PROBLEM SET 3.1

1. Six candidates ran for governor of California in 2002. The following table contains the vote totals.

Candidate	Votes
Gray Davis	3,469,025
Bill Simon	3,105,477
Reinhold S. Gulke	125,338
Peter Miguel Camejo	381,700
Gary David Copeland	158,161
Iris Adam	86,432
Total	7,326,133

a. If California required a majority to win, what is the minimum number of votes that would be required? Did any candidate win a majority of the vote?

b. What percentage of the vote did each candidate receive?

c. Who won the election under the plurality method?

2. Three candidates ran for Governor of Wyoming in 2002. The following table contains the vote totals.

Candidate	Votes
Dave Freudenthal	92,545
Eli Bebout	88,741
Dave Dawson	3909
Total	185,195

a. If Wyoming required a majority to win, what is the minimum number of votes that would be required? Did any candidate win a majority of the vote?

b. What percentage of the vote did each candidate receive?

c. Who won the election under the plurality method?

3. The nine-member city council is choosing from three possible locations for a new fire station. They rank each of the alternatives in their order of preference. The preferences are summarized in the following table.

	Number of Councilors			
Ranking	4	2	2	1
1st	Davis Ave.	9th St.	Beca Blvd.	Beca Blvd.
2nd	9th St.	Beca Blvd.	9th St.	Davis Ave.
3rd	Beca Blvd.	Davis Ave.	Davis Ave.	9th St.

a. How many first-place votes would be required for a majority win?

b. How many first-place votes did each location receive?

c. Under the plurality method, which location is selected?

4. The members of the football team selected their team captain from three of the seniors: Jorgensen, Petrini, and Ramirez. Each player ranked his choices from first to third, and 88 players completed ballots. There are six ways to rank the choices; the following table shows the number of players who ranked the candidates in a given order.

	Number of Players					
Ranking	11	18	20	14	15	10
1st	Jorgensen	Jorgensen	Petrini	Ramirez	Petrini	Ramirez
2nd	Petrini	Ramirez	Jorgensen	Jorgensen	Ramirez	Petrini
3rd	Ramirez	Petrini	Ramirez	Petrini	Jorgensen	Jorgensen

a. How many first-place votes would be required for a majority win?

b. How many first-place votes did each person receive?

c. Under the plurality method, who is selected as team captain?

5. The planning commission is going to select a consultant for a management study. Each of the seven members on the commission ranked the consultants from first to third, with the following results.

	Number of Members		
Ranking	2	1	4
1st	Finster	Gorman	Yamada
2nd	Gorman	Yamada	Finster
3rd	Yamada	Finster	Gorman

a. If a first-place vote is awarded 3 points, second-place 2 points, and third-place 1 point, calculate the point total for each consultant.

b. Under the Borda count method, which consultant is selected?

6. The senior class is selecting a president and vice-president from among four candidates. Voters ranked the candidates first through fourth. The candidates receiving the two highest point totals will be elected president and vice-president, respectively.

Candidates	1st-Place Votes	2nd-Place Votes	3rd-Place Votes	4th-Place Votes
Aaron	135	223	127	105
Denise	185	164	139	102
Garth	106	168	176	140
Kermit	164	35	148	243

a. If a first-place vote is awarded 4 points, second-place 3 points, third-place 2 points, and fourth-place 1 point, calculate the point total for each candidate.

b. Under the Borda count method, who is selected as president and who is selected as vice-president?

Problems 7 and 8

In Major League Baseball, a variation of the Borda count method is used to select the most valuable player (MVP). In this case, a tenth-place vote is worth 1 point, a ninth-place vote is worth 2 points, . . . , a second-place vote is worth 9 points, and a first-place vote is worth 14 points.

7. The top ten players in the National League in 2002 are listed in the following table along with the number of MVP votes each player received for first through tenth places from the 32 voters.

a. What is the maximum number of points that a player can receive?

b. Find the modified Borda count total for each of the ten players.

c. Who is the winner under the modified Borda count method?

Player	Votes Received For Each Place									
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Johnson	0	0	5	3	4	4	2	4	1	0
Berkman	0	1	7	5	6	5	2	3	1	1
Green	0	0	3	8	4	3	2	3	3	4
Bonds	32	0	0	0	0	0	0	0	0	0
Kent	0	0	3	2	4	8	5	2	3	1
Sosa	0	0	0	2	3	1	1	4	4	2
Schilling	0	0	0	1	2	3	2	1	3	2
Pujols	0	26	4	0	1	0	1	0	0	0
Smoltz	0	1	3	5	2	1	6	2	3	3
Guerrero	0	4	5	3	3	5	2	3	5	1

Player	Votes Received For Each Place
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Ramirez	0	0	0	0	0	2	5	1	2	2
Anderson	0	4	5	7	7	1	2	0	2	0
Thome	0	0	0	0	2	3	3	8	1	4
Tejada	21	6	1	0	0	0	0	0	0	0
Giambi	0	0	2	8	10	4	1	1	1	1
Williams	0	0	0	0	1	0	3	2	3	2
Rodriguez	5	7	11	4	0	1	0	0	0	0
Ordonez	0	0	0	0	1	4	5	3	0	4
Hunter	0	0	0	5	5	8	1	5	4	1
Soriano	2	11	9	4	1	0	0	0	0	0

8. The top ten players in the American League in 2002 are listed in the preceding table along with the number of votes for MVP each player received for first through tenth places from the 28 voters.

   a. What is the maximum number of points that a player can receive?

   b. Find the modified Borda count total for each of the ten players.

   c. Who is the winner under the modified Borda count method?

9. Referring to the preference table in problem 3, which location is selected if the Borda count method is used?

10. Referring to the preference table in problem 4, who is selected as team captain if the Borda count method is used?

11. Consider an election in which there are 5 candidates and 30 voters. Determine the total number of Borda count points possible.

12. Consider an election in which there are 8 candidates and 100 voters. Determine the total number of Borda count points possible.

Problems 13 and 14

The Heisman Memorial Trophy was originally presented in 1935 under the name Downtown Athletic Club award (DAC); it was awarded to the outstanding college football player east of the Mississippi. Currently, the Heisman Trophy is awarded to the outstanding college football player in the United States. Voting for the Heisman Trophy uses a variation of the Borda count system. Voters submit their choices for first, second, and third places, and 3 points are awarded for every first-place vote, 2 for every second-place vote, and 1 for every third-place vote. The winner is the player with the most points.

13. Ten players received votes for the 2002 Heisman trophy. The following table contains the votes each player received for first, second, and third place.

Player	1st-Place Votes	2nd-Place Votes	3rd-Place Votes
Larry Johnson	108	130	142
Ken Dorsey	122	89	99
Carson Palmer	242	224	154
Quentin Griffin	1	8	9
Byron Leftwich	22	26	34
Kliff Kingsbury	6	2	11
Brad Banks	199	173	152
Willis McGahee	101	118	121
Jason Gesser	5	22	15
Chris Brown	5	11	11

Source: cnnsi.com

   a. Calculate the Borda count total for each player.

   b. Who won the Heisman trophy in 2002 using this modified Borda count method?

14. Ten players received votes for the 2001 Heisman trophy. The following table contains the votes each player received for first, second, and third place.

Player	1st-Place Votes	2nd-Place Votes	3rd-Place Votes
David Carr	34	60	58
Dwight Freeney	7	6	24
Eric Crouch	162	86	88
Ken Dorsey	109	122	67
Roy Williams	13	36	35
Antwaan Randle El	46	39	51
Bryant McKinnie	26	12	14
Rex Grossman	137	105	87
Julius Peppers	2	10	15
Joey Harrington	54	68	66

Source: cnnsi.com

a. Calculate the Borda count total for each player.

b. Who won the Heisman trophy in 2001 using this modified Borda count method?

15. There are three candidates running for president of the senior class: Peter, Carmen, and Shawna. Voters mark their ballots to indicate their first, second, and third choices (only ballots with all three choices marked are valid). The results are summarized as follows:

Candidate	1st-Place Votes	2nd-Place Votes	3rd-Place Votes
Peter	33	68	34
Carmen	53	28	54
Shawna	49	39	47

a. Who is elected president using the Borda count method?

b. Who is elected president using the plurality method?

16. Three candidates, Able, Boastful, and Charming are running for the office of mayor of Tinytown. Voters mark their ballots to indicate their first, second, and third choices (only ballots with all three choices marked are valid). The results are summarized as follows:

Candidate	1st-Place Votes	2nd-Place Votes	3rd-Place Votes
Able	33	33	34
Boastful	39	19	42
Charming	28	48	24

a. Who wins using the Borda count method?

b. Who wins using the plurality method?

17. Returning to the situation presented in problem 3, the nine-member city council is choosing from three possible locations for a new fire station. They rank each of the alternatives in their order of preference. The results are summarized as follows.

Number of Councilors				
Ranking	1	2	2	4
1st	Davis Ave.	9th St.	Beca Blvd	Beca Blvd
2nd	9th St.	Beca Blvd	9th St.	Davis Ave.
3rd	Beca Blvd	Davis Ave.	Davis Ave.	9th St.

a. Which location received the fewest first-place votes?

b. Eliminate the location that received the fewest first-place votes and create a new preference table.

c. Under the plurality with elimination method, which location is selected?

18. Returning to the situation presented in problem 5, the planning commission is going to select a consultant for a management study. Each of the seven members on the commission ranks the consultants from first to third. The results are shown in the following preference table.

Ranking	Number of Members		
	2	1	4
1st	Finster	Gorman	Yamada
2nd	Gorman	Yamada	Finster
3rd	Yamada	Finster	Gorman

a. Which consultant received the fewest first-place votes?

b. Eliminate the consultant who received the fewest first-place votes and create a new preference table.

c. Under the plurality with elimination method, which consultant is selected?

19. A new president is being elected by the ceramics guild, and three members, Ann, Eno, and Pat, have been nominated. The members of the guild are asked to rank the candidates from first to third. The 48 ballots are grouped in the following preference table.

Ranking	Number of Members					
	12	8	6	10	8	4
1st	Ann	Ann	Eno	Eno	Pat	Pat
2nd	Eno	Pat	Ann	Pat	Ann	Eno
3rd	Pat	Eno	Pat	Ann	Eno	Ann

a. Who is elected president using the plurality with elimination method?

b. Which member is selected if the pairwise comparison method is used?

20. The board of directors of a large company is choosing a site for a new branch office in the southwest. The cities being considered are Albuquerque (A), Phoenix (P), Sante Fe (S), and Tucson (T). The members of the board are asked to rank the cities from first to fourth. The results are summarized in the following preference table. Although there are 24 ways to rank the cities, only those shown in the table are used.

Ranking	Number of Members											
	4	3	3	3	3	2	2	1	1	1	1	1
1st	P	A	A	S	T	S	T	A	P	P	S	S
2nd	T	S	T	A	P	A	A	P	P	T	P	T
3rd	A	P	S	P	A	T	P	S	A	A	A	A
4th	S	T	P	T	S	P	S	T	S	A	T	P

a. Which city is chosen using the plurality with elimination method?

b. Which city is selected if the pairwise comparison method is used?

21. The Jimenez family is deciding which national park to visit next summer; the choices are Yellowstone (Y), the Grand Canyon (G), and Mount St. Helens (M). The family's preferences are as follows:

Ranking	Dad	Mom	Boy 1	Boy 2	Boy 3	Girl 1	Girl 2
1st	Y	G	M	G	Y	Y	M
2nd	M	M	G	Y	G	M	Y
3rd	G	Y	Y	M	M	G	G

a. Using the pairwise comparison method, there are three pairs to consider. For each pair, indicate which park is preferred and by what margin.

b. Using the pairwise comparison method, how many points are awarded to each park, and which park is selected to visit for next summer?

22. After the last performance, the cast and crew of the school play are going out for dinner. The choices of restaurants are Chinese (C), Italian (I), and Mexican (M). The choices are summarized in the following preference table.

Ranking	Number of Students					
	5	4	4	5	3	6
1st	C	C	I	I	M	M
2nd	I	M	C	M	C	I
3rd	M	I	M	C	I	C

a. Using the pairwise comparison method, there are three pairs to consider. For each pair, indicate which restaurant is preferred and by what margin.

b. Using the pairwise comparison method, how many points are awarded to each restaurant, and which restaurant is selected?

23. a. In applying the pairwise comparison method, how many pairwise comparisons must be made if there are five candidates? How many will be needed with eight candidates?

b. If there are four candidates in an election, in how many different ways can they be arranged in order of preference?

24. a. In applying the pairwise comparison method, how many pairwise comparisons must be made if there are seven candidates? How many will be needed with nine candidates?

b. If there are five candidates in an election, in how many different ways can they be arranged in order of preference?

Problems 25 through 32

There are many situations in which there is a need to know not only the winner of the election, but also how the other candidates ranked. Each of the methods we have studied can be extended to produce a winner and rank the other candidates for second place, third place, etc.

25. The following table contains preferences of faculty members for students being considered for scholarships. The first-place candidate will receive a $5000 scholarship while second-, third-, and fourth-place candidates will receive $4000, $3000, and $2000, respectively. Rank the candidates in order according to their Borda count totals and determine which candidate receives which scholarship.

Number of Faculty					
Ranking	6	4	2	1	1
1st	Peterson	Mitchell	Bryan	Bryan	Davison
2nd	Mitchell	Bryan	Mitchell	Davison	Mitchell
3rd	Davison	Peterson	Peterson	Mitchell	Bryan
4th	Bryan	Davison	Davison	Peterson	Peterson

26. The members of the football team will select a senior captain and a junior captain from three candidates. Use the preference table from problem 4 and extend the Borda count method to rank the candidates according to their Borda count totals. The first-place candidate will be the senior captain, and the second-place candidate will be the junior captain. What are the results of the election?

27. An extension of the plurality method can be used to rank candidates according to the number of first-place votes received. How would the candidates from problem 25 be ranked using this extension of the plurality method?

28. An extension of the plurality method can be used to rank candidates according to the number of first-place votes received. How would the candidates from problem 4 be ranked using this extension of the plurality method?

29. To use the plurality with elimination method to rank candidates, the order of elimination is important. The first candidate to be eliminated will be ranked last. The second candidate to be eliminated will be eliminated second to last. This will continue until we are left with one candidate, who will be ranked first. Use this method to rank the candidates from problem 18.

30. An extension of the plurality with elimination method is explained in the previous problem. Use this extension to rank the cities from problem 20.

31. To use the pairwise comparison method to rank candidates, use the point totals. The candidate with the greatest point total is ranked first. The candidate with the next-largest point total is ranked second, and so on. Use this method to rank the consultants from problem 18.

32. Extend the pairwise comparison method, explained in the previous problem, to rank the candidates from problem 25.

Problems 33 and 34

A modified Borda count method is used in many types of contests other than elections, such as athletic events. A fairly common practice is to emphasize first-place finishes, or to de-emphasize other finishes, by changing the way in which points are awarded (such as 5 points for first place, 3 for second place, 2 for third place, and 1 for fourth place).

33. How many points would each candidate receive in problem 15, and who would be elected senior class president, if 4 points are given for first-place votes, 2 for second place, and 1 for third place?

34. How many points would each candidate receive in problem 16, and who would be elected mayor of Tinytown, if 4 points are given for first-place votes, 2 for second place, and 1 for third place?

Problems 35 and 36

Four teams, the Raiders (R), the Spartans (S), the Titans (T), and the Vikings (V) are competing for the gymnastics team championship in four events. (*Note:* Each team was allowed two competitors in each event.) The results are as follows.

Ranking	Events			
	Beam	Horse	Bars	Floor
1st	R	S	V	S
2nd	S	T	T	V
3rd	T	V	R	R
4th	V	R	V	V

35. Assume the four events are of equal importance. How do the teams rank in this competition if the finishing positions are scored 4, 3, 2, and 1, respectively, using the Borda count method?

36. How do the teams rank if the finishing positions are scored 5, 3, 1, and 0, respectively, using a modified Borda count method?

37. The Board of Commissioners for Baker County must pick a site for a new jail. Three locations have been determined to be suitable, and each of the 15 commissioners rank their preferences in order. The preferences are listed in the following table.

Ranking	Number of Commissioners				
	4	4	4	2	1
1st	A	B	C	A	B
2nd	C	C	B	B	A
3rd	B	A	A	C	C

a. Using the plurality with elimination method, in which the alternative with the most *last*-place votes is eliminated, which site is selected?

b. Which site is selected using the plurality with elimination method?

c. Which site is selected using the Borda count method?

d. Which site is selected using a modified Borda count method with assigned values of 4 points for first place, 2 points for second place, and 1 point for third place?

38. Four seniors on the baseball team, Joe Aaron (A), Billy Bonds (B), Mike Griffey (G), and Tim Ruth (R), are being considered for team captain. The 21 other members of the team are asked to rank them in order of preference from first to fourth. The ballots are grouped as follows.

Ranking	Number of Players										
	4	3	2	2	2	2	2	1	1	1	1
1st	A	B	R	G	G	A	A	B	G	G	R
2nd	B	G	B	A	B	B	R	R	R	R	G
3rd	G	R	A	R	R	R	G	G	A	B	B
4th	R	A	G	B	A	G	B	A	B	A	A

a. Who is selected as team captain using the plurality method?

b. Who is selected as team captain using the plurality method with elimination?

c. Who is selected as team captain using the Borda count method?

d. Who is selected as team captain using a modified Borda count method with assigned values of 6 points for first place, 3 points for second, 2 points for third, and 1 point for fourth?

39. Refer to the preference table in problem 38.

a. Suppose that the plurality with elimination method is modified as follows. First, a runoff election takes place between the candidates who rank second and third in terms of first-place votes; then, the winner of the runoff is matched against the candidate who originally had the most first-place votes. Who is selected as team captain if this method is implemented?

b. Who would be selected as baseball team captain if the plurality with elimination method is used, and the candidate with the most *last*-place votes is eliminated at each step? This method eliminates candidates who are *least* preferred.

40. Refer to the preference table from problem 38.

a. Suppose that the plurality with elimination method is modified by using the following runoff elections. The first runoff election is between the two candidates with the lowest number of first-place votes; the second uses the winner of the runoff contest and the candidate with the second-highest number of first-place votes; and finally, the third uses the winner of the second runoff contest and the candidate with the most original first-place votes. If the number of first-place votes is a tie, the order is determined by the number of second-place votes. Who is selected as team captain if this method is implemented?

b. Who would be selected if plurality with elimination is used and the two candidates with the most first-place votes are matched in a runoff election?

Problems 41 and 42

The 2000 Presidential election brought national attention to electoral rules, outdated voting equipment, and plurality voting. After 35 days of controversy, and a Supreme Court decision, George Bush gained Florida's 25 disputed electoral votes, giving him 271 votes in the Electoral College to Al Gore's 266. Thus, George Bush became the President of the United States despite securing only 47.87% of the popular vote. According to the Federal Election Commission, the top four candidates for United States President in 2000 received the following vote totals.

Candidate	Popular Vote Total
Al Gore (Democrat)	50,999,897
George Bush (Republican)	50,456,002
Ralph Nader (Green)	2,882,955
Patrick Buchanan (Reform/Ind.)	448,895
Total	104,787,749

Suppose the plurality with elimination method had been used in the 2000 presidential election. In each round, eliminate the candidate with the lowest vote total and redistribute their votes to the other candidates until a majority of votes for one candidate has been achieved.

41. Suppose 93.9% of the Buchanan voters would vote for Bush, 3.9% would vote for Gore, and 2.2% would vote for Nader if Buchanan were not in the race; and that 53.1% of the Nader supporters would vote for Bush, and 46.9% would vote for Gore if Nader were not in the race. Who would have won the election and what percentage of the popular vote would that candidate have received?

42. Suppose 85.5% of the Buchanan voters would vote for Bush, 10.5% would vote for Gore, and 4% would vote for Nader if Buchanan were not in the race; and 9.8% of the Nader supporters would vote for Bush, and 90.2% would vote for Gore if Nader were not in the race. Who would have won the election and what percentage of the popular vote would that candidate have received?

Extended Problems

43. In November 1998, information was leaked to the press that International Olympic Committee (IOC) members were taking bribes in exchange for votes for cities seeking to host the Olympic Games. As a result, the process by which a city becomes the host for the Olympic Games was overhauled. Research the process that a city must now go through to become the host city. How long does the process take? Once a city is selected, how many years does the city have to prepare for the games? Describe the voting process. How many IOC members may vote? What type of voting system is used? On the Internet, go to www.olympic.org for more information. Summarize your findings in a report.

Problems 44 and 45

When there are several candidates or alternatives in an election, and there is no clear winner with a majority, runoff elections are often used to determine the winner. Sometimes the order of the runoffs is determined by the number of votes received, while at other times it may be determined by some other method (even as simple as drawing straws).

44. Ten members of the city council are voting on three budget options, referred to as A, B, and C. The preferences of the council members are summarized as follows:

    Four members prefer A to B, and B to C.

    Three members prefer B to C, and C to A.

    Three members prefer C to A, and A to B.

    a. Which option is selected if the council first chooses between A and B, and then chooses between the winner and C? (*Note:* You should assume that if voters prefer A to B and B to C, then they would prefer A to C when selecting between those two choices. This principle is referred to as transitivity, and underlies all our work with preference schedules.)

    b. Which option is selected if A and C are considered first, with the winner against B?

    c. Which option is selected if B and C are considered first, with the winner against A?

45. The school board is considering three options for a new career-counseling program. The preferences of the board members are summarized as follows:

    Two members prefer A to B, and B to C.

    Five members prefer A to C, and C to B.

    Four members prefer B to A, and A to C.

    Four members prefer C to B, and B to A.

    a. Which option is selected if the council first chooses between A and B, and then chooses between the winner and C?

    b. Which option is selected if A and C are considered first, with the winner against B?

    c. Which option is selected if B and C are considered first, with the winner against A?

46. Create a formula to calculate the number of pairwise comparisons needed in an election with n candidates. First create a table listing the number of comparisons needed if there are 2, 3, 4, 5, 6, and 7 candidates. Find a pattern and express the result in terms of n.

47. The 76th Annual Academy Awards was held in February 2004, with "The Lord of the Rings" taking the Oscar for the best motion picture of the year. How are nominations made in each category and how many different categories are there? How many voting members of the Academy of Motion Picture Arts and Sciences are there? How are the final ballots cast and what method is used to determine a winner in each category? How are ties handled? Research the Academy Awards and write a report to summarize your findings. On the Internet, go to www.oscars.org for information.

48. Countries around the world use different voting methods to elect their government officials. Often, in local elections, one method is used, while in national elections a different method is used. One good source for voting information is www.electionworld.org. Use the index and click on the name of a country to get a description of the voting process.

    a. Find five examples, if possible, of countries that use the plurality or the plurality with elimination method.

    b. Find five examples, if possible, of countries that use a preferential ballot.

    c. Find five examples, if possible, of countries that use the Borda count method.

    d. Find three examples of voting methods other than the ones discussed in this chapter. In each case, describe the method and give the names of the countries in which it is used.

    e. Investigate several countries listed as dictatorships and describe how government officials are selected.