



EXAMPLE 5 Find the zeros of quadratic functions

UNDERSTAND REPRESENTATIONS

If a real number k is a zero of the function $y = ax^2 + bx + c$, then k is an x -intercept of this function's graph and k is also a root of the equation $ax^2 + bx + c = 0$.

Find the zeros of the function by rewriting the function in intercept form.

a. $y = x^2 - x - 12$

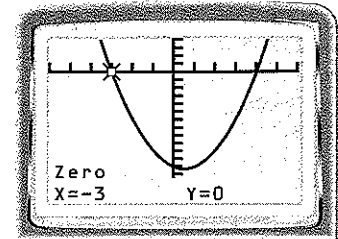
b. $y = x^2 + 12x + 36$

Solution

a. $y = x^2 - x - 12$ Write original function.
 $= (x + 3)(x - 4)$ Factor.

The zeros of the function are -3 and 4 .

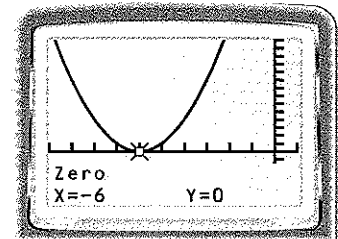
CHECK Graph $y = x^2 - x - 12$. The graph passes through $(-3, 0)$ and $(4, 0)$.



b. $y = x^2 + 12x + 36$ Write original function.
 $= (x + 6)(x + 6)$ Factor.

The zero of the function is -6 .

CHECK Graph $y = x^2 + 12x + 36$. The graph passes through $(-6, 0)$.



GUIDED PRACTICE for Example 5

Find the zeros of the function by rewriting the function in intercept form.

10. $y = x^2 + 5x - 14$

11. $y = x^2 - 7x - 30$

12. $f(x) = x^2 - 10x + 25$

4.3 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS8 for Exs. 33, 47, and 67
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 41, 56, 58, 63, and 71
- ◆ = MULTIPLE REPRESENTATIONS Ex. 68

SKILL PRACTICE

- VOCABULARY** What is a zero of a function $y = f(x)$?
- ★ WRITING** Explain the difference between a monomial, a binomial, and a trinomial. Give an example of each type of expression.

EXAMPLE 1

on p. 252
for Exs. 3–14

FACTORING Factor the expression. If the expression cannot be factored, say so.

- | | | |
|---------------------|---------------------|---------------------|
| 3. $x^2 + 6x + 5$ | 4. $x^2 - 7x + 10$ | 5. $a^2 - 13a + 22$ |
| 6. $r^2 + 15r + 56$ | 7. $p^2 + 2p + 4$ | 8. $q^2 - 11q + 28$ |
| 9. $b^2 + 3b - 40$ | 10. $x^2 - 4x - 12$ | 11. $x^2 - 7x - 18$ |
| 12. $c^2 - 9c - 18$ | 13. $x^2 + 9x - 36$ | 14. $m^2 + 8m - 65$ |

EXAMPLE 2

on p. 253
for Exs. 15–23

EXAMPLE 3

on p. 254
for Exs. 24–41

EXAMPLE 4

on p. 254
for Exs. 42–43

EXAMPLE 5

on p. 255
for Exs. 44–55

FACTORING WITH SPECIAL PATTERNS Factor the expression.

15. $x^2 - 36$

16. $b^2 - 81$

17. $x^2 - 24x + 144$

18. $t^2 - 16t + 64$

19. $x^2 + 8x + 16$

20. $c^2 + 28c + 196$

21. $n^2 + 14n + 49$

22. $s^2 - 26s + 169$

23. $z^2 - 121$

SOLVING EQUATIONS Solve the equation.

24. $x^2 - 8x + 12 = 0$

25. $x^2 - 11x + 30 = 0$

26. $x^2 + 2x - 35 = 0$

27. $a^2 - 49 = 0$

28. $b^2 - 6b + 9 = 0$

29. $c^2 + 5c + 4 = 0$

30. $n^2 - 6n = 0$

31. $t^2 + 10t + 25 = 0$

32. $w^2 - 16w + 48 = 0$

33. $z^2 - 3z = 54$

34. $r^2 + 2r = 80$

35. $u^2 = -9u$

36. $m^2 = 7m$

37. $14x - 49 = x^2$

38. $-3y + 28 = y^2$

ERROR ANALYSIS Describe and correct the error in solving the equation.

39.

$$x^2 - x - 6 = 0$$

$$(x - 2)(x + 3) = 0$$
~~$$x - 2 = 0 \text{ or } x + 3 = 0$$

$$x = 2 \text{ or } x = -3$$~~

40.

$$x^2 + 7x + 6 = 14$$

$$(x + 6)(x + 1) = 14$$
~~$$x + 6 = 14 \text{ or } x + 1 = 14$$

$$x = 8 \text{ or } x = 13$$~~

41. **★ MULTIPLE CHOICE** What are the roots of the equation $x^2 + 2x - 63 = 0$?**(A)** 7, -9**(B)** -7, -9**(C)** -7, 9**(D)** 7, 9**WRITING EQUATIONS** Write an equation that you can solve to find the value of x .42. A rectangular picnic site measures 24 feet by 10 feet. You want to double the site's area by adding the same distance x to the length and the width.43. A rectangular performing platform in a park measures 10 feet by 12 feet. You want to triple the platform's area by adding the same distance x to the length and the width.**FINDING ZEROS** Find the zeros of the function by rewriting the function in intercept form.

44. $y = x^2 + 6x + 8$

45. $y = x^2 - 8x + 16$

46. $y = x^2 - 4x - 32$

47. $y = x^2 + 7x - 30$

48. $f(x) = x^2 + 11x$

49. $g(x) = x^2 - 8x$

50. $y = x^2 - 64$

51. $y = x^2 - 25$

52. $f(x) = x^2 - 12x - 45$

53. $g(x) = x^2 + 19x + 84$

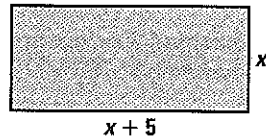
54. $y = x^2 + 22x + 121$

55. $y = x^2 + 2x + 1$

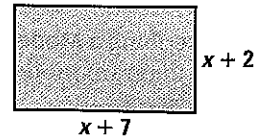
56. **★ MULTIPLE CHOICE** What are the zeros of $f(x) = x^2 + 6x - 55$?**(A)** -11, -5**(B)** -11, 5**(C)** -5, 11**(D)** 5, 1157. **REASONING** Write a quadratic equation of the form $x^2 + bx + c = 0$ that has roots 8 and 11.58. **★ SHORT RESPONSE** For what integers b can the expression $x^2 + bx + 7$ be factored? Explain.

GEOMETRY Find the value of x .

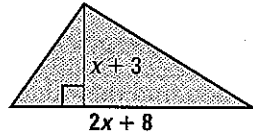
59. Area of rectangle = 36



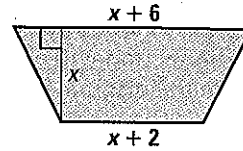
60. Area of rectangle = 84



61. Area of triangle = 42



62. Area of trapezoid = 32



63. **★ OPEN-ENDED MATH** Write a quadratic function with zeros that are equidistant from 10 on a number line.
64. **CHALLENGE** Is there a formula for factoring the *sum* of two squares? You will investigate this question in parts (a) and (b).
- Consider the sum of two squares $x^2 + 16$. If this sum can be factored, then there are integers m and n such that $x^2 + 16 = (x + m)(x + n)$. Write two equations that m and n must satisfy.
 - Show that there are no integers m and n that satisfy both equations you wrote in part (a). What can you conclude?

PROBLEM SOLVING

EXAMPLE 4
on p. 254
for Exs. 65–67

65. **SKATE PARK** A city's skate park is a rectangle 100 feet long by 50 feet wide. The city wants to triple the area of the skate park by adding the same distance x to the length and the width. Write and solve an equation to find the value of x . What are the new dimensions of the skate park?

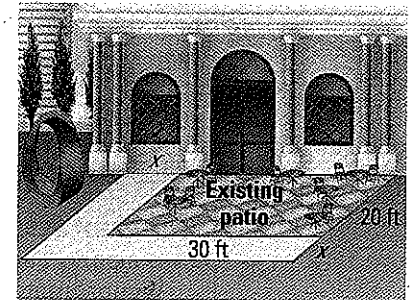
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66. **ZOO** A rectangular enclosure at a zoo is 35 feet long by 18 feet wide. The zoo wants to double the area of the enclosure by adding the same distance x to the length and the width. Write and solve an equation to find the value of x . What are the new dimensions of the enclosure?

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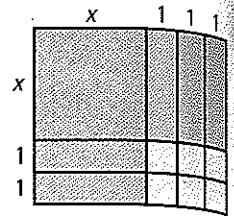
67. **MULTI-STEP PROBLEM** A museum has a café with a rectangular patio. The museum wants to add 464 square feet to the area of the patio by expanding the existing patio as shown.

- Find the area of the existing patio.
- Write a verbal model and an equation that you can use to find the value of x .
- Solve your equation. By what distance x should the length and the width of the patio be expanded?



68. **MULTIPLE REPRESENTATIONS** Use the diagram shown.

- Writing an Expression** Write a quadratic trinomial that represents the area of the diagram.
- Describing a Model** Factor the expression from part (a). *Explain* how the diagram models the factorization.
- Drawing a Diagram** Draw a diagram that models the factorization $x^2 + 8x + 15 = (x + 5)(x + 3)$.

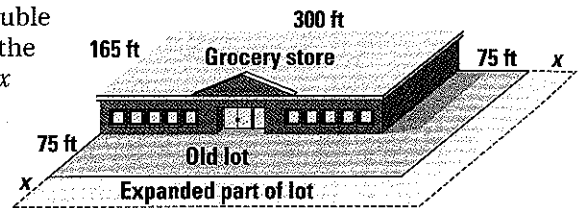


69. **SCHOOL FAIR** At last year's school fair, an 18 foot by 15 foot rectangular section of land was roped off for a dunking booth. The length and width of the section will each be increased by x feet for this year's fair in order to triple the original area. Write and solve an equation to find the value of x . What is the length of rope needed to enclose the new section?

70. **RECREATION CENTER** A rectangular deck for a recreation center is 21 feet long by 20 feet wide. Its area is to be halved by subtracting the same distance x from the length and the width. Write and solve an equation to find the value of x . What are the deck's new dimensions?

71. **★ SHORT RESPONSE** A square garden has sides that are 10 feet long. A gardener wants to double the area of the garden by adding the same distance x to the length and the width. Write an equation that x must satisfy. Can you solve the equation you wrote by factoring? *Explain* why or why not.

72. **CHALLENGE** A grocery store wants to double the area of its parking lot by expanding the existing lot as shown. By what distance x should the lot be expanded?



MIXED REVIEW

PREVIEW

Prepare for
Lesson 4.4
in Exs. 73–81.

Solve the equation.

73. $2x - 1 = 0$ (p. 18)

74. $3x + 4 = 0$ (p. 18)

75. $-8x + 7 = 0$ (p. 18)

76. $6x + 5 = 0$ (p. 18)

77. $4x - 5 = 0$ (p. 18)

78. $3x + 1 = 0$ (p. 18)

79. $|x - 6| = 7$ (p. 51)

80. $|2x - 5| = 10$ (p. 51)

81. $|4 - 3x| = 8$ (p. 51)

Graph the function.

82. $y = 3x - 1$ (p. 89)

83. $f(x) = -\frac{1}{4}x + 2$ (p. 89)

84. $y = |x - 4| - 4$ (p. 123)

85. $y = \frac{1}{2}|x + 1| + 2$ (p. 123)

86. $y = -2x^2 + 8x + 7$ (p. 236)

87. $g(x) = -2(x + 1)^2 - 4$ (p. 245)

88. $f(x) = (x + 4)(x - 2)$ (p. 245)

89. $y = -(x - 3)(x - 7)$ (p. 245)

90. **PARK DESIGN** A city plans to place a playground in a triangular region of a park. The vertices of the triangle are $(0, 0)$, $(14, 3)$, and $(6, 25)$ where the coordinates are given in feet. Find the area of the triangular region. (p. 203)

4.4 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS
on p. WS8 for Exs. 27, 39, and 63

★ = STANDARDIZED TEST PRACTICE
Exs. 2, 12, 64, 65, and 67

SKILL PRACTICE

1. **VOCABULARY** What is the greatest common monomial factor of the terms of the expression $12x^2 + 8x + 20$?

2. ★ **WRITING** Explain how the values of a and c in $ax^2 + bx + c$ help you determine whether you can use a perfect square trinomial factoring pattern.

EXAMPLES

1 and 2

on p. 259
for Exs. 3–12

FACTORING Factor the expression. If the expression cannot be factored, say so.

3. $2x^2 + 5x + 3$

4. $3n^2 + 7n + 4$

5. $4r^2 + 5r + 1$

6. $6p^2 + 5p + 1$

7. $11z^2 + 2z - 9$

8. $15x^2 - 2x - 8$

9. $4y^2 - 5y - 4$

10. $14m^2 + m - 3$

11. $9d^2 - 13d - 10$

12. ★ **MULTIPLE CHOICE** Which factorization of $5x^2 + 14x - 3$ is correct?

(A) $(5x - 3)(x + 1)$

(B) $(5x + 1)(x - 3)$

(C) $5(x - 1)(x + 3)$

(D) $(5x - 1)(x + 3)$

EXAMPLE 3

on p. 260
for Exs. 13–21

FACTORING WITH SPECIAL PATTERNS Factor the expression.

13. $9x^2 - 1$

14. $4r^2 - 25$

15. $49n^2 - 16$

16. $16s^2 + 8s + 1$

17. $49x^2 + 70x + 25$

18. $64w^2 + 144w + 81$

19. $9p^2 - 12p + 4$

20. $25t^2 - 30t + 9$

21. $36x^2 - 84x + 49$

EXAMPLE 4

on p. 260
for Exs. 22–31

FACTORING MONOMIALS FIRST Factor the expression.

22. $12x^2 - 4x - 40$

23. $18z^2 + 36z + 16$

24. $32v^2 - 2$

25. $6u^2 - 24u$

26. $12m^2 - 36m + 27$

27. $20x^2 + 124x + 24$

28. $21x^2 - 77x - 28$

29. $-36n^2 + 48n - 15$

30. $-8y^2 + 28y - 60$

31. **ERROR ANALYSIS** Describe and correct the error in factoring the expression.

$$4x^2 - 36 = 4(x^2 - 36)$$

$$= 4(x + 6)(x - 6)$$

EXAMPLE 5

on p. 261
for Exs. 32–40

SOLVING EQUATIONS Solve the equation.

32. $16x^2 - 1 = 0$

33. $11q^2 - 44 = 0$

34. $14s^2 - 21s = 0$

35. $45n^2 + 10n = 0$

36. $4x^2 - 20x + 25 = 0$

37. $4p^2 + 12p + 9 = 0$

38. $15x^2 + 7x - 2 = 0$

39. $6r^2 - 7r - 5 = 0$

40. $36z^2 + 96z + 15 = 0$

EXAMPLE 7

on p. 262
for Exs. 41–49

FINDING ZEROS Find the zeros of the function by rewriting the function in intercept form.

41. $y = 4x^2 - 19x - 5$

42. $g(x) = 3x^2 - 8x + 5$

43. $y = 5x^2 - 27x - 18$

44. $f(x) = 3x^2 - 3x$

45. $y = 11x^2 - 19x - 6$

46. $y = 16x^2 - 2x - 5$

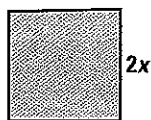
47. $y = 15x^2 - 5x - 20$

48. $y = 18x^2 - 6x - 4$

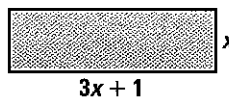
49. $g(x) = 12x^2 + 5x - 7$

GEOMETRY Find the value of x .

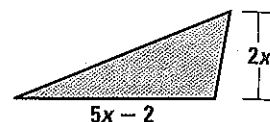
50. Area of square = 36



51. Area of rectangle = 30



52. Area of triangle = 115



SOLVING EQUATIONS Solve the equation.

53. $2x^2 - 4x - 8 = -x^2 + x$

54. $24x^2 + 8x + 2 = 5 - 6x$

55. $18x^2 - 22x = 28$

56. $13x^2 + 21x = -5x^2 + 22$

57. $x = 4x^2 - 15x$

58. $(x + 8)^2 = 16 - x^2 + 9x$

CHALLENGE Factor the expression.

59. $2x^3 - 5x^2 + 3x$

60. $8x^4 - 8x^3 - 6x^2$

61. $9x^3 - 4x$

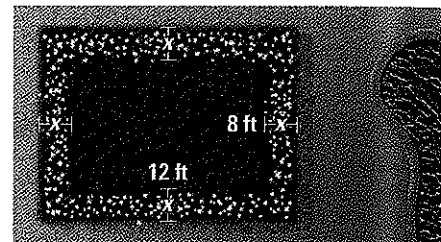
PROBLEM SOLVING

EXAMPLE 6
on p. 261
for Exs. 62–63

62. **ARTS AND CRAFTS** You have a rectangular stained glass window that measures 2 feet by 1 foot. You have 4 square feet of glass with which to make a border of uniform width around the window. What should the width of the border be?

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63. **URBAN PLANNING** You have just planted a rectangular flower bed of red roses in a city park. You want to plant a border of yellow roses around the flower bed as shown. Because you bought the same number of red and yellow roses, the areas of the border and flower bed will be equal. What should the width of the border of yellow roses be?



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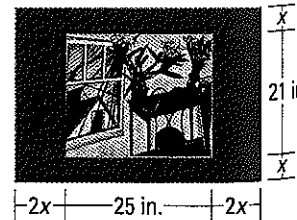
EXAMPLE 7
on p. 262
for Exs. 64–65

64. **★ MULTIPLE CHOICE** A surfboard shop sells 45 surfboards per month when it charges \$500 per surfboard. For each \$20 decrease in price, the store sells 5 more surfboards per month. How much should the shop charge per surfboard in order to maximize monthly revenue?

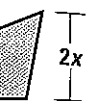
- (A) \$340 (B) \$492 (C) \$508 (D) \$660

65. **★ SHORT RESPONSE** A restaurant sells about 330 sandwiches each day at a price of \$6 each. For each \$.25 decrease in price, 15 more sandwiches are sold per day. How much should the restaurant charge to maximize daily revenue? *Explain* each step of your solution. What is the maximum daily revenue?

66. **PAINTINGS** You place a mat around a 25 inch by 21 inch painting as shown. The mat is twice as wide at the left and right of the painting as it is at the top and bottom of the painting. The area of the mat is 714 square inches. How wide is the mat at the left and right of the painting? at the top and bottom of the painting?

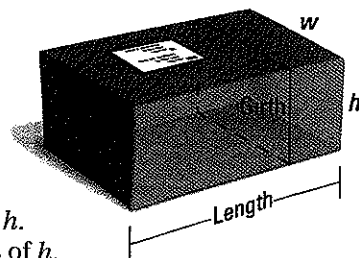


$e = 115$



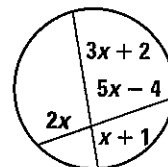
$$x^2 + 9x$$

67. ★ **EXTENDED RESPONSE** A U.S. Postal Service guideline states that for a rectangular package like the one shown, the sum of the length and the girth cannot exceed 108 inches. Suppose that for one such package, the length is 36 inches and the girth is as large as possible.



- What is the girth of the package?
- Write an expression for the package's width w in terms of h . Write an equation giving the package's volume V in terms of h .
- What height and width maximize the volume of the package? What is the maximum volume? *Explain* how you found it.

68. **CHALLENGE** Recall from geometry the theorem about the products of the lengths of segments of two chords that intersect in the interior of a circle. Use this theorem to find the value of x in the diagram.



MIXED REVIEW

PREVIEW

Prepare for
Lesson 4.5
in Exs. 69–74.

Solve the equation. (p. 18)

69. $11 + 12x = 3(4x + 7)$

70. $6x - 19 = 5(3 + 2x)$

71. $-9(5x + 3) = 9x - 42$

72. $6(x - 7) = 15(2x - 4)$

73. $9(x - 3) = 3(5x - 17)$

74. $4(3x - 11) = 3(11 - x) + x$

Solve the system of linear equations using Cramer's rule. (p. 203)

75. $4x + 9y = -14$
 $3x + 5y = -7$

76. $8x + 5y = -2$
 $2x + 3y = 14$

77. $5x - 8y = -50$
 $2x - 3y = -25$

Graph the function.

78. $y = x^2 - 3x - 18$ (p. 236)

79. $f(x) = 2x^2 + 11x + 5$ (p. 236)

80. $y = 3(x + 6)^2 - 2$ (p. 245)

81. $g(x) = (x + 4)(x - 5)$ (p. 245)

QUIZ for Lessons 4.1–4.4

Graph the function. Label the vertex and axis of symmetry. (p. 236)

1. $y = x^2 - 6x + 14$

2. $y = 2x^2 + 8x + 15$

3. $f(x) = -3x^2 + 6x - 5$

Write the quadratic function in standard form. (p. 245)

4. $y = (x - 4)(x - 8)$

5. $g(x) = -2(x + 3)(x - 7)$

6. $y = 5(x + 6)^2 - 2$

Solve the equation.

7. $x^2 + 9x + 20 = 0$ (p. 252)

8. $n^2 - 11n + 24 = 0$ (p. 252)

9. $z^2 - 3z - 40 = 0$ (p. 252)

10. $5s^2 - 14s - 3 = 0$ (p. 259)

11. $7a^2 - 30a + 8 = 0$ (p. 259)

12. $4x^2 + 20x + 25 = 0$ (p. 259)

13. **DVD PLAYERS** A store sells about 50 of a new model of DVD player per month at a price of \$140 each. For each \$10 decrease in price, about 5 more DVD players per month are sold. How much should the store charge in order to maximize monthly revenue? What is the maximum monthly revenue? (p. 259)