

2.5 Reason Using Properties from Algebra



- Before** You used deductive reasoning to form logical arguments.
- Now** You will use algebraic properties in logical arguments too.
- Why** So you can apply a heart rate formula, as in Example 3.

- Key Vocabulary**
- **equation**, p. 875
 - **solve an equation**, p. 875

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

KEY CONCEPT

For Your Notebook

Algebraic Properties of Equality

Let a , b , and c be real numbers.

Addition Property If $a = b$, then $a + c = b + c$.

Subtraction Property If $a = b$, then $a - c = b - c$.

Multiplication Property If $a = b$, then $ac = bc$.

Division Property If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Substitution Property If $a = b$, then a can be substituted for b in any equation or expression.

EXAMPLE 1 Write reasons for each step

Solve $2x + 5 = 20 - 3x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 5 = 20 - 3x$	Write original equation.	Given
$2x + 5 + 3x = 20 - 3x + 3x$	Add $3x$ to each side.	Addition Property of Equality
$5x + 5 = 20$	Combine like terms.	Simplify.
$5x = 15$	Subtract 5 from each side.	Subtraction Property of Equality
$x = 3$	Divide each side by 5.	Division Property of Equality

► The value of x is 3.

Distributive Property

$a(b + c) = ab + ac$, where a , b , and c are real numbers.

EXAMPLE 2 Use the Distributive Property

Solve $-4(11x + 2) = 80$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(11x + 2) = 80$	Write original equation.	Given
$-44x - 8 = 80$	Multiply.	Distributive Property
$-44x = 88$	Add 8 to each side.	Addition Property of Equality
$x = -2$	Divide each side by -44 .	Division Property of Equality

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EXAMPLE 3 Use properties in the real world

HEART RATE When you exercise, your target heart rate should be between 50% to 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a .

Solution

Equation	Explanation	Reason
$r = 0.70(220 - a)$	Write original equation.	Given
$r = 154 - 0.70a$	Multiply.	Distributive Property
$r - 154 = -0.70a$	Subtract 154 from each side.	Subtraction Property of Equality
$\frac{r - 154}{-0.70} = a$	Divide each side by -0.70 .	Division Property of Equality

**GUIDED PRACTICE** for Examples 1, 2, and 3

In Exercises 1 and 2, solve the equation and write a reason for each step.

1. $4x + 9 = -3x + 2$

2. $14x + 3(7 - x) = -1$

3. Solve the formula $A = \frac{1}{2}bh$ for b .

PROPERTIES The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for segment lengths and angle measures.

KEY CONCEPT

For Your Notebook

Reflexive Property of Equality

- Real Numbers** For any real number a , $a = a$.
- Segment Length** For any segment AB , $AB = AB$.
- Angle Measure** For any angle A , $m\angle A = m\angle A$.

Symmetric Property of Equality

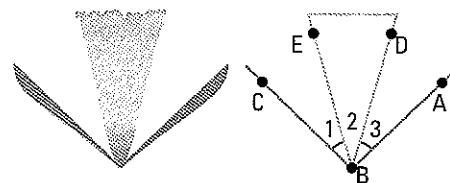
- Real Numbers** For any real numbers a and b , if $a = b$, then $b = a$.
- Segment Length** For any segments AB and CD , if $AB = CD$, then $CD = AB$.
- Angle Measure** For any angles A and B , if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

- Real Numbers** For any real numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.
- Segment Length** For any segments AB , CD , and EF , if $AB = CD$ and $CD = EF$, then $AB = EF$.
- Angle Measure** For any angles A , B , and C , if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Use properties of equality

LOGO You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$.



Solution

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	Given
$m\angle EBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle DBC$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle DBC$	Both measures are equal to the sum of $m\angle 1 + m\angle 2$.	Transitive Property of Equality