Algebra 2: Notes 6.1:

Evaluate n[th] Roots, Use Rational Exponents, and Apply Properties of Rational Exponents. **GOALS:** “”

What are the properties of exponents?

REAL nth ROOTS OF a

Let n be an integer ($n > 1$) and let a be a real number.

If n is an even integer:	If n is an odd integer:
• $a < 0$ No real nth roots.	• $a < 0$ One real nth root: $\sqrt[n]{a} = a^{1/n}$
• $a = 0$ One real nth root: $\sqrt[n]{0} = 0$	• $a = 0$ One real nth root: $\sqrt[n]{0} = 0$
• $a > 0$ Two real nth roots: $\pm\sqrt[n]{a} = \pm a^{1/n}$	• $a > 0$ One real nth root: $\sqrt[n]{a} = a^{1/n}$

PROPERTIES OF RATIONAL EXPONENTS

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those in Lesson 5.1, but now apply to rational exponents.

Property	
1. $a^m \cdot a^n = a^{m+n}$	$4^{1/2} \cdot 4^{3/2} = 4^{(1/2 + 3/2)} = 4^2 = 16$
2. $(a^m)^n = a^{mn}$	$(2^{5/2})^2 = 2^{(5/2 \cdot 2)} = 2^5 = 32$
3. $(ab)^m = a^m b^m$	$(16 \cdot 4)^{1/2} = 16^{1/2} \cdot 4^{1/2} = 4 \cdot 2 = 8$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$	$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{3^{5/2}}{3^{1/2}} = 3^{(5/2 - 1/2)} = 3^2 = 9$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{8}\right)^{1/3} = \frac{27^{1/3}}{8^{1/3}} = \frac{3}{2}$

PROPERTIES OF RADICALS

Product Property of Radicals	Quotient Property of Radicals
$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$

Rewriting nth roots:

implied as is a square root (called the root)

1. $\sqrt{7} = \sqrt[2]{7^1}$ ← all exponets not written $= 7^{\frac{1}{2}}$ ← root is always denominator

2. $\sqrt[5]{x} = \sqrt[5]{x^1} = x^{\frac{1}{5}}$

3. $\sqrt[6]{y^3} = y^{\frac{3}{6}}$ ← be sure to reduce $= y^{\frac{1}{2}}$

Rewrite the following as an nth root and simplify:

4. n=3 (root), a=-64 $\Rightarrow \sqrt[3]{-64} = \sqrt[3]{-1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[3]{-1} \cdot \sqrt[3]{2^6} = (-1)^{\frac{1}{3}} \cdot 2^{\frac{6}{3}}$ ← and reduce

$= (-1)^{\frac{3}{3}} 2^2$

- can be rewritten because $(-1)^1 = (-1)^3$

$-1 = -1 \cdot -1 \cdot -1$

$-1 = -1$

:)

$= -1 \cdot 4 = -4$

5. n=6, a=729

$\sqrt[6]{729} = \sqrt[6]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$

$= \sqrt[6]{3^6}$

rewrite

$= 3^{\frac{6}{6}}$

$= 3^1 = 3$

Evaluate an expression w/rational exponents.

Evaluate $8^{-\frac{4}{3}}$

Two Ways:

Rational exponent form:

$$8^{-\frac{4}{3}} = \frac{1}{8^{\frac{4}{3}}}$$

Prime factors of base $= \dfrac{1}{(2^3)^{\frac{4}{3}}}$

Simplify powers $= \dfrac{1}{2^4}$

calculate $= \boxed{\dfrac{1}{16}}$

Radical form:

$$8^{-\frac{4}{3}} = \frac{1}{8^{\frac{4}{3}}}$$

rewrite as radical $= \dfrac{1}{\sqrt[3]{8^4}}$

Prime factors $= \dfrac{1}{\sqrt[3]{(2^3)^4}}$

simplify $= \dfrac{1}{\sqrt[3]{2^{12}}} = \dfrac{1}{2^{\frac{12}{3}}} = \dfrac{1}{2^4}$

reduce $= \boxed{\dfrac{1}{16}}$

Solve equations with nth roots.

Goals

1. Isolate exponent
2. Take the root of both sides.
3. Solve for x (keep in mind that even roots have two solutions "±" and odd only one)

a. $2x^6 = 1458$

Isolate Exp. $x^6 = 729$

Take Root $\sqrt[6]{x^6} = \sqrt[6]{729}$

$x = \sqrt[6]{3^6}$

$x = 3$

b. $(x + 4)^3 = 12$

Treat x+4 as a group and take roots

$\sqrt[3]{(x+4)^3} = \sqrt[3]{12}$

$x + 4 = \sqrt[3]{12}$

$x = -4 + \sqrt[3]{12}$

$x \approx -4 + 2.29$

$x \approx -1.71$

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