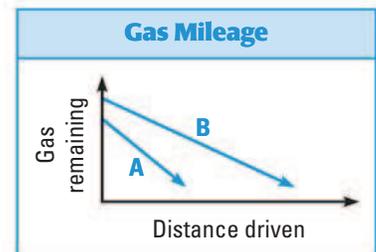
EXAMPLE 5

on p. 174
for Exs. 34–37

34. **★ MULTIPLE CHOICE** Which car has better gas mileage?

- (A) A (B) B
(C) Same rate (D) Cannot be determined

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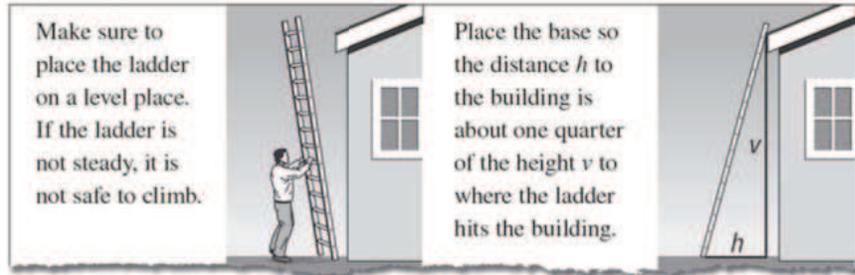
35. **★ SHORT RESPONSE** Compare the graphs of the three lines described below. Which is most steep? Which is the least steep? Include a sketch in your answer.

Line a: through the point $(3, 0)$ with a y -intercept of 4

Line b: through the point $(3, 0)$ with a y -intercept greater than 4

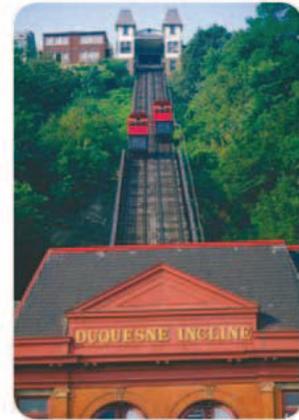
Line c: through the point $(3, 0)$ with a y -intercept between 0 and 4

36. **MULTI-STEP PROBLEM** Ladder safety guidelines include the following recommendation about ladder placement. The horizontal distance h between the base of the ladder and the object the ladder is resting against should be about one quarter of the vertical distance v between the ground and where the ladder rests against the object.



- Find the recommended slope for a ladder.
 - Suppose the base of a ladder is 6 feet away from a building. The ladder has the recommended slope. Find v .
 - Suppose a ladder is 34 feet from the ground where it touches a building. The ladder has the recommended slope. Find h .
37. **MULTIPLE REPRESENTATIONS** The Duquesne (pronounced “du-KAYN”) Incline was built in 1888 in Pittsburgh, Pennsylvania, to move people up and down a mountain there. On the incline, you move about 29 feet vertically for every 50 feet you move horizontally. When you reach the top of the hill, you have moved a horizontal distance of about 700 feet.

- Making a Table** Make a table showing the vertical distance that the incline moves for each 50 feet of horizontal distance during its climb. How high is the incline at the top?
- Drawing a Graph** Write a fraction that represents the slope of the incline’s climb path. Draw a graph to show the climb path.
- Comparing Slopes** The Burgenstock Incline in Switzerland moves about 144 vertical feet for every 271 horizontal feet. Write a fraction to represent the slope of this incline’s path. Which incline is steeper, the *Burgenstock* or the *Duquesne*?



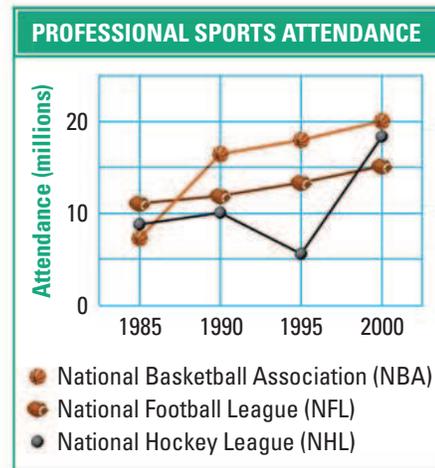
38. **PROVING THEOREM 3.7** Use slopes of lines to write a paragraph proof of the Transitive Property of Parallel Lines on page 164.

AVERAGE RATE OF CHANGE In Exercises 39 and 40, slope can be used to describe an *average rate of change*. To write an average rate of change, rewrite the slope fraction so the denominator is one.

- BUSINESS** In 2000, a business made a profit of \$8500. In 2006, the business made a profit of \$15,400. Find the average rate of change in dollars per year from 2000 to 2006.
- ROCK CLIMBING** A rock climber begins climbing at a point 400 feet above sea level. It takes the climber 45 minutes to climb to the destination, which is 706 feet above sea level. Find the average rate of change in feet per minute for the climber from start to finish.

41. ★ **EXTENDED RESPONSE** The line graph shows the regular season attendance (in millions) for three professional sports organizations from 1985 to 2000.

- During which five-year period did the NBA attendance increase the most? Estimate the rate of change for this five-year period in people per year.
- During which five-year period did the NHL attendance increase the most? Estimate the rate of change for this five-year period in people per year.
- Interpret** The line graph for the NFL seems to be almost linear between 1985 and 2000. Write a sentence about what this means in terms of the real-world situation.



42. **CHALLENGE** Find two values of k such that the points $(-3, 1)$, $(0, k)$, and $(k, 5)$ are collinear. *Explain* your reasoning.

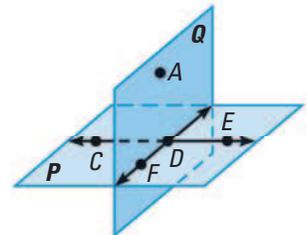
MIXED REVIEW

43. Is the point $(-1, -7)$ on the line $y = 2x - 5$? *Explain.* (p. 878)

44. Find the intercepts of the graph of $y = -3x + 9$. (p. 879)

Use the diagram to write two examples of each postulate. (p. 96)

- Through any two points there exists exactly one line.
- Through any three noncollinear points there exists exactly one plane.



Solve the equation for y . Write a reason for each step. (p. 105)

47. $6x + 4y = 40$

48. $\frac{1}{2}x - \frac{5}{4}y = -10$

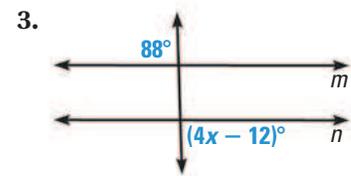
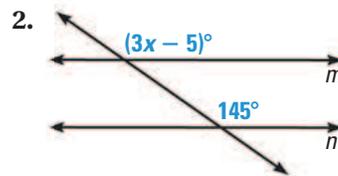
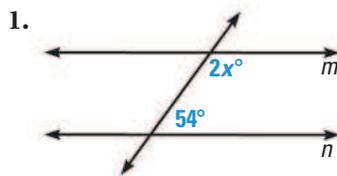
49. $16 - 3y = 24x$

PREVIEW

Prepare for Lesson 3.5 in Exs. 47–49.

QUIZ for Lessons 3.3–3.4

Find the value of x that makes $m \parallel n$. (p. 161)



Find the slope of the line that passes through the given points. (p. 171)

4. $(1, -1), (3, 3)$

5. $(1, 2), (4, 5)$

6. $(-3, -2), (-7, -6)$

3.4 Investigate Slopes

MATERIALS • graphing calculator or computer

QUESTION How can you verify the Slopes of Parallel Lines Postulate?

You can verify the postulates you learned in Lesson 3.4 using geometry drawing software.

EXAMPLE Verify the Slopes of Parallel Lines Postulate

STEP 1 Show axes Show the x -axis and the y -axis by choosing Hide/Show Axes from the F5 menu.

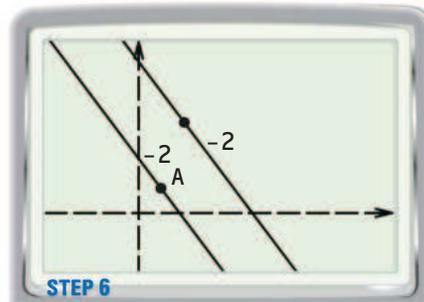
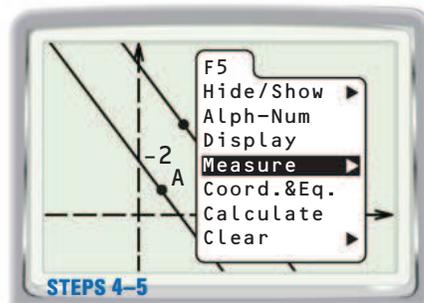
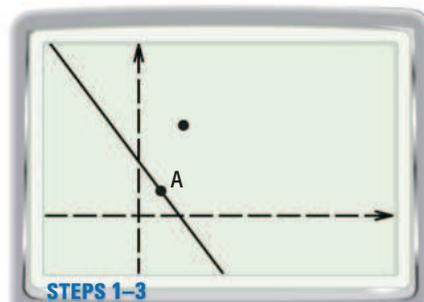
STEP 2 Draw line Draw a line by choosing Line from the F2 menu. Do not use one of the axes as your line. Choose a point on the line and label it A .

STEP 3 Graph point Graph a point not on the line by choosing Point from the F2 menu.

STEP 4 Draw parallel line Choose Parallel from the F3 menu and select the line. Then select the point not on the line.

STEP 5 Measure slopes Select one line and choose Measure Slope from the F5 menu. Repeat this step for the second line.

STEP 6 Move line Drag point A to move the line. What do you expect to happen?



PRACTICE

- Use geometry drawing software to verify the Slopes of Perpendicular Lines Postulate.
 - Construct a line and a point not on that line. Use Steps 1–3 from the Example above.
 - Construct a line that is perpendicular to your original line and passes through the given point.
 - Measure the slopes of the two lines. Multiply the slopes. What do you expect the product of the slopes to be?
- WRITING** Use the arrow keys to move your line from Exercise 1. Describe what happens to the product of the slopes when one of the lines is vertical. Explain why this happens.