

3.5 Write and Graph Equations of Lines



Before

You found slopes of lines.

Now

You will find equations of lines.

Why?

So you can find monthly gym costs, as in Example 4.

Key Vocabulary

- slope-intercept form
- standard form
- x-intercept, p. 879
- y-intercept, p. 879

Linear equations may be written in different forms. The general form of a linear equation in **slope-intercept form** is $y = mx + b$, where m is the slope and b is the y-intercept.

EXAMPLE 1 Write an equation of a line from a graph

Write an equation of the line in slope-intercept form.

Solution

STEP 1 Find the slope. Choose two points on the graph of the line, $(0, 4)$ and $(3, -2)$.

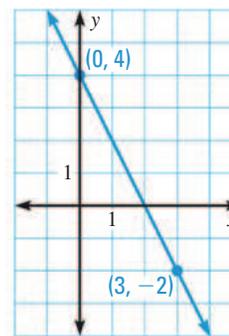
$$m = \frac{4 - (-2)}{0 - 3} = \frac{6}{-3} = -2$$

STEP 2 Find the y-intercept. The line intersects the y-axis at the point $(0, 4)$, so the y-intercept is 4.

STEP 3 Write the equation.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$y = -2x + 4 \quad \text{Substitute } -2 \text{ for } m \text{ and } 4 \text{ for } b.$$



EXAMPLE 2 Write an equation of a parallel line

Write an equation of the line passing through the point $(-1, 1)$ that is parallel to the line with the equation $y = 2x - 3$.

Solution

STEP 1 Find the slope m . The slope of a line parallel to $y = 2x - 3$ is the same as the given line, so the slope is 2.

STEP 2 Find the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$1 = 2(-1) + b \quad \text{Substitute for } x, y, \text{ and } m.$$

$$3 = b \quad \text{Solve for } b.$$

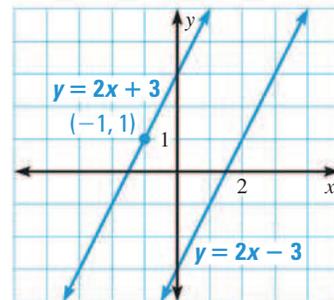
► Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$.

LINEAR EQUATIONS

The graph of a linear equation represents all the solutions of the equation. So, the given point must be a solution of the equation.

CHECKING BY GRAPHING You can check that equations are correct by graphing. In Example 2, you can use a graph to check that $y = 2x - 3$ is parallel to $y = 2x + 3$.

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EXAMPLE 3 Write an equation of a perpendicular line

Write an equation of the line j passing through the point $(2, 3)$ that is perpendicular to the line k with the equation $y = -2x + 2$.

Solution

STEP 1 Find the slope m of line j . Line k has a slope of -2 .

$$-2 \cdot m = -1 \quad \text{The product of the slopes of } \perp \text{ lines is } -1.$$

$$m = \frac{1}{2} \quad \text{Divide each side by } -2.$$

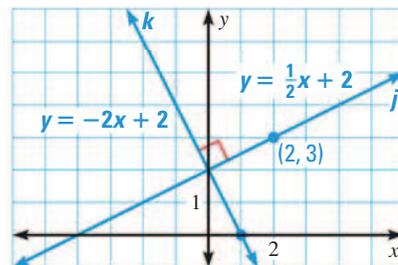
STEP 2 Find the y -intercept b by using $m = \frac{1}{2}$ and $(x, y) = (2, 3)$.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$3 = \frac{1}{2}(2) + b \quad \text{Substitute for } x, y, \text{ and } m.$$

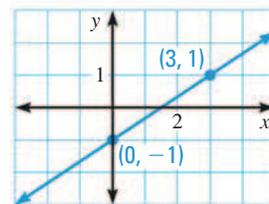
$$2 = b \quad \text{Solve for } b.$$

► Because $m = \frac{1}{2}$ and $b = 2$, an equation of line j is $y = \frac{1}{2}x + 2$. You can check that the lines j and k are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.



GUIDED PRACTICE for Examples 1, 2, and 3

- Write an equation of the line in the graph at the right.
- Write an equation of the line that passes through $(-2, 5)$ and $(1, 2)$.
- Write an equation of the line that passes through the point $(1, 5)$ and is parallel to the line with the equation $y = 3x - 5$. Graph the lines to check that they are parallel.
- How do you know the lines $x = 4$ and $y = 2$ are perpendicular?



EXAMPLE 4 Write an equation of a line from a graph

GYM MEMBERSHIP The graph models the total cost of joining a gym. Write an equation of the line. Explain the meaning of the slope and the y -intercept of the line.



Solution

STEP 1 Find the slope.

$$m = \frac{363 - 231}{5 - 2} = \frac{132}{3} = 44$$

STEP 2 Find the y -intercept. Use the slope and one of the points on the graph.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$231 = 44 \cdot 2 + b \quad \text{Substitute for } x, y, \text{ and } m.$$

$$143 = b \quad \text{Simplify.}$$

STEP 3 Write the equation. Because $m = 44$ and $b = 143$, an equation of the line is $y = 44x + 143$.

► The equation $y = 44x + 143$ models the cost. The slope is the monthly fee, \$44, and the y -intercept is the initial cost to join the gym, \$143.

STANDARD FORM Another form of a linear equation is *standard form*. In **standard form**, the equation is written as $Ax + By = C$, where A and B are not both zero.

EXAMPLE 5 Graph a line with equation in standard form

Graph $3x + 4y = 12$.

Solution

The equation is in standard form, so you can use the intercepts.

STEP 1 Find the intercepts.

To find the x -intercept, let $y = 0$.

$$3x + 4y = 12$$

$$3x + 4(0) = 12$$

$$x = 4$$

To find the y -intercept, let $x = 0$.

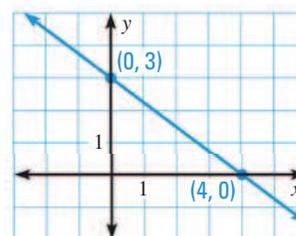
$$3x + 4y = 12$$

$$3(0) + 4y = 12$$

$$y = 3$$

STEP 2 Graph the line.

The intercepts are $(4, 0)$ and $(0, 3)$. Graph these points, then draw a line through the points.



CHOOSE A METHOD

Another way you could graph the equation is to solve the equation for y . Then the equation will be in slope-intercept form. Use rise and run from the point where the line crosses the y -axis to find a second point. Then graph the line.

**GUIDED PRACTICE** for Examples 4 and 5

5. The equation $y = 50x + 125$ models the total cost of joining a climbing gym. What are the meaning of the slope and the y -intercept of the line?

Graph the equation.

6. $2x - 3y = 6$ 7. $y = 4$ 8. $x = -3$

WRITING EQUATIONS You can write linear equations to model real-world situations, such as comparing costs to find a better buy.

EXAMPLE 6 Solve a real-world problem

DVD RENTAL You can rent DVDs at a local store for \$4.00 each. An Internet company offers a flat fee of \$15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

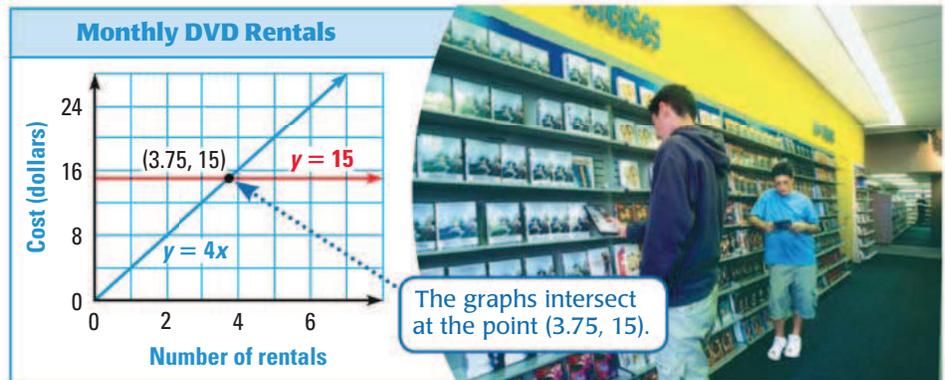
Solution

STEP 1 **Model** each rental with an equation.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Graph** each equation.

**ANOTHER WAY**

For alternative methods for solving the problem in Example 6, turn to page 188 for the **Problem Solving Workshop**.

READ VOCABULARY

The point at which the costs are the same is sometimes called the *break-even point*.

▶ The point of intersection is $(3.75, 15)$. Using the graph, you can see that it is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

**GUIDED PRACTICE** for Example 6

9. **WHAT IF?** In Example 6, suppose the online rental is \$16.50 per month and the local rental is \$4 each. How many DVDs do you need to rent to make the online rental a better buy?
10. How would your answer to Exercise 9 change if you had a 2-for-1 coupon that you could use once at the local store?

3.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 17, 23, and 61

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 9, 29, 64, and 65

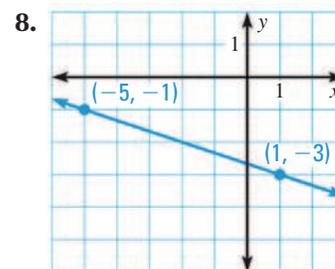
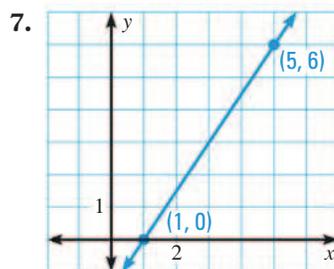
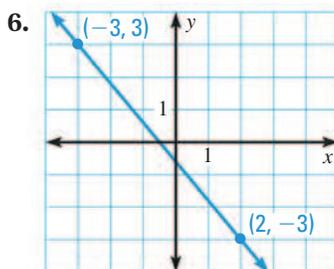
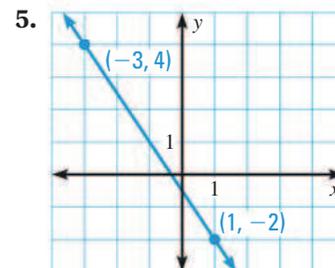
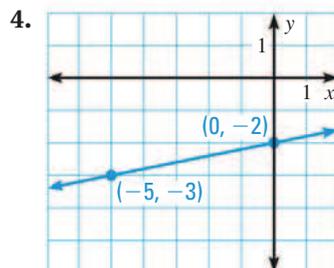
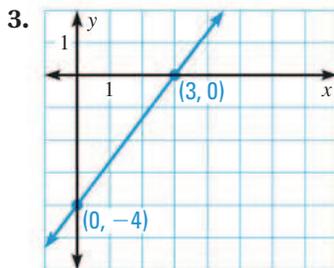
SKILL PRACTICE

- VOCABULARY** What does *intercept* mean in the expression *slope-intercept form*?
- ★ **WRITING** Explain how you can use the standard form of a linear equation to find the intercepts of a line.

EXAMPLE 1

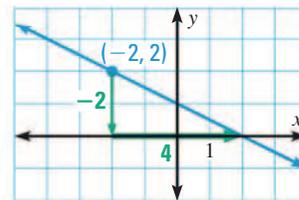
on p. 180
for Exs. 3–22

WRITING EQUATIONS Write an equation of the line shown.



9. ★ **MULTIPLE CHOICE** Which equation is an equation of the line in the graph?

- (A) $y = -\frac{1}{2}x$ (B) $y = -\frac{1}{2}x + 1$
(C) $y = -2x$ (D) $y = -2x + 1$



WRITING EQUATIONS Write an equation of the line with the given slope m and y -intercept b .

10. $m = -5, b = -12$ 11. $m = 3, b = 2$ 12. $m = 4, b = -6$
13. $m = -\frac{5}{2}, b = 0$ 14. $m = \frac{4}{9}, b = -\frac{2}{9}$ 15. $m = -\frac{11}{5}, b = -12$

WRITING EQUATIONS Write an equation of the line that passes through the given point P and has the given slope m .

16. $P(-1, 0), m = -1$ 17. $P(5, 4), m = 4$ 18. $P(6, -2), m = 3$
19. $P(-8, -2), m = -\frac{2}{3}$ 20. $P(0, -3), m = -\frac{1}{6}$ 21. $P(-13, 7), m = 0$

22. **WRITING EQUATIONS** Write an equation of a line with undefined slope that passes through the point $(3, -2)$.

EXAMPLE 2

on p. 180
for Exs. 23–29

PARALLEL LINES Write an equation of the line that passes through point P and is parallel to the line with the given equation.

23. $P(0, -1), y = -2x + 3$ 24. $P(-7, -4), y = 16$ 25. $P(3, 8), y - 1 = \frac{1}{5}(x + 4)$

26. $P(-2, 6), x = -5$ 27. $P(-2, 1), 10x + 4y = -8$ 28. $P(4, 0), -x + 2y = 12$

29. **★ MULTIPLE CHOICE** Line a passes through points $(-2, 1)$ and $(2, 9)$. Which equation is an equation of a line parallel to line a ?

(A) $y = -2x + 5$ (B) $y = -\frac{1}{2}x + 5$ (C) $y = \frac{1}{2}x - 5$ (D) $y = 2x - 5$

EXAMPLE 3

on p. 181
for Exs. 30–35

PERPENDICULAR LINES Write an equation of the line that passes through point P and is perpendicular to the line with the given equation.

30. $P(0, 0), y = -9x - 1$ 31. $P(-1, 1), y = \frac{7}{3}x + 10$ 32. $P(4, -6), y = -3$

33. $P(2, 3), y - 4 = -2(x + 3)$ 34. $P(0, -5), x = 20$ 35. $P(-8, 0), 3x - 5y = 6$

EXAMPLE 5

on p. 182
for Exs. 36–45

GRAPHING EQUATIONS Graph the equation.

36. $8x + 2y = -10$ 37. $x + y = 1$ 38. $4x - y = -8$

39. $-x + 3y = -9$ 40. $y - 2 = -1$ 41. $y + 2 = x - 1$

42. $x + 3 = -4$ 43. $2y - 4 = -x + 1$ 44. $3(x - 2) = -y - 4$

45. **ERROR ANALYSIS** Describe and correct the error in finding the x - and y -intercepts of the graph of $5x - 3y = -15$.

To find the x -intercept,
let $x = 0$:

$$5x - 3y = -15$$

$$5(0) - 3y = -15$$

$$y = 5$$

To find the y -intercept,
let $y = 0$:

$$5x - 3y = -15$$

$$5x - 3(0) = -15$$

$$x = -3$$

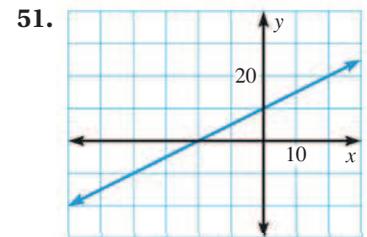
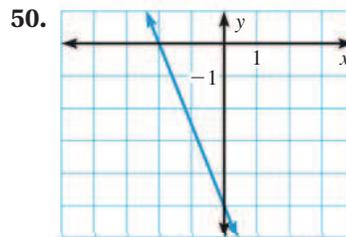
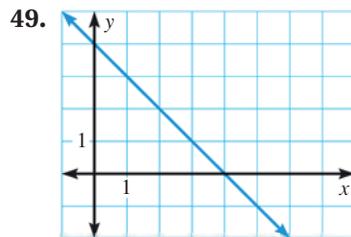
IDENTIFYING PARALLEL LINES Which lines are parallel, if any?

46. $y = 3x - 4$
 $x + 3y = 6$
 $3(x + 1) = y - 2$

47. $x + 2y = 9$
 $y = 0.5x + 7$
 $-x + 2y = -5$

48. $x - 6y = 10$
 $6x - y = 11$
 $x + 6y = 12$

USING INTERCEPTS Identify the x - and y -intercepts of the line. Use the intercepts to write an equation of the line.



52. **INTERCEPTS** A line passes through the points $(-10, -3)$ and $(6, 1)$. Where does the line intersect the x -axis? Where does the line intersect the y -axis?

SOLUTIONS TO EQUATIONS Graph the linear equations. Then use the graph to estimate how many solutions the equations share.

53. $y = 4x + 9$
 $4x - y = 1$

54. $3y + 4x = 16$
 $2x - y = 18$

55. $y = -5x + 6$
 $10x + 2y = 12$

56. **xy ALGEBRA** Solve Exercises 53–55 algebraically. (For help, see Skills Review Handbook, p. 880.) Make a conjecture about how the solution(s) can tell you whether the lines intersect, are parallel, or are the same line.

57. **xy ALGEBRA** Find a value for k so that the line through $(-1, k)$ and $(-7, -2)$ is parallel to the line with equation $y = x + 1$.

58. **xy ALGEBRA** Find a value for k so that the line through $(k, 2)$ and $(7, 0)$ is perpendicular to the line with equation $y = x - \frac{28}{5}$.

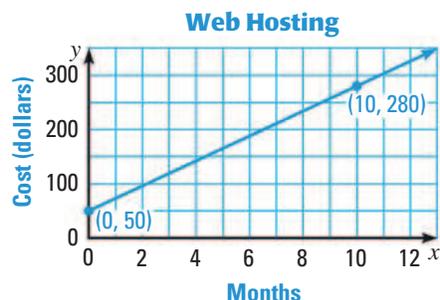
59. **CHALLENGE** Graph the points $R(-7, -3)$, $S(-2, 3)$, and $T(10, -7)$. Connect them to make $\triangle RST$. Write an equation of the line containing each side. *Explain* how you can use slopes to show that $\triangle RST$ has one right angle.

PROBLEM SOLVING

EXAMPLE 4
on p. 182
for Exs. 60–61

60. **WEB HOSTING** The graph models the total cost of using a web hosting service for several months. Write an equation of the line. Tell what the slope and y -intercept mean in this situation. Then find the total cost of using the web hosting service for one year.

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61. **SCIENCE** Scientists believe that a Tyrannosaurus Rex weighed about 2000 kilograms by age 14. It then had a growth spurt for four years, gaining 2.1 kilograms per day. Write an equation to model this situation. What are the slope and y -intercept? Tell what the slope and y -intercept mean in this situation.

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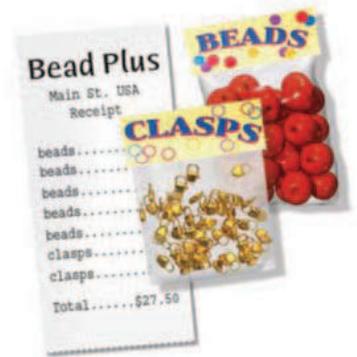
Field Museum, Chicago, Illinois

EXAMPLE 6
on p. 183
for Exs. 62–65

62. **MULTI-STEP PROBLEM** A national park has two options: a \$50 pass for all admissions during the year, or a \$4 entrance fee each time you enter.

- Model** Write an equation to model the cost of going to the park for a year using a pass and another equation for paying a fee each time.
- Graph** Graph both equations you wrote in part (a).
- Interpret** How many visits do you need to make for the pass to be cheaper? *Explain*.

63. **PIZZA COSTS** You are buying slices of pizza for you and your friends. A small slice costs \$2 and a large slice costs \$3. You have \$24 to spend. Write an equation in standard form $Ax + By = C$ that models this situation. What do the values of A , B , and C mean in this situation?
64. **★ SHORT RESPONSE** You run at a rate of 4 miles per hour and your friend runs at a rate of 3.5 miles per hour. Your friend starts running 10 minutes before you, and you run for a half hour on the same path. Will you catch up to your friend? Use a graph to support your answer.
65. **★ EXTENDED RESPONSE** Audrey and Sara are making jewelry. Audrey buys 2 bags of beads and 1 package of clasps for a total of \$13. Sara buys 5 bags of beads and 2 packages of clasps for a total of \$27.50.
- Let b be the price of one bag of beads and let c be the price of one package of clasps. Write equations to represent the total cost for Audrey and the total cost for Sara.
 - Graph the equations from part (a).
 - Explain* the meaning of the intersection of the two lines in terms of the real-world situation.
66. **CHALLENGE** Michael is deciding which gym membership to buy. Points (2, 112) and (4, 174) give the cost of gym membership at one gym after two and four months. Points (1, 62) and (3, 102) give the cost of gym membership at a second gym after one and three months. Write equations to model the cost of each gym membership. At what point do the graphs intersect, if they intersect? Which gym is cheaper? *Explain*.

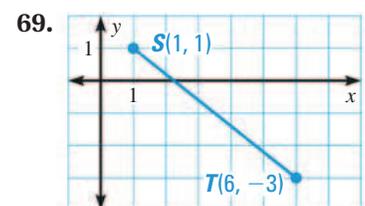
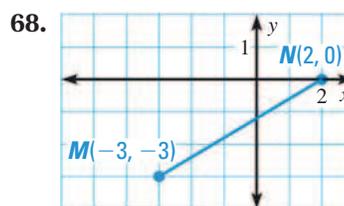
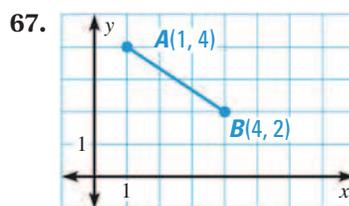


MIXED REVIEW

PREVIEW

Prepare for Lesson 3.6 in Exs. 67–69.

Find the length of each segment. Round to the nearest tenth of a unit. (p. 15)



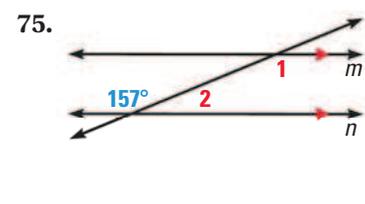
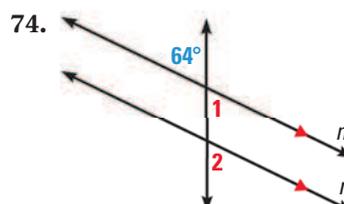
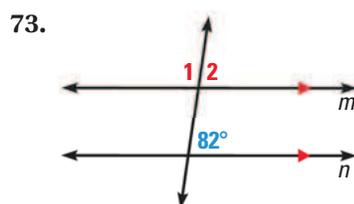
Describe the pattern in the numbers. Write the next number in the pattern. (p. 72)

70. $-2, -7, -12, -17, \dots$

71. $4, 8, 16, 32, \dots$

72. $101, 98, 95, 92, \dots$

Find $m\angle 1$ and $m\angle 2$. Explain your reasoning. (p. 154)



Another Way to Solve Example 6, page 183



MULTIPLE REPRESENTATIONS In Example 6 on page 183, you saw how to graph equations to solve a problem about renting DVDs. Another way you can solve the problem is *using a table*. Alternatively, you can use the equations to solve the problem *algebraically*.

PROBLEM

DVD RENTAL You can rent DVDs at a local store for \$4.00 each. An Internet company offers a flat fee of \$15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

METHOD 1

Using a Table You can make a table to answer the question.

STEP 1 Make a table representing each rental option.

DVDs rented	Renting locally	Renting online
1	\$4	\$15
2	\$8	\$15

STEP 2 Add rows to your table until you see a pattern.

DVDs rented	Renting locally	Renting online
1	\$4	\$15
2	\$8	\$15
3	\$12	\$15
4	\$16	\$15
5	\$20	\$15
6	\$24	\$15

STEP 3 **Analyze** the table. Notice that the values in the second column (the cost of renting locally) are less than the values in the third column (the cost of renting online) for three or fewer DVDs. However, the values in the second column are greater than those in the third column for four or more DVDs.

- It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

METHOD 2

Using Algebra You can solve one of the equations for one of its variables. Then substitute that expression for the variable in the other equation.

STEP 1 Write an equation for each rental option.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 Substitute the value of y from one equation into the other equation.

$$y = 4x$$

$$15 = 4x \quad \text{Substitute 15 for } y.$$

$$3.75 = x \quad \text{Divide each side by 4.}$$

STEP 3 Analyze the solution of the equation. If you could rent 3.75 DVDs, your cost for local and online rentals would be the same. However, you can only rent a whole number of DVDs. Look at what happens when you rent 3 DVDs and when you rent 4 DVDs, the whole numbers just less than and just greater than 3.75.

- It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

PRACTICE

- IN-LINE SKATES** You can rent in-line skates for \$5 per hour, or buy a pair of skates for \$130. How many hours do you need to skate for the cost of buying skates to be cheaper than renting them?
- WHAT IF?** Suppose the in-line skates in Exercise 1 also rent for \$12 per day. How many days do you need to skate for the cost of buying skates to be cheaper than renting them?
- BUTTONS** You buy a button machine for \$200 and supplies to make one hundred fifty buttons for \$30. Suppose you charge \$2 for a button. How many buttons do you need to sell to earn back what you spent?
- MANUFACTURING** A company buys a new widget machine for \$1200. It costs \$5 to make each widget. The company sells each widget for \$15. How many widgets do they need to sell to earn back the money they spent on the machine?
- WRITING** Which method(s) did you use to solve Exercises 1–4? *Explain* your choice(s).
- MONEY** You saved \$1000. If you put this money in a savings account, it will earn 1.5% annual interest. If you put the \$1000 in a certificate of deposit (CD), it will earn 3% annual interest. To earn the most money, does it ever make sense to put your money in the savings account? *Explain*.