PROBLEM SET 3.3

1. Suppose Martell, Arlene, Jody, and Roscoe are voters with weights 3, 7, 5, and 12, respectively.
   a. If passing a motion requires a simple majority of yes votes, then what is the smallest weight required to pass a motion?
   b. If passing a motion requires a two-thirds supermajority of yes votes, then what is the smallest weight required to pass a motion?
   c. If the quota is 17, give the notation for this weighted voting system.

2. Suppose representatives for five zones have voting weights of 4, 6, 2, 8, and 10, respectively.
   a. If passing a motion requires a simple majority of yes votes, then what is the smallest weight required to pass a motion?
   b. If passing a motion requires a two-thirds supermajority of yes votes, then what is the smallest weight required to pass a motion?
   c. If the quota is 25, give the notation for this weighted voting system.

Problems 3 and 4

Consider the weights assigned to voters in a weighted voting system. A simple majority is needed to pass a measure. Determine the quota, and express the weighted voting system using proper notation.

3. a. [6, 5, 5, 3, 3, 2, 1, 1] b. [4, 8, 5, 3, 2, 5, 2]
   c. [10, 5, 5, 5, 3, 3, 3] d. [1, 2, 2, 5, 5, 7]

4. a. [6, 12, 5, 7, 3, 2, 1] b. [18, 12, 8, 5, 5, 2, 2]
   c. [8, 8, 5, 4, 3, 2, 1] d. [6, 2, 4, 4, 5, 3]

Problems 5 and 6

The following sets of numbers represent the weights assigned to voters in a weighted voting system. Following the weights is the percentage required for measures to pass. Determine the quota in each case, and express the weighted voting system using proper notation.

5. a. [8, 5, 5, 3, 3, 2]; 60%
   b. [9, 6, 5, 5, 3, 2, 2]; 67%
   c. [7, 5, 5, 5, 3, 2]; 75%
   d. [5, 5, 4, 4, 3, 3]; 60%

6. a. [20, 15, 12, 10, 8, 5]; 60%
   b. [10, 8, 6, 4, 2]; 75%
   c. [8, 5, 5, 5, 3, 3, 3]; 70%
   d. [2, 1, 1, 1, 1, 1]; 60%

7. Consider the weighted voting system [10 | 7, 6, 6, 5].
   a. Suppose P_1, P_2, and P_3 vote yes on a motion while P_4 votes no. Will the motion pass or will it be defeated?
   b. Suppose P_1 and P_2 vote no on a motion while P_3 and P_4 vote yes. Will the motion pass or will it be defeated?
   c. Explain why this is not an acceptable weighted voting system.

8. Consider the weighted voting system [14 | 5, 4, 3, 2].
   a. Suppose P_1, P_2, and P_3 vote yes on a motion while P_4 votes no. Will the motion pass or will it be defeated?
   b. Suppose P_1 and P_2 vote yes on a motion while P_3 and P_4 vote no. Will the motion pass or will it be defeated?
   c. Explain what must happen in order to pass a motion.

9. Using the notation of a weighted voting system, give an example of a six-person committee that requires a unanimous decision on all measures.

10. Using the notation of a weighted voting system, give an example of an eight-person committee in which it takes two dissenting votes to defeat any measure.

Problems 11 through 14

For each problem, a weighted voting system is given in standard notation. Parts (a) through (d) represent coalitions of voters in favor of a measure. Determine if these coalitions are winning coalitions or losing coalitions.

11. [21 | 10, 8, 7, 7, 4, 4]
    a. $\{P_1, P_4, P_6\}$ b. $\{P_2, P_3, P_6\}$
    c. $\{P_2, P_3, P_4\}$ d. $\{P_3, P_4, P_5, P_6\}$

12. [16 | 9, 7, 6, 4, 3, 2]
    a. $\{P_1, P_4, P_6\}$ b. $\{P_2, P_3, P_6\}$
    c. $\{P_2, P_3, P_4\}$ d. $\{P_3, P_4, P_5, P_6\}$

13. [15 | 8, 4, 3, 3, 2, 2]
    a. $\{P_1, P_2, P_3\}$ b. $\{P_2, P_3, P_6\}$
    c. $\{P_2, P_3, P_4, P_5\}$ d. $\{P_2, P_3, P_4, P_5, P_6\}$

14. [10 | 4, 3, 3, 3, 2, 2, 1]
    a. $\{P_1, P_2, P_4\}$ b. $\{P_2, P_3, P_5, P_6\}$
    c. $\{P_2, P_3, P_5, P_7\}$ d. $\{P_2, P_3, P_4\}$

15. How many coalitions are possible in a weighted voting system with
    a. 8 voters?
    b. 10 voters?

16. How many coalitions are possible in a weighted voting system with
    a. 12 voters?
    b. 15 voters?

Problems 17 and 18

Make a table listing all the coalitions for each of the given weighted voting systems, and determine whether each coalition is a winning or losing coalition.

17. a. $[4 \mid 3, 2, 1]$
    b. $[26 \mid 20, 15, 10, 5]$

18. a. $[7 \mid 5, 4, 2]$
    b. $[6 \mid 4, 3, 2, 1]$

19. Consider the weighted voting system $[16 \mid 10, 5, 4]$.
    a. What fraction of the total weight does each voter control?
    b. Calculate the Banzhaf power index for each voter.
    c. Compare parts (a) and (b) and explain why the weight of a voter is not a good measure of the voter's power.

20. Consider the weighted voting system $[26 \mid 25, 3, 1]$.
    a. What fraction of the total weight does each voter control?
    b. Calculate the Banzhaf power index for each voter.
    c. Compare parts (a) and (b) and explain why the weight of a voter is not a good measure of the voter's power.

Problems 21 and 22

In Oregon in 1999, an amendment made to the state constitution allowed non-unanimous jury verdicts in murder trials. Previously, a murder conviction required a unanimous vote of all 12 jurors. The new amendment allows for an 11-to-1 jury verdict to convict. However, this new rule does not apply to aggravated murder cases.

21. Consider murder trials prior to 1999.
    a. Express the weighted voting system using the proper notation.
    b. List all the winning coalitions.
    c. Does any juror have veto power? Explain.
    d. Find the Banzhaf power index for each juror.

22. Consider murder trials after 1999.
    a. Express the weighted voting system using the proper notation.
    b. List all the winning coalitions.
    c. Does any juror have veto power? Explain.
    d. Find the Banzhaf power index for each juror.

Problems 23 and 24

Find the Banzhaf power index for each voter in each of the given weighted voting systems, and identify voters who
    (i) are dictators.
    (ii) are dummies.
    (iii) have veto power.

23. a. $[8 \mid 5, 4, 3]$
    b. $[25 \mid 14, 13, 12, 8]$
    c. $[7 \mid 7, 2, 2, 2]$

24. a. $[20 \mid 11, 10, 9]$
    b. $[26 \mid 14, 13, 12, 8]$
    c. $[4 \mid 3, 1, 1, 1]$

25. Consider each of the following quotas and the weighted voting system $[q \mid 5, 3, 1]$. Find the Banzhaf power index for each voter using the given value of q.
    a. $q = 5$ b. $q = 6$ c. $q = 7$
    d. $q = 8$ e. $q = 9$

26. Consider each of the following quotas and the weighted voting system $[q \mid 5, 3, 2, 1]$. Find the Banzhaf power index for each voter using the given value of q.
    a. $q = 6$ b. $q = 7$ c. $q = 8$
    d. $q = 9$ e. $q = 10$

Problems 27 and 28

A winning coalition is **minimal** if every member of the coalition is a critical voter. List all the minimal winning coalitions for each of the given weighted voting systems.

27. a. $[4 \mid 3, 2, 1]$
    b. $[6 \mid 4, 3, 2, 1]$

28. a. $[7 \mid 5, 4, 2]$
    b. $[26 \mid 20, 15, 10, 5]$

Problems 29 and 30

A **blocking coalition** is a set of voters who can prevent passage of a proposal. To block the passage of a proposal, the weight of the blocking coalition must be larger