

6th-graders. Therefore, the final apportionment is 3 counselors for 4th-graders, 5 for 5th-graders, and 7 for 6th-graders. Now we will solve this same problem by applying Lowndes' method. We must first compute the relative fractional parts by dividing each fractional part by the integer part of the standard quota. Those results are shown in Table 5.15.

Group of Campers	Standard Quota	Integer Part	Fractional Part	Relative Fractional Part
4th grade	3.31	3	0.31	$\frac{0.31}{3} \approx 0.103$
5th grade	5.29	5	0.29	$\frac{0.29}{5} \approx 0.058$
6th grade	6.39	6	0.39	$\frac{0.39}{6} \approx 0.065$

Table 5.15

As before, based on the whole-number parts of the standard quotas, we see that there will be 3 counselors assigned to the 4th-grade campers, 5 to the 5th-grade campers, and 6 to the 6th-grade campers, but what about the remaining counselor? When the Lowndes method is used, the *relative* fractional parts determine which group gets the remaining counselor. From Table 5.15 we see that the group with the largest relative fractional part is the group of 4th-grade campers. Therefore, by Lowndes' method of apportionment, that group should be assigned the additional counselor. This means that the counselors should be assigned as follows: 4th-grade campers will have 4 counselors, 5th-grade campers will have 5, and 6th-grade campers will have 6. Notice how the two methods of apportionment led to different assignments of counselors.

PROBLEM SET 5.1

- Suppose the total population of a country is 85,000, and the number of legislative seats to be apportioned is 25.
 - Find the standard divisor. What does this number represent?
 - If there are two states with populations 30,600 and 54,400, then find each state's standard quota. What do these numbers represent?
- Suppose the total population of a country is 364,480, and the number of legislative seats to be apportioned is 80.
 - Find the standard divisor. What does this number represent?
 - If there are three states with populations 63,784, 123,012, and 177,684, then find each state's standard quota. What do these numbers represent?

3. The total population of a city is 57,200. Twenty school board seats are apportioned to four districts. The standard quotas for the four districts are (I) 3.67, (II) 4.53, (III) 6.05, and (IV) 5.75. Find the approximate populations of each of the four districts.
4. The total population of a city is 93,462. Thirty-seven school board seats are apportioned to six districts. The standard quotas for the six districts are (I) 6.13, (II) 2.85, (III) 7.88, (IV) 5.38, (V) 4.26, and (VI) 10.50. Find the approximate populations of each of the six districts.
5. The parents of three teenagers are notified that the teens have inherited from their grandmother 36 shares of stock in a certain company. There are no instructions as to how the shares should be split up, and the parents are asked to divide the shares among the teenagers as they see fit. The parents decide to split the shares based on how many hours each teenager spent helping their grandmother each month.
 - a. The parents estimate that the teens averaged a total of 72 hours at their grandmother's house each month. Based on the parents' scheme, what division will be performed to calculate the standard divisor? Determine the standard divisor and state its meaning.
 - b. Each month at their grandmother's house, Daphne spent 44 hours doing household chores, Mike spent 8 hours on garbage detail, and Melinda spent 20 hours caring for the lawn and plants. Find and interpret each teenager's standard quota.
6. While visiting the coast, a mother buys a bag of salt-water taffy for her four children. The bag contains 96 pieces of taffy. The mother decides to apportion the taffy according to the ages of the children: 4, 7, 9, and 12 years.
 - a. What division will the mother perform to calculate the standard divisor? Determine the standard divisor and state its meaning.
 - b. Find and interpret each child's standard quota.

7. A school board allocates 10 seats for student representatives. Hamilton's method will be used to apportion the student seats to the four high school classes.

- a. Standard quotas for the high school classes are shown in the following table. Complete the remaining columns in the table.

Class	Standard Quota	Integer Part	Fractional Part
Freshman	1.39		
Sophomore	3.50		
Junior	3.23		
Senior	1.88		

- b. What is the total of the integer parts of the standard quotas, and how many more seats must be distributed?
 - c. What is the final apportionment of seats?
8. The Shady Grove condominium complex contains five buildings. According to the bylaws, the representative council is to be made up of 17 seats apportioned according to Hamilton's method.
 - a. Standard quotas for the five buildings are shown in the following table. Complete the remaining columns in the table.

Building	Standard Quota	Integer Part	Fractional Part
Cedar	3.88		
Oak	2.24		
Willow	5.25		
Pine	4.32		
Maple	1.31		

- b. What is the total of the integer parts of the standard quotas, and how many more seats must be distributed?
- c. What is the final apportionment of seats?

12. A small country with four states has 50 seats in the legislature. The populations of the states are as follows:

State	Population
Smallorado	275,000
Tinysssee	767,000
Minisota	465,000
Weesconsin	383,000

- a. Apportion the seats in the legislature according to Hamilton's method.
- b. If a mistake was discovered, and the actual total population for Smallorado was 276,000, how would the apportionment change?
13. The faculty senate at Dartvard University decided to reorganize in 2001. The new 30-seat senate will be apportioned according to Hamilton's method and will be based on enrollment in its five colleges. Apportion the seats in the faculty senate according to Hamilton's method.

College	Enrollment
Fine and Performing Arts	2540
Math and Physical Science	3580
Engineering	1410
Social Science	1830
Agriculture	750

14. The faculty senate at Dartvard University reorganizes every 3 years. Apportion the 30 seats in the 2004 faculty senate according to Hamilton's method.

College	Enrollment
Fine and Performing Arts	2930
Math and Physical Science	3320
Engineering	1290
Social Science	2140
Agriculture	1010

9. Suppose Mississippi, Alabama, Georgia, and Florida were the only states in the United States in the year 2000. According to the U.S. Census Bureau, their 2000 apportionment populations were as follows.

Mississippi:	2,852,927
Alabama:	4,461,130
Georgia:	8,206,975
Florida:	16,028,890

- a. What was the total population of the four states?
- b. Find the standard divisor, assuming that there are 435 seats to be apportioned to the House of Representatives.
- c. Find each state's standard quota.
- d. Apportion the 435 seats according to Hamilton's method.
10. Suppose California, Oregon, and Washington were the only states in the United States in the year 2000. According to the U.S. Census Bureau, their 2000 apportionment populations were as follows.

California:	33,930,798
Oregon:	3,428,543
Washington:	5,908,684

- a. What is the total population of the three states?
- b. Find the standard divisor, assuming that there are 435 seats to be apportioned to the House of Representatives.
- c. Find each state's standard quota.
- d. Apportion the 435 seats according to Hamilton's method.
11. Three friends have pooled their resources to bid on 20 bottles of vintage red wine at a wine auction. They decide to divide the bottles based on the amount each person contributed, using Hamilton's method of apportionment.

- a. If Jaron contributed \$295, Mikkell contributed \$205, and Robert contributed \$390, how should the 20 bottles be apportioned?
- b. If Mikkell had contributed an additional \$20, how would the apportionment change?

Problems 15 and 16

In 1791, the Hamilton method of apportionment for the House of Representatives was approved. However, President Washington vetoed the bill, and the Hamilton method was not used until 1852. The following table contains apportionment population totals for the states that were part of the United States in the years 1790 and 1800.

State	1790	1800
Connecticut	236,841	250,622
Delaware	55,540	61,812
Georgia	70,835	138,807
Kentucky	68,705	204,822
Maryland	278,514	306,610
Massachusetts	475,327	574,564
New Hampshire	141,822	183,855
New Jersey	179,570	206,181
New York	331,589	577,805
North Carolina	353,523	424,785
Pennsylvania	432,879	601,863
Rhode Island	68,446	68,970
South Carolina	206,236	287,131
Tennessee	Not a state until 1796	100,169
Vermont	85,533	154,465
Virginia	630,560	747,362
<i>Total</i>	3,615,920	4,889,823

Source: Balinski and Young, *Fair Representation Meeting the Ideal of One Man, One Vote*, 2nd Ed. Brookings Institution Press, 2001 Washington, DC.

15. In 1790, there were 105 seats in the House of Representatives.
- Find and interpret the standard divisor.
 - Apportion the seats according to Hamilton's method.

16. In 1800, there were 141 seats in the House of Representatives.
- Find and interpret the standard divisor.
 - Apportion the seats according to Hamilton's method.
17. The minimum ratio of population to representative as stated in Article 1, Section 2, of the U.S. Constitution is 30,000. In 1840, the apportionment population of the United States was approximately 15,908,376, and there were 223 seats in the House of Representatives.
- Find and interpret the standard divisor.
 - How many seats would have been required to maintain a standard divisor of 30,000?
18. In 2000, the apportionment population of the United States was approximately 281,424,177, and there were 435 members of the House of Representatives.
- Find and interpret the standard divisor.
 - How many seats would have been required to maintain a standard divisor of 30,000?
19. Suppose there are six seats to apportion to three states. The apportionment is to follow Hamilton's method. The states have populations of 77,500; 77,500; and 45,000.
- Find the standard divisor and interpret its meaning.
 - Find the standard quota for each state.
 - How will the seats be apportioned?
20. Suppose a seventh seat is added to the House of Representatives from the previous problem.
- Find the standard divisor and interpret its meaning.
 - Find the standard quota for each state.
 - How will the seats be apportioned under Hamilton's method?

23. Consider the following table, which shows five states and their standard quotas from the apportionment based on Census 2000.

State	Standard Quota	Integer Part	Fractional Part	Relative Fractional Part
Florida	24.78			
Mississippi	4.41			
Alabama	6.90			
Georgia	12.69			
Tennessee	8.81			

Source: U.S. Census Bureau, Census 2000

21. Eleven teaching assistants will be apportioned to five sections of an algebra class at a university on the basis of class size.
- a. The following table contains the standard quota for each class. Complete the table, then list the class sections in order from largest to smallest according to the relative fractional part.

Class Section	Standard Quota	Integer Part	Fractional Part	Relative Fractional Part
A	3.23			
B	2.25			
C	1.49			
D	2.21			
E	1.82			

- a. Complete the table for each of the five states.
- b. These five states currently hold 58 seats in the House of Representatives. Considering only these five states, how would the 58 seats be apportioned according to Lowndes' method?
- c. How would the 58 seats be apportioned according to Hamilton's method?

22. The math department at a large community college schedules classes based on preregistration numbers. The department plans to offer a total of 20 sections of three different algebra courses.
- a. Preregistration figures are listed in the following table. Complete the table, then list the algebra classes in order from largest to smallest according to the relative fractional part.
- b. How will the 20 sections be apportioned according to Lowndes' method?
- c. How will the 20 sections be apportioned according to Hamilton's method?

Class	Preregistration Numbers	Standard Quota	Integer Part	Fractional Part	Relative Fractional Part
Beginning Algebra	131				
Intermediate Algebra	275				
College Algebra	195				

24. Consider the following table, which shows five states and their standard quotas from the apportionment based on Census 2000.

State	Standard Quota	Integer Part	Fractional Part	Relative Fractional Part
Wisconsin	8.30			
Illinois	19.23			
Indiana	9.41			
Ohio	17.58			
Michigan	15.39			

Source: U.S. Census Bureau, Census 2000

- Complete the table for each of the five states.
- These five states currently hold 69 seats in the House of Representatives. Considering only these five states, how would the 69 seats be apportioned according to Lowndes' method?
- How would the 69 seats be apportioned according to Hamilton's method?

Problems 25 and 26

The following table gives the populations for the five counties of Hawaii in 1990 and in 2000.

County	1990 Population	2000 Population
Hawaii	120,317	148,677
Honolulu	836,231	876,156
Kalawao	130	147
Kauai	51,177	58,463
Maui	100,374	128,094
Total	1,108,229	1,211,537

Source: U.S. Census Bureau

25. Suppose that in 1990 Hawaii decided to create a commission to study the use of natural resources. Further, the state decided that there would be a representative for approximately every 25,000 people, and that the representative seats would be apportioned to the five counties using Lowndes' method.
- How many representatives were needed? Round to the nearest whole number.
 - What was the standard divisor and standard quota for each county in 1990?
 - List the relative fractional parts for each county.
 - How were the representative seats apportioned?

26. Suppose that in the year 2000 the representative seats needed to be reapportioned because of changes in the population. It was decided that one seat would represent approximately every 25,000 people, and that the representative seats would be apportioned using Lowndes' method.

- How many representatives were required in 2000? Round to the nearest whole number.
- What was the standard divisor and standard quota for each county in 2000?
- List the relative fractional parts for each county.
- How were the representative seats apportioned?

Problems 27 through 30

Suppose Hamilton's method had been used to apportion the number of representative seats given for the five counties in Hawaii from problems 25 and 26. How would the seats have been apportioned?

- 50 representative seats in 1990.
- 60 representative seats in 1990.
- 54 representative seats in 2000.
- 63 representative seats in 2000.

Problems 31 and 32

Use the U.S. state apportionment population totals for 1790 and 1800 from problems 15 and 16 to answer the following questions.

- In 1790, there were 105 seats in the House of Representatives. Apportion the seats according to Lowndes' method.
- In 1800, there were 141 seats in the House of Representatives. Apportion the seats according to Lowndes' method.

Problems 33 through 36

The state of Delaware is divided into three counties: Kent, New Castle, and Sussex. The population totals for each county in the years 2001 and 2002 and the area of each county are given in the following table.

County	2001 Population	2002 Population	Area in Square Miles
Kent	128,822	131,069	590
New Castle	507,085	512,370	426
Sussex	160,692	163,946	938
Total	796,599	807,385	1954

Source: U.S. Census Bureau

Problems 37 and 38

The country of Costa Rica is a democratic independent republic. Its constitution divides the government into independent executive, legislative, and judicial branches. The legislature is a national assembly made up of 57 seats, which are apportioned after each census to the country's seven provinces. The following table contains approximate 2002 population totals for each province.

Province	Approximate 2002 Population
Alajuela	716,286
Cartago	432,395
Guanacaste	264,238
Heredia	354,732
Limón	339,295
Puntarenas	357,483
San José	1,345,750
Total	3,810,179

Source: www.citypopulation.de/CostaRica.html

37. How should the 57 seats be apportioned according to Hamilton's method?

38. How should the 57 seats be apportioned according to Lowndes' method?

33. Suppose the 2001 state budget allowed for 360 state police officers.

- a. How would the police officers have been apportioned to each county using Hamilton's method if totals?

- b. How would the police officers have been apportioned to each county using Hamilton's method if apportionment was done according to county area?

34. Suppose the 2002 state budget allowed for 354 state police officers.

- a. How would the police officers have been apportioned to each county using Hamilton's method if apportionment was done according to population totals?

- b. How would the police officers have been apportioned to each county using Hamilton's method if apportionment was done according to county area?

35. Repeat problem 33 using Lowndes' method.

36. Repeat problem 34 using Lowndes' method.

39. An alternative method for finding the standard quotas is to divide each state's population, p , by the total population of the country, P , and then multiply by the number of seats to be apportioned, M . In other words, each state receives the same percentage of seats as its percentage of the country's population. Set up an algebraic expression for this calculation, and show that it is equivalent to the standard method for finding the standard quotas as was presented in this section.

40. The U.S. Constitution, in Section 2 of Article 1, specifies the way in which the U.S. House of Representatives is to be constituted. While the number of representatives is not to exceed 1 for every 30,000 people, each state must have at least one representative. There have been many arguments, changes, and controversies over apportionment in the House of Representatives. Research the history of apportionment as it relates to the House of Representatives. Who were the main figures behind the various apportionment systems we have used? Create a time

- line from 1789 to today of major events, important changes, and discoveries.

41. The apportionment of 1872 was controversial. Congress added nine seats without adopting any particular method of apportionment. Then in 1876

Rutherford B. Hayes became President of the United States based on an apportionment that some say was unconstitutional. Later it was shown by M. L. Balinski and H. P. Young that if Hamilton's method had been followed, Samuel Tilden would have been elected. Research the apportionment of 1872 and the election of 1876. What method of apportionment was used and what was the result? How would the apportionment have changed if Hamilton's method had been used? For apportionment population values, go to www.uwm.edu/~margo/apport/datasets.htm. Summarize your findings in a report.

42. Research and determine which countries currently use Hamilton's method of apportionment.

43. The country of Sweden is a parliamentary democracy. Every 4 years, the Swedish people elect representatives to the Riksdag, the Swedish parliament. There are 349 parliament seats that represent the 29 constituencies. Of the 349 seats, 310 are “fixed constituency” seats. Research the way in which the 310 fixed seats are apportioned to the 29 constituencies. How are the other 39 seats apportioned? One source for information is the Swedish government website at www.sweden.gov.se/. Write a report that summarizes your findings.
44. There was no reapportionment in the United States after the 1920 census. Research why it was never done. A presidential election was held in 1924. Would a reapportionment have affected the outcome of the 1924 election? Who was running for president in 1924? Were the results close? Summarize your findings in a report.
45. The 1990 census revealed that the population of certain U.S. states had increased more rapidly than the national average because of immigration. The results of the reapportionment after the 1990 census caused Montana and 12 other states to lose a seat each, while 8 states gained a total of 19 seats. The state of Montana filed a lawsuit over the reapportionment. Research the case made by Montana (*Montana v. U.S. Department of Commerce*) in the U.S. District Court in 1991. Why did the state of Montana believe it had a case against the government over the reapportionment? The case eventually went to the U.S. Supreme Court (*U.S. Department of Commerce v. Montana*) and was decided on March 31, 1992. How did each court rule? Were any other states trying to fight the reapportionment? Write a report that summarizes your findings.
46. A distinction has been made between “population” and “apportionment population”. On the U.S. Census Bureau website (www.census.gov), find U.S. Census data beginning from 1790. The total population values do not match the apportionment population values. Why are they different? Adjustments were made in the past according to certain criteria, and different adjustments are made today. Research the difference between total population and apportionment population and the criteria used to adjust populations. Write a report that summarizes your findings.
47. Every 10 years, based on the U.S. Census, the seats of the House of Representatives are reapportioned to the states. In the 1960s, the Supreme Court interpreted the Constitution to require that each U.S. House Legislative District have an equal number of people. Therefore, any states with more than one district must adjust their district lines based on the census. This adjustment is called redistricting. Research the process of redistricting. What are the rules that must be followed? What is gerrymandering? How does redistricting affect who is elected to the House of Representatives? Summarize your findings in a report.

5.2 The Apportionment Problem—Divisor Methods

INITIAL PROBLEM



Suppose that you, your sister, and your brother have inherited from your grandfather 85 gold coins from a sunken Spanish galleon. A firm believer in community service, your grandfather has specified that the coins should be allocated based on the number of hours that each of you has devoted to his favorite charity, the local soup kitchen, over the last calendar year. The table below shows the number of hours each of you worked at the soup kitchen during the past year. On this basis, how should the coins be apportioned? What method will benefit you the most?

Person	Hours Worked at Soup Kitchen
You	72
Your sister	43.5
Your brother	34.5

A solution of this Initial Problem is on page 314.