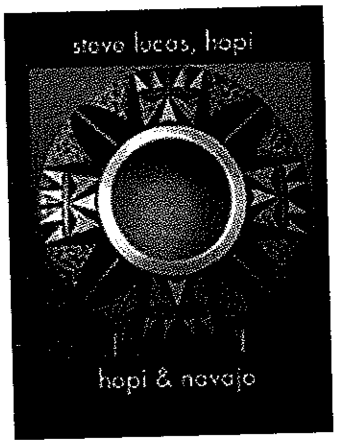
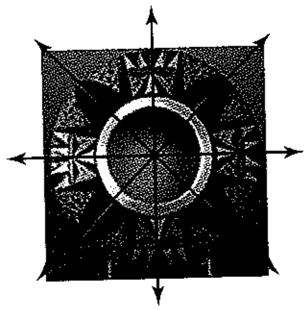
SOLUTION OF THE INITIAL PROBLEM



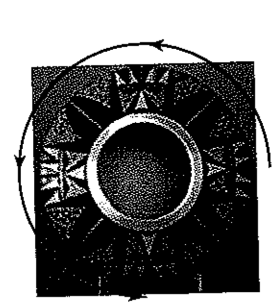
Source: Native American Collections, www.nativepots.com/Photo by Bill Bonebrake, Artist: Steve Lucas, Hopi

The art, pottery, and basketry of many cultures feature patterns of repeated geometric figures and can often be described according to their symmetries. Shown in the lower pottery figure on the left is the top view of a pot created by the Hopi artist, Steve Lucas, using traditional methods. While Native American pottery was originally designed for ceremonial or household use, today most such pottery is collected as art. Symmetry is a common design element in Hopi pottery. Describe the symmetries of the pot shown here from Native American Collections Contemporary Pottery at www.nativepots.com.

SOLUTION This pot has four reflection symmetries and three rotation symmetries as shown in the following two figures.



There are four lines of reflection symmetry.



There are three rotation symmetries: 90°, 180°, and 270°.

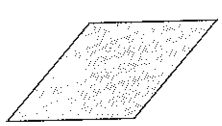
Source: Native American Collections, www.nativepots.com/Photo by Bill Bonebrake, Artist: Steve Lucas, Hopi

PROBLEM SET 2.2

1. **a.** Does the following rectangle have reflection symmetry? Sketch all lines of reflection symmetry.

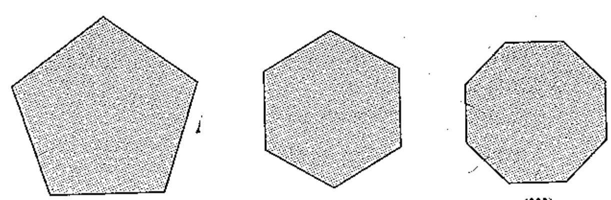
b. Does the rectangle have rotation symmetry? If so, list any rotation symmetries (that is, the angles of rotation), and identify the center of rotation.

2. **a.** Does the following rhombus have reflection symmetry? Sketch all lines of reflection symmetry.



b. Does the rhombus have rotation symmetry? If so, list any rotation symmetries (that is, the angles of rotation), and identify the center of rotation.

3. **a.** Draw all of the lines of reflection symmetry in each of the following regular n-gons. How many lines of reflection symmetry does each have?



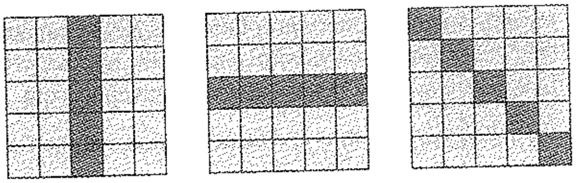
(i) (ii) (iii)

b. Use the results from part (a) to determine how many lines of reflection symmetry a regular n-gon has.

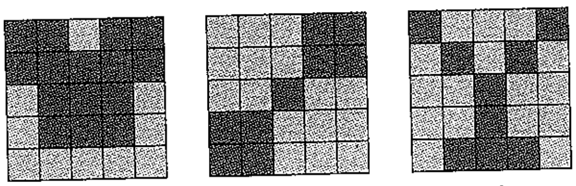
4. **a.** Trace the regular pentagon, hexagon, and octagon in the previous problem, and describe all of the rotation symmetries for those three figures.

b. How many rotation symmetries does a regular n-gon have?

5. Bingo is played on a 5-by-5 grid of squares in which certain squares must be covered in order to win, usually five squares in a row vertically, horizontally, or diagonally, as shown next.

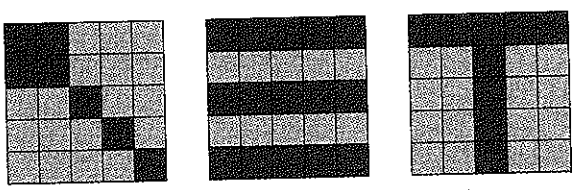


To make bingo games more interesting, other grid patterns are often chosen to be winners. Several examples are shown. For each one, tell whether the pattern has reflection symmetry, rotation symmetry, or both.



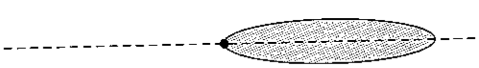
Mickey Mouse (a) | Bowtie (b) | Sherbet glass (c)

6. For each of the following Bingo patterns, tell whether the pattern has reflection symmetry, rotation symmetry, or both.



Kite and tail (a) | Layer cake (b) | Letter "T" (c)

7. Complete the figure shown next, assuming that it has the following rotation symmetries about the given point. (*Hint:* Use a protractor and a ruler or compass.)



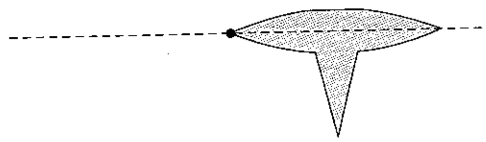
a. 180° rotation symmetry

b. 90°, 180°, and 270° rotation symmetries

c. 120° and 240° rotation symmetries

d. Which of the completed figures from parts (a) through (c) have reflection symmetry through the dashed horizontal line?

8. Complete the next figure assuming that it has the following rotation symmetries about the given point. (*Hint:* Use a protractor.)



a. 180° rotation symmetry

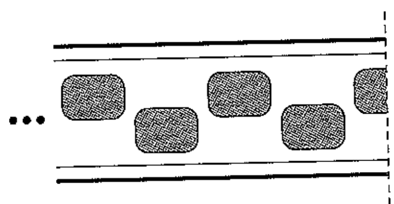
b. 90°, 180°, and 270° rotation symmetries

c. 120° and 240° rotation symmetries

d. Which of the completed figures from parts (a) through (c) have reflection symmetry through the dashed horizontal line?

9. Half of a strip pattern is shown. The pattern extends indefinitely in both directions. Complete the strip pattern given that it has reflection symmetry with respect to the dashed horizontal line. Then use crystallographic notation to classify the completed pattern.

10. Half of a strip pattern is shown. The pattern extends indefinitely to the left. Complete the strip pattern given that it has reflection symmetry with respect to the dashed vertical line. Use crystallographic notation to classify the completed pattern.



Problems 11 and 12

Consider the strip patterns shown and the symmetries described in this section. Describe the symmetries, if any, of each strip pattern and use crystallographic notation to classify the completed pattern.

11.

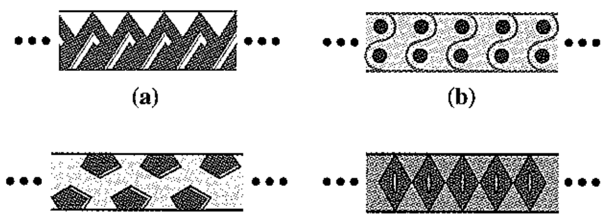
(a)

(b)

(c)

(d)

12.



(a) (b) (c) (d)

13. Consider the following letters of the alphabet.

A B C D E F G H I J K L M

a. Which letter(s) have vertical reflection symmetry?

b. Which letter(s) have horizontal reflection symmetry?

c. Which letter(s) have rotation symmetry?

14. Consider the following letters of the alphabet.

N O P Q R S T U V W X Y Z

a. Which letter(s) have vertical reflection symmetry?

b. Which letter(s) have horizontal reflection symmetry?

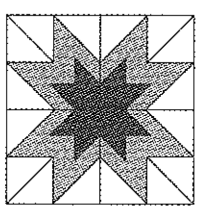
c. Which letter(s) have rotation symmetry?

Problems 15 and 16

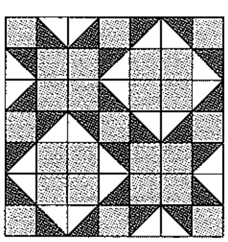
a. How many lines of reflection symmetry does the pattern have? Sketch all lines of reflection symmetry.

b. How many rotation symmetries does the pattern have? Identify the center of rotation and all angles of rotation.

15. Consider the following pattern.



16. Consider the following pattern.



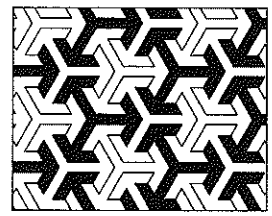
Problems 17 and 18

a. Draw four lines of reflection symmetry for the pattern.

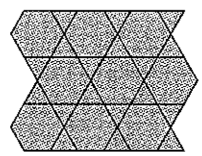
b. Specify four rotation symmetries. Be sure to identify the point(s) about which the pattern rotates and the angles of rotation.

c. Specify three translations under which the pattern is unchanged. Be sure to describe each translation using a vector.

17. Consider the following pattern. Assume it continues indefinitely in all directions.



18. Consider the following pattern. Assume it continues indefinitely in all directions.



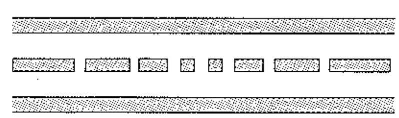
Problems 19 and 20

a. Describe or sketch all lines of reflection symmetry.

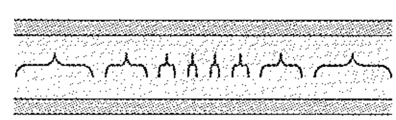
b. Describe all rotation symmetries, if any.

c. Does the strip pattern have translation symmetry? Explain.

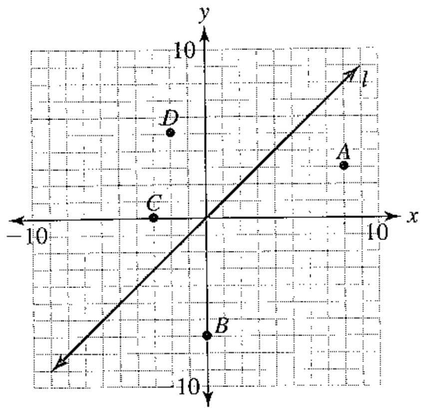
19. Consider the following strip pattern and assume that it continues indefinitely in both directions.



20. Consider the following strip pattern and assume that it continues indefinitely in both directions.



21. Consider the following points: $A(8, 3)$, $B(0, -7)$, $C(-3, 0)$, and $D(-2, 5)$.

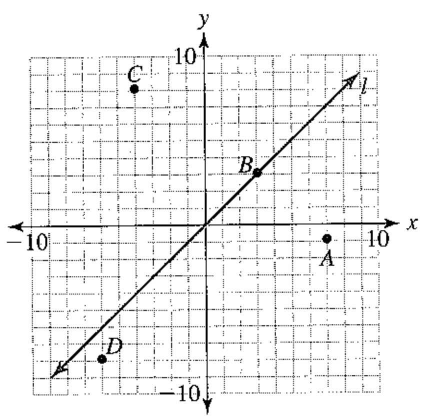


a. Find the coordinates of the images of points A, B, C, and D under a reflection with respect to the x-axis.

b. Find the coordinates of the images of points A, B, C, and D under a reflection with respect to the y-axis.

c. Find the coordinates of the images of points A, B, C, and D under a reflection with respect to the line l.

22. Consider the following points: $A(7, -1)$, $B(3, 3)$, $C(-4, 8)$, and $D(-6, -8)$.



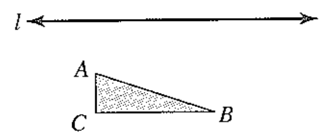
a. Find the coordinates of the images of points A, B, C, and D under a reflection with respect to the x-axis.

b. Find the coordinates of the images of points A, B, C, and D under a reflection with respect to the y-axis.

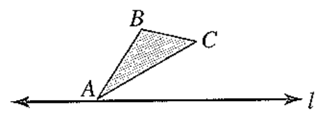
c. Find the coordinates of the images of points A, B, C, and D under a reflection with respect to the line l.

23. For each triangle shown, use a protractor, a compass, and the definition of a reflection to draw the image of $\triangle ABC$ under the reflection with respect to line l.

(a)

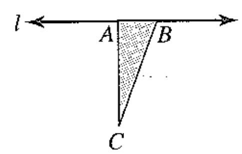


(b)

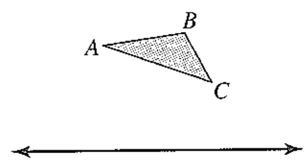


24. For each triangle shown, use a protractor, a compass, and the definition of a reflection to draw the image of $\triangle ABC$ under the reflection with respect to line l.

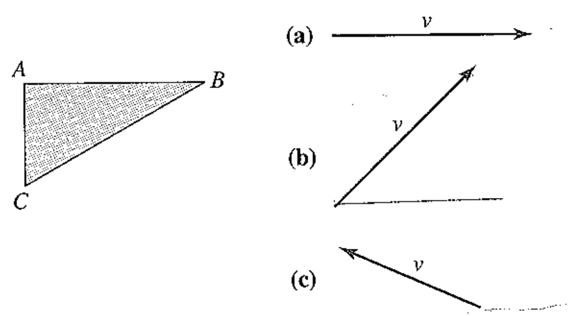
(a)



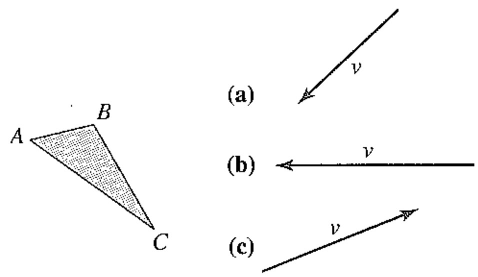
(b)



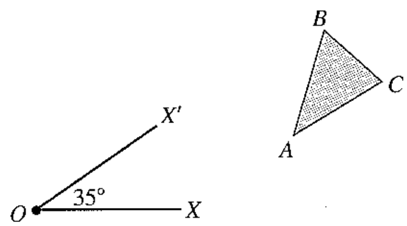
25. For each vector v, copy $\triangle ABC$ and draw the image of $\triangle ABC$ under the translation determined by the vector.



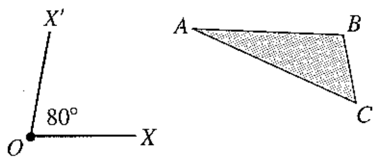
26. For each vector v, copy $\triangle ABC$ and draw the image of $\triangle ABC$ under the translation determined by the vector.



27. Use a compass and a protractor to draw the image of $\triangle ABC$ under the rotation with center O and directed angle $\angle XOX'$.

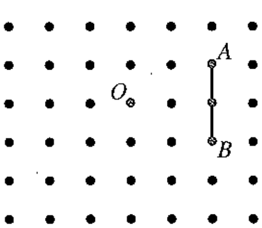


28. Use a compass and a protractor to draw the image of $\triangle ABC$ under the rotation with center O and directed angle $\angle XOX'$.



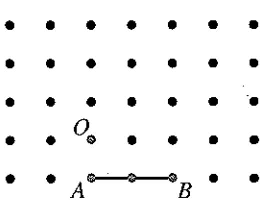
29. Find the image of $\overline{AB}$ under the rotation about point O with the given angle of rotation.

(a) $-90°$
(b) $90°$
(c) $180°$

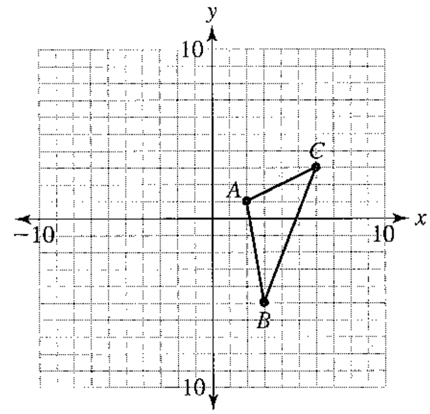


30. Find the image of $\overline{AB}$ under the rotation about point O with the given angle of rotation.

(a) $-90°$
(b) $90°$
(c) $180°$

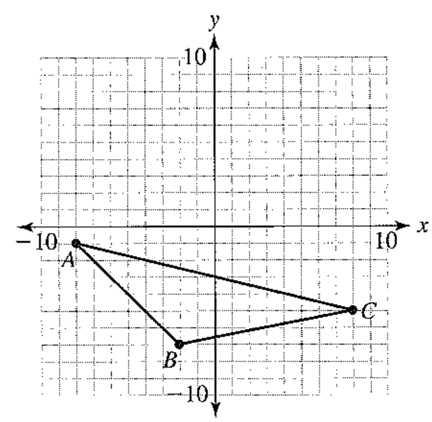


31. a. The given triangle, $\triangle ABC$, has vertex points $A(2, 1)$, $B(3, -5)$, and $C(6, 3)$. Graph the image of $\triangle ABC$ under the reflection with respect to the y-axis. Give the coordinates of the vertices after the reflection.



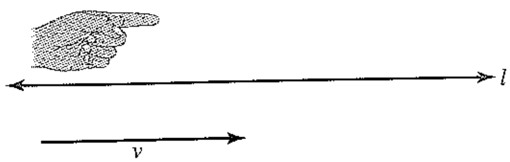
b. If a point P has coordinates (a, b), what are the coordinates of its image under the reflection with respect to the y-axis?

32. a. The given triangle, $\triangle ABC$, has vertex points $A(-8, -1)$, $B(-2, -7)$ and $C(8, -5)$. Graph the image of $\triangle ABC$ under the reflection with respect to the x-axis. Give the coordinates of the vertices after the reflection.

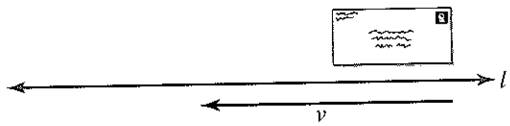


b. If a point P has coordinates (a, b), what are the coordinates of its image under the reflection with respect to the x-axis?

33. Sketch the figure that would be obtained from the glide reflection composed of a reflection with respect to line l followed by the translation to the right determined by the vector v.

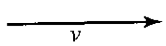


34. Sketch the figure that would be obtained from the glide reflection composed of a reflection with respect to line l followed by the translation to the left determined by the vector v.



35. Make a copy of the following figures.

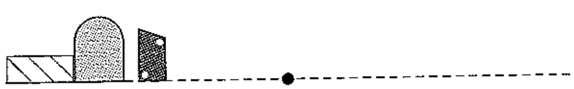
a. Translate the figures to the right three times, each time using the translation determined by the vector v. Leave a copy of the resulting figures in place.



b. Using the completed pattern from part (a), reflect the pattern over the dashed horizontal line leaving a copy of the pattern in place.

c. Describe the symmetries of the strip pattern obtained in part (b).

36. Make a copy of the following figure.

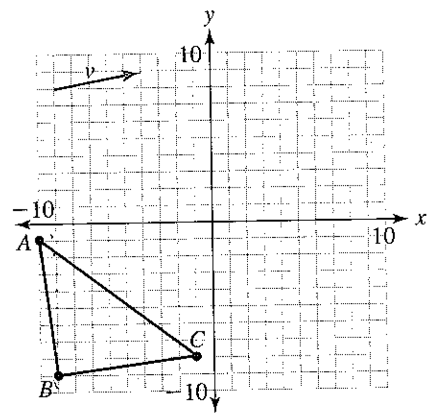


a. Rotate the figures 180°, with the black dot as the center of rotation. Leave a copy of the resulting figures in place.

b. Using the completed pattern from part (a), reflect the pattern over a vertical line that passes through the black dot that is the center of rotation. Leave a copy of the pattern in place.

c. Describe the symmetries of the strip pattern obtained in part (b).

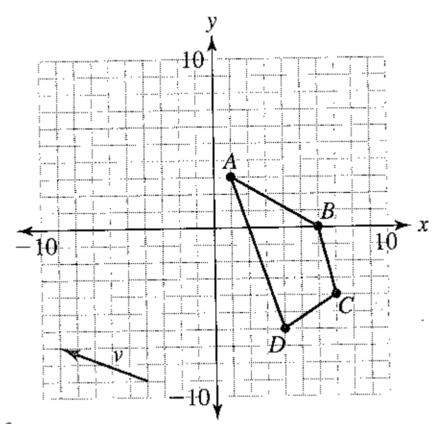
37. Consider $\triangle ABC$ and vector v.



a. Translate $\triangle ABC$ according to vector v and give the coordinates of the vertices of $\triangle A'B'C'$.

b. Perform the glide reflection composed of a reflection of $\triangle ABC$ with respect to the x-axis followed by a translation defined by vector v. Give the coordinates of the vertices of $\triangle A''B''C''$.

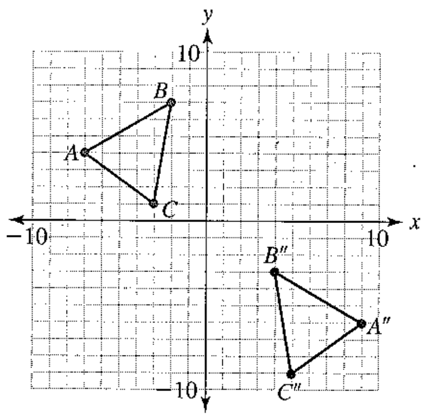
38. Consider quadrilateral $ABCD$ and vector v.



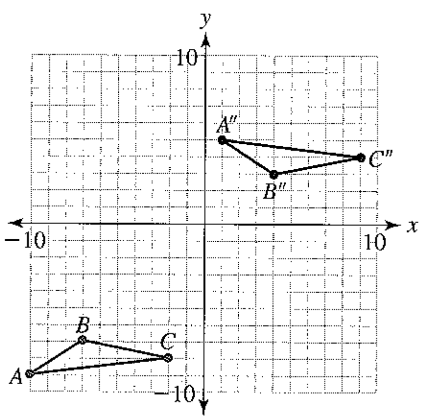
a. Translate quadrilateral $ABCD$ according to vector v and give the coordinates of the vertices of quadrilateral $A'B'C'D'$.

b. Perform the glide reflection composed of a reflection of quadrilateral $ABCD$ with respect to the y-axis followed by a translation determined by vector v. Give the coordinates of the vertices of quadrilateral $A''B''C''D''$.

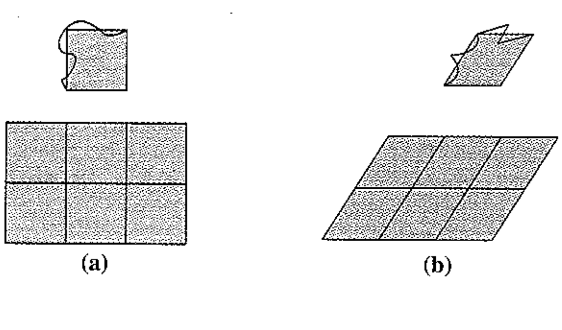
39. Specify a glide reflection that will transform $\triangle ABC$ to $\triangle A''B''C''$. Indicate the line of reflection and use a vector to indicate the distance and direction of the translation.



40. Specify a glide reflection that will transform $\triangle ABC$ to $\triangle A''B''C''$. Indicate the line of reflection and use a vector to indicate the distance and direction of the translation.

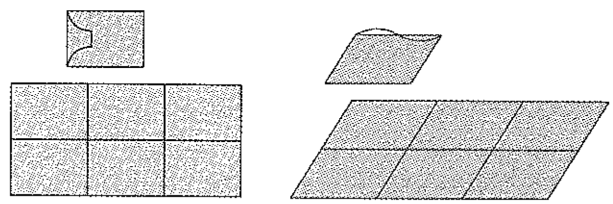


41. Create two Escher-type tilings with irregular shapes. Shown above each of the following grids is a basic shape with a curve on the top and a curve on the left side. To create a new basic shape, translate the left curve and the top curve in the figure to their opposite sides to create a shape that tiles the plane. Use a grid like the ones shown to verify that the new shape will tile the plane. Add color to your finished design.



(a) (b)

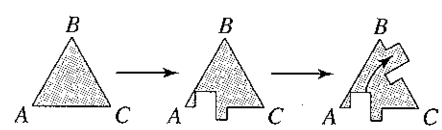
42. Create Escher-type tilings with irregular shapes. Shown above each of the following grids is a basic shape with a curve on the left or a curve on the top. For each figure shown, translate the curve to the opposite side of the figure to create a shape that tiles the plane. Use a grid like the one shown to verify that the shape will tile the plane. Add color to your finished design.



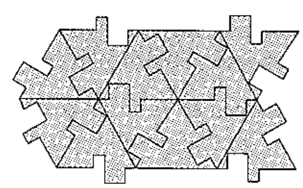
(a) (b)

Problems 43 and 44

Rotations can also be used to make Escher-type drawings. The following figure shows a triangle that has been altered and the alteration rotated to produce an Escher-type pattern.



(a) (b) (c)



Side $\overline{AC}$ of $\triangle ABC$ in (a) is altered arbitrarily in (b). Notice that points A and C have not been moved. Then, using point C as the center of a rotation, the altered version of $\overline{AC}$ is rotated so that A is rotated to B in (c). The result is an alteration of $\overline{BC}$. The shape will tile the plane as shown.

43. Using the preceding figure as an example, alter $\overline{AC}$ in a different way, and then rotate it to $\overline{BC}$ using C as center of rotation. Cut out at least 12 copies of your design and paste them together to demonstrate how they will tile the plane.