6.2 Use Proportions to Solve Geometry Problems

Before You wrote and solved proportions.

Now You will use proportions to solve geometry problems.

Why? So you can calculate building dimensions, as in Ex. 22.

Key Vocabulary
- **scale drawing**
- **scale**

In Lesson 6.1, you learned to use the Cross Products Property to write equations that are equivalent to a given proportion. Three more ways to do this are given by the properties below.

REVIEW RECIPROCALS
For help with reciprocals, see p. 869.

KEY CONCEPT *For Your Notebook*

Additional Properties of Proportions

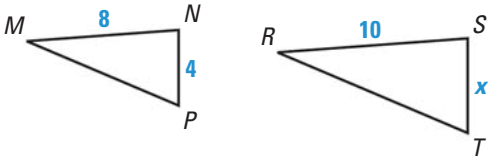
2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. If you interchange the means of a proportion, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

EXAMPLE 1 Use properties of proportions

In the diagram, $\frac{MN}{RS} = \frac{NP}{ST}$. Write four true proportions.



Solution

Because $\frac{MN}{RS} = \frac{NP}{ST}$, then $\frac{8}{10} = \frac{4}{x}$.

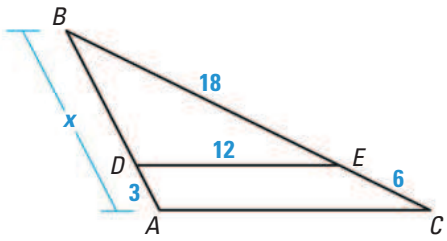
By the Reciprocal Property, the reciprocals are equal, so $\frac{10}{8} = \frac{x}{4}$.

By Property 3, you can interchange the means, so $\frac{8}{4} = \frac{10}{x}$.

By Property 4, you can add the denominators to the numerators, so $\frac{8+10}{10} = \frac{4+x}{x}$, or $\frac{18}{10} = \frac{4+x}{x}$.

EXAMPLE 2 Use proportions with geometric figures

ALGEBRA In the diagram, $\frac{BD}{DA} = \frac{BE}{EC}$. Find BA and BD.



Solution

$\frac{BD}{DA} = \frac{BE}{EC}$	Given
$\frac{BD + DA}{DA} = \frac{BE + EC}{EC}$	Property of Proportions (Property 4)
$\frac{x}{3} = \frac{18 + 6}{6}$	Substitution Property of Equality
$6x = 3(18 + 6)$	Cross Products Property
$x = 12$	Solve for x.

▶ So, $BA = 12$ and $BD = 12 - 3 = 9$.

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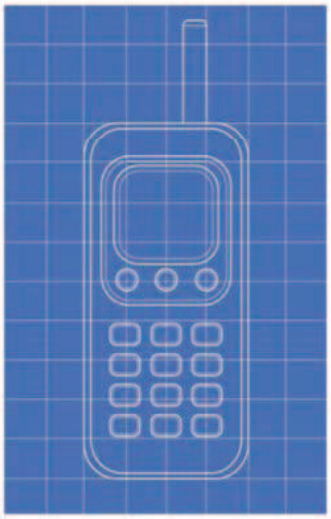
SCALE DRAWING A **scale drawing** is a drawing that is the same shape as the object it represents. The **scale** is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

EXAMPLE 3 Find the scale of a drawing

BLUEPRINTS The blueprint shows a scale drawing of a cell phone. The length of the antenna on the blueprint is 5 centimeters. The actual length of the antenna is 2 centimeters. What is the scale of the blueprint?



Solution

To find the scale, write the ratio of a length in the drawing to an actual length, then rewrite the ratio so that the denominator is 1.

$$\frac{\text{length on blueprint}}{\text{length of antenna}} = \frac{5 \text{ cm}}{2 \text{ cm}} = \frac{5 \div 2}{2 \div 2} = \frac{2.5}{1}$$

▶ The scale of the blueprint is 2.5 cm : 1 cm.

GUIDED PRACTICE for Examples 1, 2, and 3

1. In Example 1, find the value of x.
2. In Example 2, $\frac{DE}{AC} = \frac{BE}{BC}$. Find AC.
3. **WHAT IF?** In Example 3, suppose the length of the antenna on the blueprint is 10 centimeters. Find the new scale of the blueprint.

EXAMPLE 4 Use a scale drawing

MAPS The scale of the map at the right is 1 inch : 26 miles. Find the actual distance from Pocahontas to Algona.

Solution

Use a ruler. The distance from Pocahontas to Algona on the map is about 1.25 inches. Let x be the actual distance in miles.

$\frac{1.25 \text{ in.}}{x \text{ mi}} = \frac{1 \text{ in.}}{26 \text{ mi}}$ ← distance on map ← actual distance

$x = 1.25(26)$ Cross Products Property

$x = 32.5$ Simplify.

▸ The actual distance from Pocahontas to Algona is about 32.5 miles.

EXAMPLE 5 Solve a multi-step problem

SCALE MODEL You buy a 3-D scale model of the Reunion Tower in Dallas, TX. The actual building is 560 feet tall. Your model is 10 inches tall, and the diameter of the dome on your scale model is about 2.1 inches.

a. What is the diameter of the actual dome?

b. About how many times as tall as your model is the actual building?

Solution

a. $\frac{10 \text{ in.}}{560 \text{ ft}} = \frac{2.1 \text{ in.}}{x \text{ ft}}$ ← measurement on model ← measurement on actual building

$10x = 1176$ Cross Products Property

$x = 117.6$ Solve for x.

▸ The diameter of the actual dome is about 118 feet.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{560 \text{ ft}}{10 \text{ in.}} = \frac{560 \text{ ft}}{10 \text{ in.}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = 672$$

▸ The actual building is 672 times as tall as the model.

GUIDED PRACTICE for Examples 4 and 5

4. Two cities are 96 miles from each other. The cities are 4 inches apart on a map. Find the scale of the map.

5. **WHAT IF?** Your friend has a model of the Reunion Tower that is 14 inches tall. What is the diameter of the dome on your friend's model?

6.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 13, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, and 24

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A ? is a drawing that has the same shape as the object it represents.

2. ★ **WRITING** Suppose the scale of a model of the Eiffel Tower is 1 inch : 20 feet. *Explain* how to determine how many times taller the actual tower is than the model.

EXAMPLE 1 on p. 364 for Exs. 3–10

REASONING **Copy and complete the statement.**

3. If $\frac{8}{x} = \frac{3}{y}$, then $\frac{8}{3} = \frac{?}{?}$.

4. If $\frac{x}{9} = \frac{y}{20}$, then $\frac{x}{y} = \frac{?}{?}$.

5. If $\frac{x}{6} = \frac{y}{15}$, then $\frac{x+6}{6} = \frac{?}{?}$.

6. If $\frac{14}{3} = \frac{x}{y}$, then $\frac{17}{3} = \frac{?}{?}$.

REASONING **Decide whether the statement is *true* or *false*.**

7. If $\frac{8}{m} = \frac{n}{9}$, then $\frac{8+m}{m} = \frac{n+9}{9}$.

8. If $\frac{5}{7} = \frac{a}{b}$, then $\frac{7}{5} = \frac{a}{b}$.

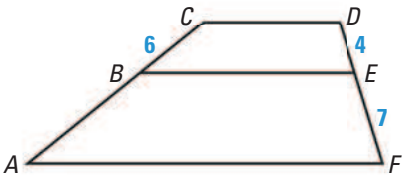
9. If $\frac{d}{2} = \frac{g+10}{11}$, then $\frac{d}{g+10} = \frac{2}{11}$.

10. If $\frac{4+x}{4} = \frac{3+y}{y}$, then $\frac{x}{4} = \frac{3}{y}$.

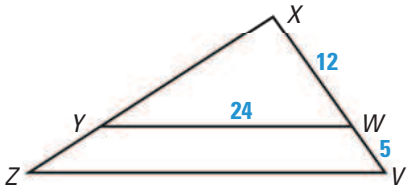
EXAMPLE 2 on p. 365 for Exs. 11–12

PROPERTIES OF PROPORTIONS **Use the diagram and the given information to find the unknown length.**

11. Given $\frac{CB}{BA} = \frac{DE}{EF}$, find BA.



12. Given $\frac{XW}{XV} = \frac{YW}{ZV}$, find ZV.



EXAMPLES 3 and 4 on pp. 365–366 for Exs. 13–14

SCALE DIAGRAMS **In Exercises 13 and 14, use the diagram of the field hockey field in which 1 inch = 50 yards. Use a ruler to approximate the dimension.**

13. Find the actual length of the field.

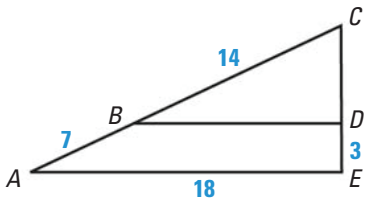
14. Find the actual width of the field.

15. **ERROR ANALYSIS** *Describe* and correct the error made in the reasoning.

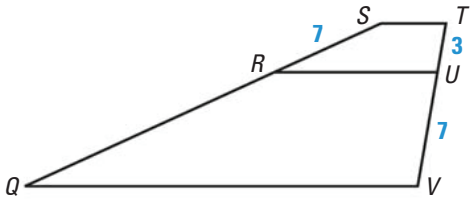
If $\frac{a}{3} = \frac{c}{4}$, then $\frac{a+3}{3} = \frac{c+3}{4}$. ✗

PROPERTIES OF PROPORTIONS **Use the diagram and the given information to find the unknown length.**

16. Given $\frac{CA}{CB} = \frac{AE}{BD}$, find BD.



17. Given $\frac{SQ}{SR} = \frac{TV}{TU}$, find RQ.



18. ★ **MULTIPLE CHOICE** If x, y, z, and q are four different numbers, and the proportion $\frac{x}{y} = \frac{z}{q}$ is true, which of the following is false?

Ⓐ $\frac{y}{x} = \frac{q}{z}$ Ⓑ $\frac{x}{z} = \frac{y}{q}$ Ⓒ $\frac{y}{x} = \frac{z}{q}$ Ⓓ $\frac{x+y}{y} = \frac{z+q}{q}$

CHALLENGE **Two number patterns are *proportional* if there is a nonzero number k such that $(a_1, b_1, c_1, \ldots) = k(a_2, b_2, c_2, \ldots) = ka_2, kb_2, kc_2, \ldots$.**

19. Given the relationship $(8, 16, 20) = k(2, 4, 5)$, find k.

20. Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

21. Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} = k$.

PROBLEM SOLVING

EXAMPLE 5 on p. 366 for Ex. 22

22. **ARCHITECTURE** A basket manufacturer has headquarters in an office building that has the same shape as a basket they sell.

a. The bottom of the basket is a rectangle with length 15 inches and width 10 inches. The base of the building is a rectangle with length 192 feet. What is the width of the base of the building?

b. About how many times as long as the bottom of the basket is the base of the building?

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Longaberger Company Home Office
Newark, Ohio

23. **MAP SCALE** A street on a map is 3 inches long. The actual street is 1 mile long. Find the scale of the map.

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24. ★ **MULTIPLE CHOICE** A model train engine is 12 centimeters long. The actual engine is 18 meters long. What is the scale of the model?

Ⓐ 3 cm : 2 m Ⓑ 1 cm : 1.5 m Ⓒ 1 cm : 3 m Ⓓ 200 cm : 3 m

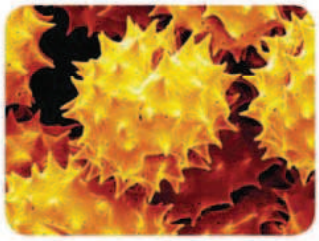
◯ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

MAP READING **The map of a hiking trail has a scale of 1 inch : 3.2 miles. Use a ruler to approximate the actual distance between the two shelters.**

25. Meadow View and Whispering Pines

26. Whispering Pines and Blueberry Hill

27. **POLLEN** The photograph shows a particle of goldenrod pollen that has been magnified under a microscope. The scale of the photograph is 900 : 1. Use a ruler to estimate the width in millimeters of the particle.



RAMP DESIGN **Assume that the wheelchair ramps described each have a slope of $\frac{1}{12}$, which is the maximum slope recommended for a wheelchair ramp.**

28. A wheelchair ramp has a 21 foot run. What is its rise?

29. A wheelchair ramp rises 4 feet. What is its run?

30. **STATISTICS** Researchers asked 4887 people to pick a number between 1 and 10. The results are shown in the table below.

Answer	1	2	3	4	5
Percent	4.2%	5.1%	11.4%	10.5%	10.7%
Answer	6	7	8	9	10
Percent	10.0%	27.2%	8.8%	6.0%	6.1%

a. Estimate the number of people who picked the number 3.

b. You ask a participant what number she picked. Is the participant more likely to answer 6 or 7? *Explain*.

c. Conduct this experiment with your classmates. Make a table in which you compare the new percentages with the ones given in the original survey. Why might they be different?

ALGEBRA **Use algebra to verify the property of proportions.**

31. Property 2

32. Property 3

33. Property 4

REASONING **Use algebra to *explain* why the property of proportions is true.**

34. If $\frac{a-b}{a+b} = \frac{c-d}{c+d}$, then $\frac{a}{b} = \frac{c}{d}$.

35. If $\frac{a+c}{b+d} = \frac{a-c}{b-d}$, then $\frac{a}{b} = \frac{c}{d}$.

36. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = \frac{a}{b}$. (*Hint:* Let $\frac{a}{b} = r$.)

37. **CHALLENGE** When fruit is dehydrated, water is removed from the fruit. The water content in fresh apricots is about 86%. In dehydrated apricots, the water content is about 75%. Suppose 5 kilograms of raw apricots are dehydrated. How many kilograms of water are removed from the fruit? What is the approximate weight of the dehydrated apricots?

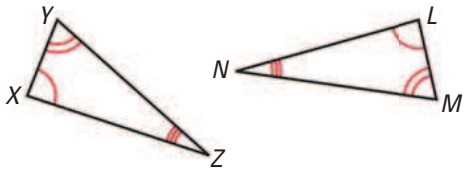
MIXED REVIEW

38. Over the weekend, Claudia drove a total of 405 miles, driving twice as far on Saturday as on Sunday. How far did Claudia travel each day? *(p. 65)*

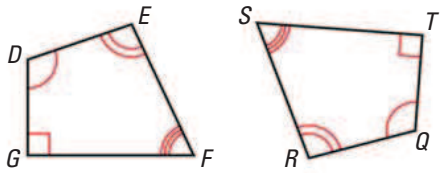
PREVIEW
Prepare for Lesson 6.3 in Exs. 39–40.

Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures. *(p. 225)*

39. $\triangle XYZ \cong \triangle LMN$



40. $DEFG \cong QRST$



QUIZ *for Lessons 6.1–6.2*

Solve the proportion. *(p. 356)*

1. $\frac{10}{y} = \frac{5}{2}$ **2.** $\frac{x}{6} = \frac{9}{3}$ **3.** $\frac{1}{a+3} = \frac{4}{16}$ **4.** $\frac{6}{d-6} = \frac{4}{8}$

Copy and complete the statement. *(p. 364)*

5. If $\frac{9}{x} = \frac{5}{2}$, then $\frac{9}{5} = \frac{?}{?}$.

6. If $\frac{x}{15} = \frac{y}{21}$, then $\frac{x}{y} = \frac{?}{?}$.

7. If $\frac{x}{8} = \frac{y}{12}$, then $\frac{x+8}{8} = \frac{?}{?}$.

8. If $\frac{32}{5} = \frac{x}{y}$, then $\frac{37}{5} = \frac{?}{?}$.

9. In the diagram, $AD = 10$, B is the midpoint of $\overline{AD}$, and AC is the geometric mean of AB and AD. Find AC. *(p. 364)*

EXTRA PRACTICE for Lesson 6.2, p. 906 **ONLINE QUIZ** at classzone.com