

Granger considers both pieces 1 and 2 unacceptable because they are worth less than one-third of the estate's total value. To Granger, piece 3 contains more than one-third of the estate's value, so that piece is acceptable.

STEP 3: Choosers reveal which pieces are acceptable.

Both Oswald and Granger consider piece 3 acceptable. Oswald also considers piece 2 acceptable.

STEP 4: Consider choosers' acceptable pieces and assign pieces.

Because both Oswald and Granger agree that piece 1 is worth less than one-third of the value of the estate, Drewhan gets that piece. Because Oswald considers piece 2 acceptable and Granger does not, Oswald gets piece 2. Granger gets piece 3, which is the only piece acceptable to him, but to him it is worth more than half of the total value of the estate. This arrangement, of course, is quite acceptable to Granger.

PROBLEM SET 4.1

Problems 1 and 2

For each part listed, explain which kind of fair-division problem applies: continuous, discrete, or mixed.

1. a. An inheritance of an acre of land, a diamond

necklace, and a car

b. A collection of antique paintings

c. The family pets: a dog, a cat, and a hamster

d. A peanut butter pie

2. a. A set of novels signed by the author

b. Fifty acres of land and a farmhouse

c. A two-week vacation

d. A television set, a stereo, and a DVD player

3. A pastry bar, half chocolate and half maple, is to be

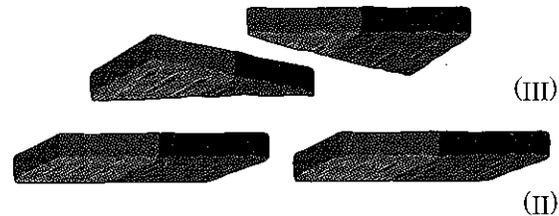
shared by Madeline and Graham.



Madeline likes chocolate and maple equally, while Graham prefers chocolate to maple. Using the divide-and-choose method for two players, consider three possible divisions of the pastry bar made by Madeline.



(I)

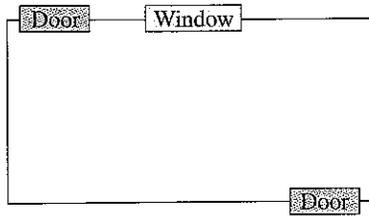


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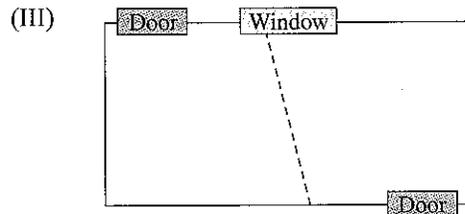
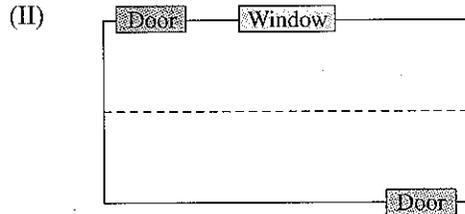
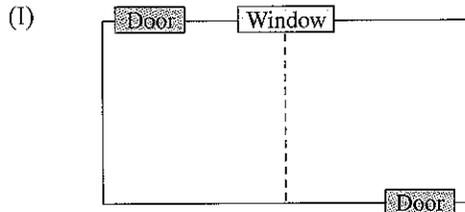
(II)

- Visually inspect the two pieces in (I), and explain which half Graham would select and why. You need not assign numerical values to the pieces.
- Visually inspect the two pieces in (II), and explain which half Graham would select and why. You need not assign numerical values to the pieces.
- Visually inspect the two pieces in (III), and explain which half Graham would select and why. You need not assign numerical values to the pieces.
- Suppose in division (I), Graham selects the chocolate half, leaving the maple half for Madeline. How would each judge the fairness of the portion they received compared with the value of the entire pastry bar?

4. Monty and Albert place a divider in their shared bedroom.



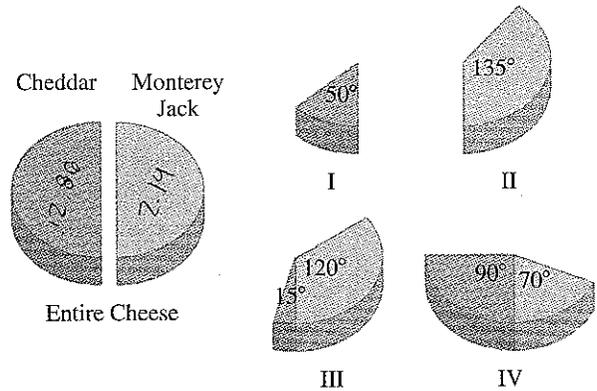
Monty thinks that dividing the room in half is fair, and he does not care which half he gets. Albert would prefer a window on his half of the room. Using the divide-and-choose method for two players, consider three possible divisions Monty could make in the room



- Visually inspect the two halves in (I), and explain which half Albert would select and why. You need not assign numerical values to the pieces.
- Visually inspect the two halves in (II), explain which half Albert would select and why. You need not assign numerical values to the pieces.
- Visually inspect the two halves in (III), explain which half Albert would select and why. You need not assign numerical values to the pieces.
- Suppose in division (I), Albert selects the left half, leaving the right half for Monty. How would each judge the fairness of the portion they received compared to the value of the entire room?

Problems 5 and 6

You purchase a cheese wheel at the grocery store for \$15. The cheese wheel is made up of 32 ounces of cheddar and 32 ounces of Monterey jack, as shown.



- Suppose you like cheddar cheese twice as much as Monterey jack cheese.
 - Determine your cheddar cheese to Monterey jack cheese preference ratio.
 - What fraction of the cheese wheel is the cheddar cheese?
 - Based on your cheese preference, what fraction of the value of the whole cheese wheel is the value of the cheddar cheese? Recall that one complete revolution is 360° .
 - If you cut off a wedge containing 20 ounces of cheddar cheese, what dollar value would you place on the wedge?
 - What dollar value would you place on wedges I and III?
- Suppose you like cheddar cheese six times as much as Monterey jack cheese.
 - Determine your cheddar cheese to Monterey jack cheese preference ratio.
 - What fraction of the cheese wheel is the Monterey jack cheese?
 - Based on your cheese preference, what fraction of the value of the cheese wheel is the value of the Monterey jack cheese?
 - If you cut off a wedge containing 25 ounces of Monterey jack cheese, what dollar value would you place on the wedge?
 - What dollar value would you place on wedges II and IV?

9. A cell-phone company offers plans for two phones divided between the two phones. If Patrick prefers daytime minutes to evening minutes in a 3-to-1 ratio, which of the following plans would be a fair division according to Patrick? Explain.

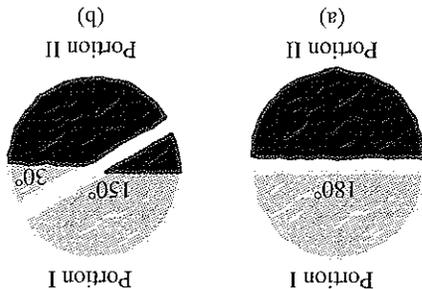
Plan A:	Phone 1: 300 day 150 evening	Phone 2: 200 day 350 evening
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Plan B:	Phone 1: 400 day 100 evening	Phone 2: 100 day 400 evening
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Plan C:	Phone 1: 300 day 100 evening	Phone 2: 200 day 400 evening
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Plan D:	Phone 1: 100 day 500 evening	Phone 2: 400 day 0 evening
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10. You buy a sugar cookie that has been dipped halfway into chocolate. If you like a plain sugar cookie three times as much as a chocolate-coated cookie, which of the following divisions represents a fair division of the cookie according to your preferences?



7. A manufacturer shrink-wrapped a bottle of laundry soap and a bottle of fabric softener together and suggested a retail price of \$6.40.

a. Donna never uses fabric softener, but she uses laundry soap regularly. Determine Donna's fabric softener to laundry soap preference ratio. What dollar value would she place on each item?

b. Pierce values laundry soap and fabric softener equally. Determine Pierce's fabric softener to laundry soap preference ratio. What dollar value would he place on each item?

c. John values laundry soap over fabric softener in a 3-to-1 ratio. Describe, in words, what John's preference ratio means. What dollar value would he place on each item?

d. Bethany values fabric softener over laundry soap in a 4-to-1 ratio. Describe, in words, what Bethany's preference ratio means. What dollar value would she place on each item?

8. A \$3.00 package of dinner rolls contains 6 wheat and 6 white rolls.

a. Brian likes each kind of roll equally well. Determine Brian's wheat roll to white roll preference ratio. What dollar value would he place on each set of 6 rolls?

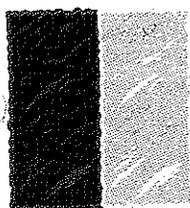
b. Roger eats only white rolls. Determine Roger's preference ratio for wheat rolls to white rolls. What dollar value would he place on each set of 6 rolls?

c. Suzanne prefers wheat rolls to white rolls in a 5-to-1 ratio. Describe, in words, what Suzanne's preference ratio means. What dollar value would she place on each set of 6 rolls?

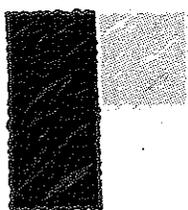
d. Spencer prefers white rolls to wheat rolls in a 2-to-1 ratio. Describe, in words, what Spencer's preference ratio means. What dollar value would he place on each set of 6 rolls?

Problems 11 through 18

A bakery frosted an 8-inch-by-8-inch square cake on one side with vanilla frosting and on the other side with chocolate frosting, as shown next. Brian prefers vanilla frosting over chocolate in a 5-to-1 ratio. Jody likes chocolate slightly more than vanilla, for a vanilla-to-chocolate ratio of 4 to 5. Kurt will eat only chocolate frosting, and Francis will eat only vanilla frosting. Consider the following cake and the cake portions shown.

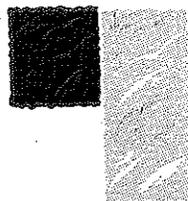


Entire Cake



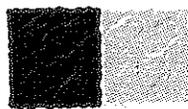
I

Cake with half of the vanilla frosted side removed.



II

Cake with half of the chocolate frosted side removed.



III

Cake with half of chocolate frosted side and half of vanilla frosted side removed.



IV

Cake with one-fourth of the chocolate frosted side and two-thirds of the vanilla frosted side removed.

11. Suppose Brian bought the entire cake for \$9.00.
- What monetary value does he place on the vanilla half of the entire cake?
 - What monetary value does he place on the chocolate half of the entire cake?
 - What monetary value does Brian place on cake portions I, II, III, and IV?

12. Suppose Jody bought the entire cake for \$9.00.
- What monetary value does she place on the vanilla half of the entire cake?
 - What monetary value does she place on the chocolate half of the entire cake?
 - What monetary value does Jody place on cake portions I, II, III, and IV?
13. Suppose Kurt bought the entire cake for \$9.00.
- What monetary value does he place on the vanilla half of the entire cake?
 - What monetary value does he place on the chocolate half of the entire cake?
 - What monetary value does Kurt place on cake portions I, II, III, and IV?
14. Suppose Francis bought the entire cake for \$9.00.
- What monetary value does she place on the vanilla half of the entire cake?
 - What monetary value does she place on the chocolate half of the entire cake?
 - What monetary value does Francis place on cake portions I, II, III, and IV?
15. Consider your results from problem 11. For each part below, describe a new portion of cake containing both chocolate and vanilla, to which Brian would assign the given dollar amount.
- \$3.50
 - \$2.50
 - \$6.75
16. Consider your results from problem 12. For each of the following, describe a new portion of cake containing both chocolate and vanilla to which Jody would assign the given dollar amount.
- \$3.50
 - \$2.50
 - \$6.75
17. Consider your results from problem 13. For each of the following, describe a new portion of cake containing both chocolate and vanilla, to which Kurt would assign the given dollar amount.
- \$3.50
 - \$2.50
 - \$6.75
18. Consider your results from Problem 14. For each of the following, describe a new portion of cake containing both chocolate and vanilla, to which Francis would assign the given dollar amount.
- \$3.50
 - \$2.50
 - \$6.75

19. A bowl contains 5 ounces of vanilla and 2 ounces of chocolate ice cream. Jeanne prefers vanilla ice cream to chocolate ice cream in a 2-to-1 ratio. Suppose that Jeanne will use points to assign value to portions of the ice cream.
- How many points might Jeanne assign to each ounce of vanilla ice cream?
 - How many points might Jeanne assign to each ounce of chocolate ice cream?
 - What is the total point value Jeanne would assign to the bowl of ice cream?
 - Give two different divisions of the ice cream that Jeanne would consider to be fair.
20. A bowl contains 7 ounces of strawberry and 5 ounces of vanilla ice cream. Jeremy prefers strawberry ice cream to vanilla in a 5-to-4 ratio. Suppose that Jeremy will use points to assign value to portions of the ice cream.
- How many points might Jeremy assign to each ounce of strawberry ice cream?
 - How many points might Jeremy assign to each ounce of vanilla ice cream?
 - What is the total point value Jeremy would assign to the bowl of ice cream?
 - Give two different divisions of the ice cream that Jeremy would consider to be fair.
21. A dessert is made up of 6 ounces of fudge cake and 4 ounces of ice cream. Marcus prefers ice cream to fudge cake in a 5-to-2 ratio. Suppose that Marcus will use points to assign value to portions of the dessert.
- How many points might Marcus assign to each ounce of ice cream?
 - How many points might Marcus assign to each ounce of fudge cake?
 - What is the total point value Marcus would assign to the dessert?
 - Describe two different divisions of the dessert that Marcus would consider to be fair.
 - Marcus' friend Max prefers fudge cake to ice cream in a 3-to-1 ratio. For each of the divisions from part (d), decide which portion Max would select and why.
22. A dessert is made up of 8 ounces of apple pie and 4 ounces of whipped cream. Rena prefers apple pie to whipped cream in a 4-to-1 ratio. Suppose that Rena will use points to assign value to portions of the dessert.
- How many points might Rena assign to each ounce of apple pie?
 - How many points might Rena assign to each ounce of whipped cream?
 - What is the total point value Rena would assign to the dessert?
 - Give two different divisions of the dessert that Rena would consider to be fair.
 - Rena's friend Judy prefers apple pie to whipped cream in a 2-to-1 ratio. For each of the divisions from part (d), decide which portion Judy would select and why.
23. Two students, Casey and Tran, try to schedule time with a tutor during the week. The tutor has a total of 7 hours available, 3 hours in the morning and 4 hours in the afternoon. Casey prefers morning meeting times to afternoon times in a 3-to-1 ratio. Tran likes afternoon hours twice as much as morning hours. Neither student is willing to create divisions that are the same. They agree to use the divide-and-choose method for two players.
- Determine how the tutor's time could be fairly divided if Casey is the divider. Then indicate which option Tran would choose, and explain why.
 - Determine how the tutor's time could be fairly divided if Tran is the divider. Then indicate which option Casey would choose, and explain why.
24. Two siblings, Eric and Alex, must divide the 10 hours of computer time available to them for the week. Seven hours are weekday hours, and the remaining 3 hours are on the weekend. Eric prefers weekday hours to weekend hours in a 2-to-1 ratio. Alex prefers weekend hours over weekday hours by a ratio of 3 to 1. Neither boy is willing to create divisions that are the same. They agree to use the divide-and-choose method for two players.
- How could Eric fairly divide the computer time if he were the divider? Then indicate which option Alex would choose, and explain why.
 - How could Alex fairly divide the computer time if he were the divider? Then indicate which option Eric would choose, and explain why.

25. Three people must share weekday, weeknight, and weekend time on a computer and decide to divide the time fairly using the divide-and-choose method for three players. Arnel will be the divider; Paula and Marc will be the choosers. Arnel divides the time into three plans: plan A, plan B, and plan C.
- Suppose that Paula thinks plan A and plan C are acceptable, while Marc thinks only plan A is acceptable. Describe a fair division of the time among Arnel, Paula, and Marc.
 - Suppose that Paula thinks plan B is acceptable, while Marc thinks plan C is acceptable. Describe a fair division of the time among Arnel, Paula, and Marc.
 - Suppose that Paula and Marc both think plan B is the only acceptable plan. Describe how the players could create a fair division of the time among Arnel, Paula, and Marc.
26. Three teenagers have access to the family car for a few hours each week. The car is available before school on some days, after school on some days, and for a few hours on the weekend. Janice divides the available time into what she believes are three fair plans: plan I, plan II, and plan III. Jill and Jayce judge the fairness of each plan.
- Suppose that Jill thinks plan I and plan II are acceptable, while Jayce thinks plan III is acceptable. Describe a fair division of the time among Janice, Jill, and Jayce.
 - Suppose that both Jill and Jayce believe plans I and II are acceptable. Describe a fair division of the time among Janice, Jill, and Jayce.
 - Suppose that both Jill and Jayce believe that plan I is the only fair plan. Explain how the process that leads to a fair division may cause Janice to value her piece less than she values one of Jill's or Jayce's pieces.
27. A cell-phone company advertises a popular phone plan. For \$24.30, the user can have 1000 weekday minutes, 1000 weeknight minutes, and 1000 weekend minutes. Duane, Doreen, and Kaylee will divide the minutes using the divide-and-choose method for three players.
- If Duane has no preference, and he values weekday, weeknight, and weekend minutes equally, then what monetary value would he place on each set of minutes? What value would he require in order to consider a share of minutes to be a fair share?
 - If Doreen's weekday to weeknight to weekend ratio is 5 to 3 to 1, then what monetary value would she place on each set of minutes? What value would she require in order to consider a share of minutes to be a fair share?
 - If Kaylee's weekday to weeknight to weekend ratio is 2 to 3 to 1, then what monetary value would she place on each set of minutes? What value would she require in order to consider a share of minutes to be a fair share?
 - Duane is the divider, and he creates the following three plans:
Plan I: 500 weekday minutes, 200 weeknight minutes, 300 weekend minutes
Plan II: 100 weekday minutes, 700 weeknight minutes, 200 weekend minutes
Plan III: 400 weekday minutes, 100 weeknight minutes, 500 weekend minutes
Determine the result of the divide-and-choose method for three players.

29. Three beachcombers collect shells and other items from the beach each morning to sell in their shops. They decide that it would be fair to divide the beach into three regions so that they each hunt in their own areas. The beach from which they collect is made up of 600 linear feet of sandy beach, 600 linear feet of rocky beach, and 600 linear feet of grassy beach.
- a. Sondra finds things to sell in each of the three areas, so she values each area equally. How many points might she assign to each foot of sandy beach, rocky beach, and grassy beach? Based on this system, what is the total value Sondra would place on the entire beach, and what would be her fair-share requirement?
- b. Lanie values the shells he finds on the sandy beach more than anything else, and his sandy beach to rocky beach to grassy beach preference ratio is 2 to 1 to 1. How many points might he assign to each foot of sandy beach, rocky beach, and grassy beach? Based on this system, what is the total value Lanie would place on the entire beach, and what would be his fair-share requirement?
- c. Lisa values sandy beach to rocky beach to grassy beach in a 1-to-3-to-2 preference ratio. How many points might she assign to each foot of sandy beach, rocky beach, and grassy beach? Based on this system, what is the total value Lisa would place on the entire beach, and what would be her fair-share requirement?
- d. If Sondra is the divider and decides that each person should have a beach to himself or herself, determine the result of the divide-and-choose method for three players.
- e. If Lanie is the divider and decides on the following options, then determine the result of the divide-and-choose method for three players.
- Option I: 100 feet of sandy beach, 300 feet of rocky beach, 300 feet of grassy beach
 Option II: 250 feet of sandy beach, 100 feet of rocky beach, 200 feet of grassy beach
 Option III: 250 feet of sandy beach, 200 feet of rocky beach, 100 feet of grassy beach

28. A new cell-phone company advertises a phone plan. For \$30.60, the user can have 1100 weekday minutes, 1100 weekend minutes, and 1100 weekend minutes. Duane, Doreen, and Kaylee will divide the minutes using the divide-and-choose method for three players.
- a. If Duane has no preference, and he values weekday, weekend, and weekend minutes equally, what monetary value would he place on each set of minutes? What value would he require in order to consider a share of minutes to be a fair share?
- b. If Doreen prefers weekday and weekend minutes over weekend minutes in a ratio of 2 to 2 to 1, what monetary value would she place on each set of minutes? What value would she require in order to consider a share of minutes to be a fair share?
- c. If Kaylee's weekday to weekend to weekend ratio is 1-to-2-to-3, what monetary value would she place on each set of minutes? What value would she require in order to consider a share of minutes to be a fair share?
- d. Duane is the divider, and he creates the following three plans:
- Plan I: 400 weekday minutes, 150 weekend minutes, 550 weekend minutes
 Plan II: 300 weekday minutes, 350 weekend minutes, 450 weekend minutes
 Plan III: 400 weekday minutes, 600 weekend minutes, 100 weekend minutes
- Determine the result of the divide-and-choose method for three players.

30. Suppose the three beachcombers from problem 29 decide to move to another beach to collect shells and other items each morning to sell in their shops. The beach from which they collect is made up of 900 linear feet of sandy beach, 600 linear feet of rocky beach, and 300 linear feet of grassy beach.

- What is the total value Sondra would place on the entire beach, and what would be her fair-share requirement?
- What is the total value Lanie would place on the entire beach, and what would be his fair-share requirement?
- What is the total value Elsa would place on the entire beach, and what would be her fair-share requirement?
- If Sondra is the divider and decides on the following options, determine the result of the divide-and-choose method for three players.
 Option I: 300 feet of sandy beach, 200 feet of rocky beach, 100 feet of grassy beach
 Option II: 250 feet of sandy beach, 250 feet of rocky beach, 100 feet of grassy beach
 Option III: 350 feet of sandy beach, 150 feet of rocky beach, 100 feet of grassy beach
- If Elsa is the divider and decides on the following options, determine the result of the divide-and-choose method for three players.
 Option I: 400 feet of sandy beach, 150 feet of rocky beach, 125 feet of grassy beach
 Option II: 200 feet of sandy beach, 250 feet of rocky beach, 75 feet of grassy beach
 Option III: 300 feet of sandy beach, 200 feet of rocky beach, 100 feet of grassy beach

31. Consider the divide-and-choose method for three players. Eric, Suzanne, and Janet will divide a giant submarine sandwich that is part ham, part vegetarian, and part tuna. The divider sliced the sandwich into three portions; the point values placed on each portion by the players are displayed in the following table.

Player	Portion 1	Portion 2	Portion 3
Eric	17	10	30
Suzanne	15	15	15
Janet	20	10	10

- Who was the divider? Explain.
- Determine which portions each player thinks are fair.
- Determine a fair division of the sandwich.

32. For the sandwich division in problem 31, consider the following table.

Player	Portion 1	Portion 2	Portion 3
Eric	15	25	35
Suzanne	45	25	5
Janet	10	10	10

- Who was the divider? Explain.
 - Determine which portions each player thinks are fair.
 - Determine a fair division of the sandwich.
33. Three friends meet for dinner at an expensive restaurant. After the meal, the management gives them a platter containing three complimentary desserts: a 12-ounce portion of pudding, a 12-ounce portion of cobbler, and a 12-ounce portion of cheesecake. Sharon values pudding to cobbler to cheesecake in a ratio of 1 to 2 to 3. Ally prefers cheesecake to either of the other two desserts by a ratio of 1 to 1 to 3. Bev likes all the desserts equally well, for a 1-to-1-to-1 ratio.
- What point value might each woman assign to 1 ounce of each dessert? Fill in the following table with the point values that each woman would place on each dessert.

Dessert	Sharon's Point Value	Ally's Point Value	Bev's Point Value
12 oz pudding			
12 oz cobbler			
12 oz cheesecake			

- What total value would Sharon place on all the desserts? Determine a fair share based on her values.
- What total value would Ally place on all the desserts? Determine a fair share based on her values.
- What total value would Bev place on all the desserts? Determine a fair share based on her values.
- If Bev is the divider and decides not to subdivide the desserts, but leaves the three desserts intact on the three plates, then what is the result of applying the divide-and-choose method for three players?

34. Suppose the three friends in problem 33 decide they would like to sample each of the desserts. Bev is the divider and puts some of each dessert on each of three plates. Plate 1 contains 5 ounces of pudding, 3 ounces of cobbler, and 4 ounces of cheesecake. Plate 2 contains 2 ounces of pudding, 4 ounces of cobbler, and 6 ounces of cheesecake. Plate 3 contains 5 ounces of pudding, 5 ounces of cobbler, and 2 ounces of cheesecake.
- a. Using the preference ratios given in problem 33, what point value might each woman assign to 1 ounce of each dessert? Fill in the following table with the point value that each woman would assign to each plate.

Dessert	Sharon's Point Value	Ally's Point Value	Bev's Point Value
Plate 1 5 oz pudding 3 oz cobbler 4 oz cheesecake			
Plate 2 2 oz pudding 4 oz cobbler 6 oz cheesecake			
Plate 3 5 oz pudding 5 oz cobbler 2 oz cheesecake			

- b. What total value would Sharon place on all the desserts? Determine a fair share based on her values.
- c. What total value would Ally place on all the desserts? Determine a fair share based on her values.
- d. What total value would Bev place on all the desserts? Determine a fair share based on her values.
- e. If Bev is the divider and created the plates as indicated above, what is the result of applying the divide-and-choose method for three players?

35. Suppose four players will divide a cake using the divide-and-choose method for four players. Player 1 is the divider and cuts the cake into four pieces that each represent one-fourth of the value. The following table shows the opinions of the other three players: acceptable cuts are marked "accept" and unacceptable cuts are marked "reject." Explain how to obtain a fair division of the cake based on the players' expressed preferences.

	Slice A	Slice B	Slice C	Slice D
Player 2	Accept	Accept	Reject	Reject
Player 3	Reject	Accept	Accept	Reject
Player 4	Reject	Reject	Accept	Reject

36. Suppose four players will divide a cake using the divide-and-choose method for four players. Player 1 is the divider and cuts the cake into four pieces that each represent one-fourth of the value. The following table shows the opinions of the other three players: acceptable cuts are marked "accept" and unacceptable cuts are marked "reject." Explain how to obtain a fair division of the cake based on the players' expressed preferences.

	Slice A	Slice B	Slice C	Slice D
Player 2	Accept	Accept	Reject	Reject
Player 3	Accept	Reject	Accept	Reject
Player 4	Reject	Reject	Accept	Reject

Problems 37 through 44

The following problems use the last-diminisher method of fair division.

37. Three kids, Tia, Tom, and Tara, will partition the back yard so they can each build a fort. For each of the following scenarios, describe a fair division of the back yard using the last-diminisher method.
- a. Tia partitions a portion of the yard. Tom moves the divider to make the area smaller. Tara makes no changes.
- b. Tia partitions a portion of the yard. Tom and Tara make no changes.
- c. Tia partitions a portion of the yard. Tom makes the area smaller, then Tara makes the area even smaller.

38. Cleo, Rita, and Leon will share a platter of Chinese food. For each of the following scenarios, describe a fair division of the platter of food using the last-diminisher method.
- Cleo serves a portion of the food. Rita makes no changes. Leon removes a bit of food from the portion and returns it to the platter.
 - Cleo serves a portion of the food. Rita makes no changes. Leon makes no changes.
 - Cleo serves a portion of the food. Rita removes some food from the portion and returns it to the platter. Leon makes no changes.

39. Using the last-diminisher method, four players will divide a pie. Player 1 cuts a slice of pie that player 1 believes is valued at one-fourth of the entire pie. Each player, in order, either will judge the piece acceptable or will cut it smaller. The results of the first round are in the following table.

Player 1	Player 2	Player 3	Player 4
Cuts piece of pie	Believes piece is less than a fair share and makes no cuts	Believes piece is too big and trims the piece	Believes piece is less than a fair share and makes no cuts

- Which player keeps the piece of pie at the end of the first round?
 - Player 1 begins the second round by cutting a piece of pie. What value will player 1 put on the piece cut?
 - If player 2 and player 4 then both trim the piece of pie, who will keep the piece?
 - Who are the final two players, and how will they divide the rest of the pie equitably?
40. Five people will divide a pizza using the last-diminisher method. Player 1 begins by cutting a slice that is valued at one-fifth of the value of the entire pizza.
- If no player diminishes the piece cut by player 1, what happens?
 - In the second round, who will cut a piece of pizza and what value will it have compared with the pizza that is left?
 - If in round two player 3 and player 5, in turn, think the piece is too large and both trim it, what happens?
 - Who is still participating in round 3? Who will cut a piece of pizza? How will the rest of the pizza be fairly divided if no player trims the pizza?

41. Six people will use the last-diminisher method to divide a cake. Player 1 will begin and will cut a piece that is valued at one-sixth of the entire cake. Each player will decide, in turn, whether to trim the slice. The following table lists the players who trim the cake in each round.

	Player 1	Player 2	Player 3	Player 4	Player 5	Player 6
Round 1	Cut slice	Trim			Trim	
Round 2		Trim	Trim			
Round 3						
Round 4						Trim

- Which players are left after round 1?
 - Who cut the piece of cake in round 3?
 - List the players who keep the slice in rounds 1, 2, 3, and 4.
 - Who are the last two remaining players, and how will they fairly divide the remaining cake?
42. Six people will use the last-diminisher method to divide a cake. Player 1 will begin and will cut a piece that is valued at one-sixth of the entire cake. Each player will decide, in turn, whether to trim the slice. The following table lists the players who trim the cake in each round.

	Player 1	Player 2	Player 3	Player 4	Player 5	Player 6
Round 1	Cut slice					
Round 2			Trim	Trim		
Round 3			Trim		Trim	Trim
Round 4			Trim		Trim	

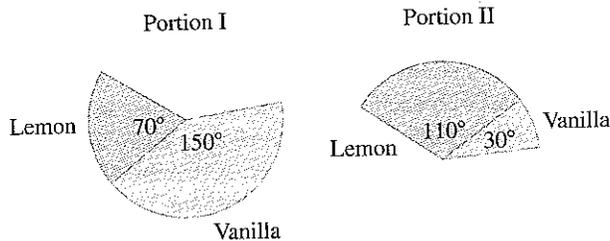
- Which players are left after round 1?
- List the players who cut the slice in each round.
- List the players who keep the slice in each round.
- Who are the last two remaining players, and how will they fairly divide the remaining cake?

43. Three friends, Galen, Leland, and Sandie, each have 15-day vacations scheduled in their time-share vacation homes. One is in the mountains at a ski resort. Another is in an oceanfront condominium. The third is at a cabin in the woods near a lake. They decide to divide the vacations so they each can enjoy all three locations. Galen's preference ratio for mountain to ocean to woods is 2 to 1 to 2, Leland's is 3 to 1 to 1, and Sandie's is 1 to 3 to 2. They will use the last-diminisher method to divide the days in the vacation homes fairly.
- Using points to represent the value a player places on each vacation day, determine how each player would value the three time-share locations. Also, for each player, determine what point value would constitute a fair share.
 - Suppose Galen is the divider and he creates the following package: 5 days in the mountains, 3 days at the ocean, and 6 days in the woods. Verify that this package represents a fair share according to Galen's values.
 - The package from part (b) will pass to Leland and then to Sandie. Explain whether either of them will "trim" the package, how the package will be trimmed, and who gets the package or the trimmed package.
 - Determine how the remaining two players can split the time fairly using the divide-and-choose method for two players. Many answers are possible.
44. Three friends, Galen, Leland, and Sandie, each have vacations scheduled in their time-share vacation homes. One is 14 days in the mountains at a ski resort. Another is 20 days in an oceanfront condominium. The third is 21 days in a cabin in the woods near a lake. They decide to divide the vacations so they each can enjoy all three locations. Galen has a broken leg and will not be able to ski, so his mountain to ocean to woods ratio is 1 to 5 to 4. Leland's is 3 to 1 to 1, and Sandie's is 6 to 3 to 1. They will use the last-diminisher method to divide the days in the vacation homes fairly.
- Using points to represent the value a player places on each vacation day, determine how each player would value the three time-share locations. Also, for each player, determine what point value would constitute a fair share.
 - Suppose Galen is the divider and he creates the following package: 6 days in the mountains, 8 days at the ocean, and 5 days in the woods. Verify that this package represents a fair share according to Galen's values.
 - The package from part (b) will pass to Sandie and then to Leland. Explain whether either of them will trim the package, how the package is trimmed, and who gets the package or the trimmed package.
 - Determine how the remaining two players can split the time fairly using the divide-and-choose method for two players.
45. This section discussed fair-division methods for continuous problems, in which one player was the divider, as in the divide-and-choose method for two or three players, and also in which each player had a chance to act as the divider and the chooser, as in the last-diminisher method. Another fair-division method for three players is called the lone-chooser method. In this case, two players divide the cake into two pieces using the divide-and-choose method. Each of the dividers then cuts his or her piece of cake into what he or she feels are three equally valued pieces. The player who is the lone chooser will select one piece from each divider. Explain how this results in a fair division of the cake.

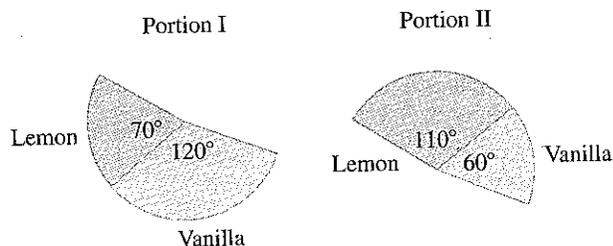
Extended Problems

46. Terry, Sean, and Nora want to share a cake that is half lemon and half vanilla. Terry prefers lemon to vanilla in a 3-to-1 ratio. Sean's lemon-to-vanilla preference ratio is 3 to 2. Nora values lemon over vanilla in a ratio of 4 to 1. They will use the lone-chooser method, and Nora will be the lone chooser. See problem 45 for details about the lone-chooser method.

- a. Terry and Sean are the dividers. If Terry cuts the cake into the following portions, which portion does Sean select?



- b. Terry and Sean are the dividers. If Sean cuts the cake into the following portions, which portion does Terry select?



- c. Verify that the portions all players end up with from parts (a) and (b) are fair shares according to their own values.

47. The Washington State University Math Department has created a website in which you can participate in and learn about fair-division strategies. You will find three interactive games at www.sci.wsu.edu/math/Lessons/FairDivision.

- a. In the Two-Person Lake Front Property game, you are one of two people who want to divide a continuous item. In this case, the item is property. Play the two-person game several times. Explain how to play the game and how to give both yourself and the computer an acceptable piece of land.
- b. In the Divorce Settlement game, you and the computer are divorcing. You must divide the home, car, boat, and dog. There are photos of these assets so you can decide how much you value each item. Explain how to play the game and describe a satisfactory division of the items.
- c. The Estate Division game allows for more than two people. Explain how to play the game and how to divide the estate fairly.

48. Another interactive program on the Internet is "The Fair Division Calculator," created by Francis Su from Harvey Mudd College. On this site, you will find an explanation of "fair division" and links to papers and other references. The Fair Division Calculator is interactive and allows the user to practice fair-division strategies for dividing continuous objects such as cake and undesirable items such as chores. Visit the site at www.math.hmc.edu/~su/fairdivision/ or search keywords "fair division calculator." Decide whether you would like to divide "goods," "burdens," or "rent," and practice the method several times. In an essay, explain how the method used led to a fair division.

49. The Spratly Islands are a group of more than 230 islands and reefs in the South China Sea. China, Taiwan, Vietnam, the Philippines, Malaysia, and Brunei have made claim to all or part of the land and surrounding water. Research the controversy surrounding the Spratly Islands and write a report summarizing the problem and the fair-division strategies that have been proposed to solve the problem.

50. The ocean is rich in resources, and conflicts between countries have arisen over pollution, fish, and the ocean floor's resources. On November 1, 1967, Malta's Ambassador to the United Nations asked the nations to focus on the escalating conflicts. It was clear that a comprehensive treaty was needed for the oceans, and thus the United Nations Convention on the Law of the Sea was held. During the drafting of the Convention, some countries were opposed in principle to a binding settlement to be decided by third-party judges or arbitrators. They insisted that issues could best be resolved by direct negotiations between states without requiring them to bring in outsiders. Research and write a report on how conflicts are settled over rights to ocean resources. Also, summarize the history behind the use of ocean resources. Many sources of information about this topic are available on the Internet. Go to www.un.org/depts/los/index.htm or search keywords "Law of the Sea Convention."