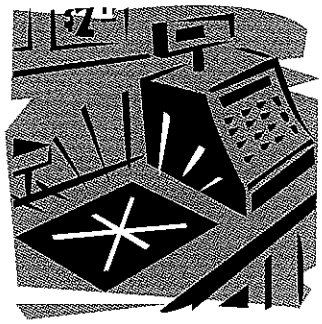
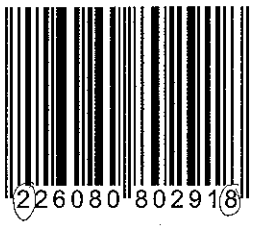
SOLUTION OF THE INITIAL PROBLEM



Assuming the grocery store scanner is working properly, can you be confident that you will be charged the correct price for the item with the Universal Product Code shown next?



PRICE PER LB.	NET WT LBS.
3.88	0.75
TOTAL PRICE	2.91

SOLUTION The UPC for this item is 2 26080 80291 8. According to Table 1.3, the initial digit of 2 indicates that the item is sold by weight, which you can also tell by reading that it costs \$3.88 per pound and has a net weight of 0.75 pound. When the product was weighed, a machine produced a UPC label that indicates the price of \$2.91 with the digits 291 in positions 9 through 11. To verify that the last 8 is the correct digit, we can use the UPC check-digit scheme. We find that

$$\begin{aligned} 3(2 + 6 + 8 + 8 + 2 + 1) + 1(2 + 0 + 0 + 0 + 9 + 8) &= 3(27) + 1(19) \\ &= 81 + 19 \\ &= 100, \end{aligned}$$

is divisible by 10, as it should be. Thus, you can be confident that the price will be read correctly because of the presence of the check digit 8 in position 12 of the UPC.

This sample UPC also illustrates the flexibility of the UPC. In this case, the digits in positions 9 through 11 represent the cost of the item instead of being merely 3 digits of a 5-digit group that specifies a particular product.

PROBLEM SET 1.1

Problems 1 and 2

Determine whether each of the social security numbers could be a valid number issued today. For any number that is valid, what state would be listed in the mailing address of the application for the social security number?

1. **a.** 409-32-2174 **b.** 543-12-1926
   c. 885-04-3179 **d.** 615-00-4454
   e. 030-09-5397 **f.** 700-32-3231
2. **a.** 408-71-0000 **b.** 585-71-3179
   c. 615-03-4454 **d.** 001-01-0001
   e. 466-32-3231 **f.** 800-42-6389

3. Suppose a four-digit code is created for each person in a group so that the fourth digit is assigned to make the sum of all four digits divisible by 7. The digits 0 through 9 may be used.
   a. Is 3380 a valid identification number under this system?
   b. What would the fourth digit have to be to make 891 a valid four-digit identification number under this system?
   c. How will a four-digit identification number be recognizable as an invalid number?
   d. If a single-digit error is made in the number 2453, how will the error be detected?
   e. Under what circumstance will a single-digit error not be detected? Give an example.

4. Suppose a four-digit code is created for each person in a group so that the fourth digit is assigned to make the sum of all four digits divisible by 8. The digits 0 through 8 may be used.
   a. Is 2934 a valid identification number under this system?
   b. What would the fourth digit have to be to make 861 a valid four-digit identification number under this system?
   c. How will a four-digit identification number be recognizable as an invalid number?
   d. If a single-digit error is made in the number 7522, how will the error be detected?
   e. Under what circumstance will a single-digit error not be detected? Give an example.

5. Suppose that a five-digit code consists of four digits and a check digit assigned so that the sum of the five digits is divisible by 9. Assume that the digits 0 through 9 may be used in the first four digits of the code. When the sum of the first four digits is already divisible by 9, assume that 0 is always selected for the check digit. Determine whether the following errors would be detected and explain why they could or could not be detected.
   a. The third and fourth digits have been transposed.
   b. The third digit was supposed to be a 9 but was recorded as a 0.
   c. The first digit is recorded as an 8 rather than a 3.
   d. The check digit is recorded as a 9.

6. Suppose that a five-digit code consists of four digits and a check digit that is assigned so that the sum of the five digits is divisible by 8. Assume that the digits 0 through 8 may be used in the code. When the sum of the first four digits is already divisible by 8, assume that 0 is always selected for the check digit. Determine whether the following errors would be detected and explain why.
   a. The first and second digits have been transposed.
   b. The third digit was supposed to be a 0 but was recorded as an 8.
   c. The fourth digit is recorded as a 1 rather than a 5.
   d. The check digit is recorded as an 8.

7. A company creates a five-digit identification number using the digits 0 through 9. The fifth digit is the check digit. The check digit will be chosen to be the smallest number that will make the sum of the digits even.
   a. Determine whether all single-digit errors will be detected. Justify your answer.
   b. Determine whether all adjacent transpositions will be detected. Justify your answer.

8. A company creates a five-digit identification number using the digits 0 through 9. The fifth digit is the check digit. The check digit will be chosen to be the smallest number that will make the sum of all the digits divisible by 5.
   a. Determine whether all single-digit errors will be detected. Justify your answer.
   b. Determine whether all adjacent transpositions will be detected. Justify your answer.

Problems 9 through 14

Suppose you create a seven-digit identification number so that the seventh digit is the check digit. The check digit is the remainder when a weighted sum of the first six digits is divided by 7. The weighted sum is formed by multiplying each of the first six digits by the digit's position. Thus, the weighted sum $= 1d_1 + 2d_2 + 3d_3 + 4d_4 + 5d_5 + 6d_6$. For example, if the first six digits of an identification number are 208455, then the weighted sum is $1(2) + 2(0) + 3(8) + 4(4) + 5(5) + 6(5) = 2 + 0 + 24 + 16 + 25 + 30 = 97$. When the weighted sum is divided by 7, the remainder is 6. The complete identification number is 2084556.

9. The first six digits of an identification number are given. Calculate the check digit in each case.
   a. 212648 b. 977425 c. 105063

10. The first six digits of an identification number are given. Calculate the check digit in each case.
    a. 534712 b. 906725 c. 446091

11. Use the check-digit scheme defined above to create an example of a valid identification number with the given requirement(s).
    a. The check digit is 5.
    b. No zeroes are used in the number, and the check digit is 3.
    c. All the digits are the same.

12. Use the check-digit scheme defined above to create an example of a valid identification number with the given requirement(s).
    a. The check digit is 6.
    b. No digits are repeated, and the check digit is 1.
    c. The first six digits are the same.

13. You made a mistake when recording an identification number. The number 9104733 was recorded rather than 2104733.
    a. What type of error was made?
    b. Will this error be detected? Why or why not?

14. You made a mistake when recording an identification number. The number 0724311 was recorded rather than 7024311.
    a. What type of error was made?
    b. Will this error be detected? Why or why not?

Problems 15 through 20

The ISBN uses a weighted sum of the first nine digits in the number to create the check digit, which is used to detect single-digit errors or the transposition of two digits. Each digit is weighted, by multiplying it by a constant, according to its position in the number. The weighted sum is found by the following formula:

$$\text{Weighted sum} = 10d_1 + 9d_2 + 8d_3 + 7d_4 + 6d_5 + 5d_6 + 4d_7 + 3d_8 + 2d_9$$

The check digit is the remainder when the weighted sum is divided by 11. In the case of a remainder of 10, the check digit is an "X."

15. Calculate the check digit for each of the following partial ISBNs.

a. 0-24-361427 **b.** 3-92-392206

c. 1-56-554108 **d.** 0-13-639444

16. Calculate the check digit for each of the following partial ISBNs.

a. 2-09-202101 **b.** 0-06-028548

c. 1-87-760346 **d.** 0-87-580218

17. In each of the following ISBNs, a scanner made a single-digit error, but the check digit is correct. Correct the error in as many ways as possible.

a. 2-08-152852-X **b.** 1-88-342354-0

18. In each of the following ISBNs, a scanner made a single-digit error, but the check digit is correct. Correct the error in as many ways as possible.

a. 9-68-380983-9 **b.** 0-08-720514-3

19. a. In the ISBN 0-31-011690-7, a bookstore employee made an adjacent transposition error, but the check digit is correct. Correct the error.

b. For an ISBN, suppose the check digit was calculated as the remainder when the weighted sum is divided by 5. What single-digit errors would not be detected?

20. a. In the ISBN 0-13-891448-X, a bookstore employee made an adjacent transposition error, but the check digit is correct. Correct the error.

b. For an ISBN, suppose the check digit was calculated as the remainder when the weighted sum is divided by 8. What single-digit errors would not be detected?

Problems 21 through 24

Many credit cards use a weighted even/odd code called the **LUHN formula**. Both MasterCard and Discover use a 16-digit code. The check digit is the 16th digit and is the number that must be added to the weighted sum to make it divisible by 10. To find the weighted sum, begin with the second digit from the right (the digit next to the check digit) and multiply every other digit by 2. When multiplying by 2, if a two-digit number results, then add the two digits before finding the weighted sum.

For example, the check digit, d, for the Discover card number 6011 2465 0103 721d can be found by calculating the following weighted sum.

$$(2 \times 6) + 0 + (2 \times 1) + 1 + (2 \times 2) + 4 + (2 \times 6) + 5 + (2 \times 0) + 1 + (2 \times 0) + 3 + (2 \times 7) + 2 + (2 \times 1) = (12) + 0 + (2) + 1 + (4) + 4 + (12) + 5 + (0) + 1 + (0) + 3 + (14) + 2 + (2)$$

Notice that there are three two-digit numbers, namely 12, 12, and 14, that resulted from multiplying by 2. Add the digits in each of those cases.

$$(3) + 0 + (2) + 1 + (4) + 4 + (3) + 5 + (0) + 1 + (0) + 3 + (5) + 2 + (2) = 35$$

Because the weighted sum is 35, the check digit must be 5 to yield $35 + 5 = 40$, which is divisible by 10.

21. a. Is the credit card number, 6011 9826 3451 7117, valid under the check-digit scheme we just defined?

b. Use the check-digit scheme we just defined to find the check digit for the credit card number 6011 4533 8956 875.

22. a. Is the credit card number, 5423 9011 8372 1312, valid under the check-digit scheme we just defined?

b. Use the check-digit scheme we just defined to find the check digit for the credit card number 5532 9014 7389 237.

23. a. Use the check-digit scheme we just defined to find the missing digit for the credit card number 5582 19?4 4232 8673.

b. For the credit card number 6011 9783 0912 1359, will a transposition of the digits in the 9th and 10th positions be detected? Why or why not?

24. a. Use the check-digit scheme we just defined to find the missing digit for the credit card number 6011 3224 5?13 0952.

b. For the credit card number 6011 9783 0912 1359, will a transposition of the digits in the 4th and 5th positions be detected? Why or why not?

Problems 25 and 26

The bar codes and the first 11 digits of the UPCs for three products are given. For each partial UPC, (i) identify the manufacturer, (ii) identify the product number, and (iii) calculate the missing check digit.

25.

(a) (b) (c)

26.

(a) (b) (c)

27. A digit is missing from each of the following UPC numbers. Find the missing digit in each case.

a. ? 47000 21300 2

b. 2 97030 1823? 1

c. 0 21000 3?851 0

28. A digit is missing from each of the following UPC numbers. Find the missing digit in each case.

a. ? 73909 11220 6

b. 3 81130 18?34 3

c. 0 ?3600 32118 9

29. For the UPC number, 0 70330 50606 0, will every single-digit error be detected? Explain.

30. For the UPC number, 0 70330 50606 0, will every adjacent transposition error be detected? Explain.

Problems 31 and 32

The UPC and many credit card check digits are calculated using a two-weight scheme. Consider other two-weight schemes. Suppose a five-digit identification number is created so that the fifth digit is the remainder when the weighted sum of the first four digits is divided by 9.

31. The weighted sum is $7d_1 + d_2 + 7d_3 + d_4$.

a. Will all single-digit errors be detected? Explain why or why not.

b. Give an example of an adjacent transposition error that will not be detected.

32. The weighted sum is $2d_1 + 3d_2 + 2d_3 + 3d_4$.

a. Will all single-digit errors be detected? Explain why or why not.

b. Give an example of an adjacent transposition error that will not be detected.

Problems 33 through 36

Personal checks have a **magnetic ink character recognition line (MICR)** across the bottom of the check. This line includes the routing number, the account number, and the number of the check. The first nine digits across the bottom of a check make up the routing number. Of those nine digits, the first four represent the Federal Reserve routing number, the second four identify the institution, and the ninth is the check digit. The check digit of the routing number is the last digit in the following weighted sum:

$$7d_1 + 3d_2 + 9d_3 + 7d_4 + 3d_5 + 9d_6 + 7d_7 + 3d_8.$$

33. Determine the check digit for each of the following routing numbers where the first eight digits have been given.

a. 06311482 **b.** 12200048

34. Determine the check digit for each of the following routing numbers where the first eight digits have been given.

a. 32303047 **b.** 12303427

35. The routing number 323274210 contains an error. The correct routing number is 323274270. Describe the mistake and explain whether the mistake will be detected.

36. The routing number 323724270 contains an error. The correct routing number is 323274270. Describe the mistake and explain whether the mistake will be detected.

37. Suppose the phone company decided to add a check digit as the 11th digit in all phone numbers so that a telephone call would not ring if an error was made when placing the call. For a 10-digit phone number

consisting of a 3-digit area code followed by a 7-digit number, suppose the check digit is calculated as follows (the check digit is the last digit in the sum):

$$1d_1 + 2d_2 + 3d_3 + 1d_4 + 2d_5 + 3d_6 + 1d_7 + 2d_8 + 3d_9 + 1d_{10}.$$

a. Calculate the check digit for the phone number (583) 259-3671.

b. Will all single-digit errors be detected? Explain why or why not.

c. Will all adjacent transposition errors be detected? Explain why or why not.

38. A U.S. postal money order identification number consists of 10 digits and a check digit. The check digit is the remainder when the 10 digits of the number are added and the sum is divided by 9.

a. Calculate the check digit for the money order identification number 2448350176.

b. Will all single-digit errors be detected? Explain why or why not.

c. Will all adjacent transposition errors be detected? Explain why or why not.

Problems 39 through 42

Sorting through long lists of similar names can be tedious and time-consuming. Consider the problem of trying to locate a phone number or other information in a huge database for a "Jim Christianson" or "Jim Christensen" if you do not know the exact spelling of his last name. **Soundex** is a coding system that was created during the Franklin Roosevelt administration in the 1930s and was used by the Works Progress Administration to index names for the U.S. Census. With the Soundex system, a surname can be found even though it may have been recorded under different spellings. Genealogists use the Soundex system and it has been used by the National Archives to index census immigration records. Each Soundex code consists of a letter followed by three numbers. The letter is the first letter of the surname (last name). To determine the numerical part of the code, write out the surname and do the following:

(i) Leave the first letter alone. Then cross out all occurrences of the letters a, e, i, o, u, y, h, w.

(ii) Cross off the second of any double letters.

(iii) Replace each of the remaining letters by numbers according to the following scheme:

Letter	Number Replacement
B, P, F, V	1
C, S, K, G, J, Q, X, Z	2
D, T	3
L	4
M, N	5
R	6

(iv) Use only the first three numbers.

Consider the surnames Christianson and Christensen. They would each be encoded in the following way.

Surname	Christianson	Christensen
Leave the first letter alone. Then cross out all occurrences of the letters a, e, i, o, u, y, h, and w.	Christianson	Christensen
Cross off the second of any double letters.	C r s t n s n	C r s t n s n
Leave the first letter alone, and replace each of the other letters with the appropriate number.	C623525	C623525
Use only the first three numbers.	C623525	C623525
Soundex coded surname	C623	C623

In general, to determine the numbers in the code, note the following. If there are two or more adjacent letters in the surname that would be replaced with the same number, then use the number only once in the code. If the surname is short, or if there are fewer than three numbers to use for the code, merely add zeroes to fill out the code. For example, the surnames Thom, Baddpacks, and Amonskit would be encoded in the following way.

Surname	Thom	Baddpacks	Amonskit
Leave the first letter alone. Then cross out all occurrences of the letters a, e, i, o, u, y, h, and w.	Thom	Baddpacks	Amonskit
Cross off the second of any double letters.	Tm	Bdpcks	Amnskt
Leave the first letter alone, and replace each of the other letters with the appropriate number.	T5	B31222	A55223
Use duplicate numbers coded from adjacent letters in the original surname only once.	T5	B3122	A5523
Use only the first three numbers or fill in the code with zeroes if there are not enough numbers.	T500	B312	A552
Soundex coded surname	T500	B312	A552

39. Use the Soundex coding system to encode each of the following surnames.

a. Hildebrand

b. Walczyk

c. Marr

d. Pennington

40. Use the Soundex coding system to encode each of the following surnames.

a. Caughey **b.** Longstreet

c. Ball **d.** Prestenbacker

41. a. Use the Soundex coding system to encode the names Smithson, Saendogh, and Smythes. What do you notice?

b. The Soundex code for the name Smith is S530. Create three other surnames that would be encoded in the same way.

42. a. Use the Soundex coding system to encode the names Thomsen, Thamsun, and Thoamsynne. What do you notice?

b. The Soundex code for the name Thompson is T512. Create three other surnames that would be encoded in the same way.

Extended Problems

43. Search through your cupboards and drawers at home for items with bar codes. Look at different kinds of products such as grocery items, health products, nonfood items, and coupons. Can you find an item that has a 6 or 7 as the first digit of its UPC? Find four bar codes, making sure the first digit of the UPC is different for each bar code you use. Remove the bar code from the product if possible. Paste the bar codes to a piece of paper and carefully identify the four parts of the UPC: the type of item, the manufacturer's number, the product number, and the check digit. Verify the check digit for each UPC and be sure to show your calculations.

44. The current International Standard Book Number (ISBN) system has been in use since 1972. The ISBN number system helps publishers, book distributors, and retailers identify and track books and book products. Currently 165 countries and territories subscribe to the ISBN system. It was recently announced that the ISBN will be changed to a 13-digit format in order to ensure that there are enough numbers to identify all books available now and in the future. The change will also make the ISBN number system consistent with the European article number (EAN) bar code.

At first glance, it might seem that the ISBN 10-digit format (9 digits and a check digit) would allow, over 1 billion different identification numbers to be assigned. However, the internal structure of the system itself limits the capacity of the system. Research the ISBN numbering system. On the Internet, search keyword "ISBN" or go to www.isbn.spk-berlin.de/index.html for information about the structure of the numbering system. The ISBN is partitioned into predetermined blocks of digits. Explain what each block represents and how that limits the number of books that can be identified.

45. To convert an ISBN from the 10-digit form to the EAN 13-digit form, you must add a three-digit prefix. In retail, a bar code is placed on all items. The code for "books" is 978, so every 10-digit ISBN will have the number 978 added at the beginning when the bar code is created. In the future, another combination of three digits, 979, will also be used

to represent books and will further expand the capacity of the system. The check digit will then be recalculated to include the new first three digits. A 10-digit ISBN can be converted to a 13-digit EAN in the following way.

Step 1: Add the digits 978 at the beginning of the ISBN and remove the check digit.

Step 2: Calculate the weighted sum as 3(sum of digits in the even positions) + 1(sum of digits in the odd positions).

Step 3: Determine the new check digit as the number you would need to add to the weighted sum to make it divisible by 10.

Use the process described above to convert each of the following 10-digit ISBNs to a 13-digit EAN.

a. 0-07-037393-0

b. 0-553-27429-5

c. 0-590-45235-5

46. You may have noticed that some UPC bar codes are shorter than others. These shorter codes are called "zero-suppressed numbers" and are used for smaller packages. To obtain a shorter code, the original UPC bar code is shortened by leaving out four digits. Research on the Internet the rules for shortening bar codes. Use keyword "UPC" or go to www.howstuffworks.com/upc.htm. Explain how to derive zero-suppressed numbers from the standard UPC bar codes, and give several conversion examples.

47. In 1974, a council was created to look into developing a uniform standard numbering system for Europe similar to the UPC system used in the United States. This resulted in the creation of the European Article Number or EAN. Research the history and use of the EAN. In particular, study the use of the EAN-8 identification number, the EAN-13 identification number, the ITF-14 identification number, and the UCC/EAN-128 identification number. Pay attention to the formation of the check digit in each case. Write a report that summarizes your findings. If you search online, use the keyword "EAN" or go to the EAN International website at www.ean-int.org.

48. **Radio Frequency Identification (RFID)** is a growing technology. Radio frequencies are used to identify products, manage inventory, track perishables, and deter theft. Research this new mode of product identification. When will it be used globally? Where is it being used currently? What is the cost to the consumer? On the Internet, search keywords "radio frequency identification" or go to the EAN international website at www.ean-int.org and look for information regarding RFID.

49. The check digit is added to an identification number so that the typical errors humans make when they record data are detected. By far, the most common errors are single-digit errors, omitting or adding a digit, and transposing two adjacent digits. There are other less common errors, too. Research the types of errors made in transmitting data. What percent of the time is each type of error made? How can check digits be used to detect the less-frequent errors? Use search keywords "check-digit schemes" on the Internet and write a report to summarize your findings. Include examples of the various types of errors and schemes designed to detect the errors.

50. Research how one state assigns driver's license numbers. Show several examples of correctly coded driver's license numbers and describe the coding method. Do all states use check digits in their driver's license numbers?

1.2 Modular Arithmetic and Check-Digit Schemes

INITIAL PROBLEM

Suppose that you are considering buying a car from a seller who seems somewhat shady. The vehicle identification number visible through the windshield looks a little the worse for wear, but appears to be

1G4HP54C5KH410030

Is this number legitimate?

A solution of this Initial Problem is on page 30.

In the previous section, we discussed identification numbers in general as well as the check digits included in modern identification numbers to guard against transmission errors. In this section, we will show how modular arithmetic, originally developed by number theorists, is now used in many common check-digit schemes.