

**GUIDED PRACTICE** for Examples 3, 4, and 5

Simplify the expression. Tell which properties of exponents you used.

5.  $x^{-6}x^5x^3$

6.  $(7y^2z^5)(y^{-4}z^{-1})$

7.  $\left(\frac{s^3}{t^{-4}}\right)^2$

8.  $\left(\frac{x^4y^{-2}}{x^3y^6}\right)^3$

**5.1 EXERCISES****HOMEWORK KEY**○ = WORKED-OUT SOLUTIONS  
on p. WS9 for Exs. 17, 31, and 51★ = STANDARDIZED TEST PRACTICE  
Exs. 2, 36, 46, 51, and 53**SKILL PRACTICE**1. **VOCABULARY** State the name of the property illustrated.

a.  $a^m \cdot a^n = a^{m+n}$

b.  $a^{-m} = \frac{1}{a^m}, a \neq 0$

c.  $(ab)^m = a^m b^m$

2. ★ **WRITING** Is the number  $25.2 \times 10^{-3}$  in scientific notation? Explain.**EXAMPLE 1**on p. 330  
for Exs. 3–14**EVALUATING NUMERICAL EXPRESSIONS** Evaluate the expression. Tell which properties of exponents you used.

3.  $3^3 \cdot 3^2$

4.  $(4^{-2})^3$

5.  $(-5)(-5)^4$

6.  $(2^4)^2$

7.  $\frac{5^2}{5^5}$

8.  $\left(\frac{3}{5}\right)^4$

9.  $\left(\frac{2}{7}\right)^{-3}$

10.  $9^3 \cdot 9^{-1}$

11.  $\frac{3^4}{3^{-2}}$

12.  $\left(\frac{2}{3}\right)^{-5} \left(\frac{2}{3}\right)^4$

13.  $6^3 \cdot 6^0 \cdot 6^{-5}$

14.  $\left(\left(\frac{1}{2}\right)^{-5}\right)^2$

**EXAMPLE 2**on p. 331  
for Exs. 15–23**SCIENTIFIC NOTATION** Write the answer in scientific notation.

15.  $(4.2 \times 10^3)(1.5 \times 10^6)$

16.  $(1.2 \times 10^{-3})(6.7 \times 10^{-7})$

17.  $(6.3 \times 10^5)(8.9 \times 10^{-12})$

18.  $(7.2 \times 10^9)(9.4 \times 10^8)$

19.  $(2.1 \times 10^{-4})^3$

20.  $(4.0 \times 10^3)^4$

21.  $\frac{8.1 \times 10^{12}}{5.4 \times 10^9}$

22.  $\frac{1.1 \times 10^{-3}}{5.5 \times 10^{-8}}$

23.  $\frac{(7.5 \times 10^8)(4.5 \times 10^{-4})}{1.5 \times 10^7}$

**EXAMPLES 3 and 4**on pp. 331–332  
for Exs. 24–39**SIMPLIFYING ALGEBRAIC EXPRESSIONS** Simplify the expression. Tell which properties of exponents you used.

24.  $\frac{w^{-2}}{w^6}$

25.  $(2^2y^3)^5$

26.  $(p^3q^2)^{-1}$

27.  $(w^3x^{-2})(w^6x^{-1})$

28.  $(5s^{-2}t^4)^{-3}$

29.  $(3a^3b^5)^{-3}$

30.  $\frac{x^{-1}y^2}{x^2y^{-1}}$

31.  $\frac{3c^3d}{9cd^{-1}}$

32.  $\frac{4r^4s^5}{24r^4s^{-5}}$

33.  $\frac{2a^3b^{-4}}{3a^5b^{-2}}$

34.  $\frac{y^{11}}{4z^3} \cdot \frac{8z^7}{y^7}$

35.  $\frac{x^2y^{-3}}{3y^2} \cdot \frac{y^2}{x^{-4}}$

36. ★ **MULTIPLE CHOICE** What is the simplified form of  $\frac{2x^2y}{6xy^{-1}}$ ?

Ⓐ  $\frac{y^2}{3}$

Ⓑ  $\frac{xy^2}{3}$

Ⓒ  $\frac{x}{3}$

Ⓓ  $\frac{1}{3}$

**ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

37.  $\frac{x^{10}}{x^2} = x^5$  ✗

38.  $x^5 \cdot x^3 = x^{15}$  ✗

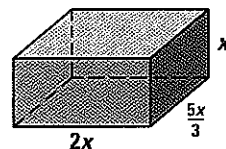
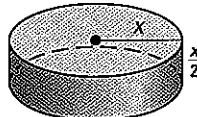
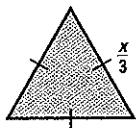
39.  $(-3)^2(-3)^4 = 9^6$  ✗

**GEOMETRY** Write an expression for the figure's area or volume in terms of  $x$ .

40.  $A = \frac{\sqrt{3}}{4}s^2$

41.  $V = \pi r^2 h$

42.  $V = lwh$



**REASONING** Write an expression that makes the statement true.

43.  $x^{15}y^{12}z^8 = x^4y^7z^{11} \cdot ?$

44.  $3x^3y^2 = \frac{12x^2y^5}{?}$

45.  $(a^5b^4)^2 = a^{14}b^{-1} \cdot ?$

46. **★ OPEN-ENDED MATH** Find three different ways to complete the following statement so that it is true:  $x^{12}y^{16} = (x^2y^2)(x^2y^2)$ .

**CHALLENGE** Refer to the properties of exponents on page 330.

47. Show how the negative exponent property can be derived from the quotient of powers property and the zero exponent property.

48. Show how the quotient of powers property can be derived from the product of powers property and the negative exponent property.

## PROBLEM SOLVING

**EXAMPLE 2**  
on p. 331  
for Exs. 49–50

49. **OCEAN VOLUME** The table shows the surface areas and average depths of four oceans. Calculate the volume of each ocean by multiplying the surface area of each ocean by its average depth. Write your answers in scientific notation.

Ocean	Surface area (square meters)	Average depth (meters)
Pacific	$1.56 \times 10^{14}$	$4.03 \times 10^3$
Atlantic	$7.68 \times 10^{13}$	$3.93 \times 10^3$
Indian	$6.86 \times 10^{13}$	$3.96 \times 10^3$
Arctic	$1.41 \times 10^{13}$	$1.21 \times 10^3$



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50. **EARTH SCIENCE** The continents of Earth move at a very slow rate. The South American continent has been moving about 0.000022 mile per year for the past 125,000,000 years. How far has the continent moved in that time? Write your answer in scientific notation.

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**EXAMPLE 5**  
on p. 332  
for Exs. 51–52

**51. ★ SHORT RESPONSE** A typical cultured black pearl is made by placing a bead with a diameter of 6 millimeters inside an oyster. The resulting pearl has a diameter of about 9 millimeters. *Compare* the volume of the resulting pearl with the volume of the bead.

**52. MULTI-STEP PROBLEM** A can of tennis balls consists of three spheres of radius  $r$  stacked vertically inside a cylinder of radius  $r$  and height  $h$ .

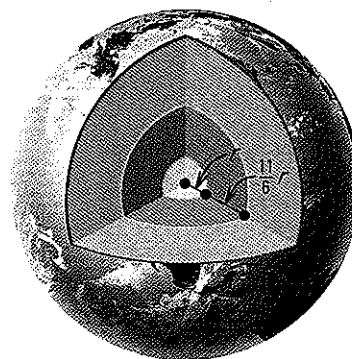
- Write an expression for the total volume of the three tennis balls in terms of  $r$ .
- Write an expression for the volume of the cylinder in terms of  $r$  and  $h$ .
- Write an expression for  $h$  in terms of  $r$  using the fact that the height of the cylinder is the sum of the diameters of the three tennis balls.
- What fraction of the can's volume is taken up by the tennis balls?

**53. ★ EXTENDED RESPONSE** You can think of a penny as a cylinder with a radius of about 9.53 millimeters and a height of about 1.55 millimeters.

- Calculate** Approximate the volume of a penny. Give your answer in cubic meters.
- Estimate** Approximate the volume of your classroom in cubic meters. *Explain* how you obtained your answer.
- Interpret** Use your results from parts (a) and (b) to estimate how many pennies it would take to fill your classroom. Do you think your answer is an overestimate or an underestimate? *Explain*.

**54. CHALLENGE** Earth's core is approximately spherical in shape and is divided into a solid inner core (the yellow region in the diagram shown) and a liquid outer core (the dark orange region in the diagram).

- Earth's radius is about 5 times as great as the radius of Earth's inner core. Find the ratio of Earth's total volume to the volume of Earth's inner core.
- Find the ratio of the volume of Earth's outer core to the volume of Earth's inner core.



## MIXED REVIEW

### PREVIEW

Prepare for  
Lesson 5.2  
in Exs. 55–60.

**Graph the function.**

55.  $y = -x + 4$  (p. 89)

56.  $y = 2x - 5$  (p. 89)

57.  $y = x^2 + 4$  (p. 236)

58.  $y = -2x^2 - 1$  (p. 236)

59.  $y = (x - 5)^2 - 3$  (p. 245)

60.  $y = 3x(x + 4)$  (p. 245)

**Use an inverse matrix to solve the linear system. (p. 210)**

61.  $x + y = 2$   
 $7x + 8y = 21$

62.  $-x - 2y = 3$   
 $2x + 8y = 1$

63.  $4x + 3y = 6$   
 $6x - 2y = 10$

**Write the expression as a complex number in standard form. (p. 275)**

64.  $(8 + 3i) - (7 + 4i)$

65.  $(5 - 2i) - (-9 + 6i)$

66.  $i(3 + i)$

67.  $(12 + 5i) - (7 - 8i)$

68.  $(5 + 4i)(2 + 3i)$

69.  $(8 - 4i)(1 + 6i)$

**GUIDED PRACTICE** for Examples 5 and 6

Graph the polynomial function.

9.  $f(x) = x^4 + 6x^2 - 3$       10.  $f(x) = -x^3 + x^2 + x - 1$       11.  $f(x) = 4 - 2x^3$

12. **WHAT IF?** If wind speed is measured in miles per hour, the model in Example 6 becomes  $E = 0.0051s^4$ . Graph this model. What wind speed is needed to generate a wave with 2000 foot-pounds of energy per square foot?

# 5.2 EXERCISES

**HOMEWORK KEY**

- = **WORKED-OUT SOLUTIONS**  
on p. WS10 for Exs. 21, 27, and 57
- ★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 24, 37, 50, 52, and 59
- ◆ = **MULTIPLE REPRESENTATIONS**  
Ex. 56

**SKILL PRACTICE**

1. **VOCABULARY** Identify the degree, type, leading coefficient, and constant term of the polynomial function  $f(x) = 6 + 2x^2 - 5x^4$ .

2. ★ **WRITING** Explain what is meant by the end behavior of a polynomial function.

**EXAMPLE 1**  
on p. 337  
for Exs. 3–8

**POLYNOMIAL FUNCTIONS** Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

3.  $f(x) = 8 - x^2$       4.  $f(x) = 6x + 8x^4 - 3$       5.  $g(x) = \pi x^4 + \sqrt{6}$
6.  $h(x) = x^3\sqrt{10} + 5x^{-2} + 1$       7.  $h(x) = -\frac{5}{2}x^3 + 3x - 10$       8.  $g(x) = 8x^3 - 4x^2 + \frac{2}{x}$

**EXAMPLE 2**  
on p. 338  
for Exs. 9–14

**DIRECT SUBSTITUTION** Use direct substitution to evaluate the polynomial function for the given value of  $x$ .

9.  $f(x) = 5x^3 - 2x^2 + 10x - 15$ ;  $x = -1$       10.  $f(x) = 8x + 5x^4 - 3x^2 - x^3$ ;  $x = 2$
11.  $g(x) = 4x^3 - 2x^5$ ;  $x = -3$       12.  $h(x) = 6x^3 - 25x + 20$ ;  $x = 5$
13.  $h(x) = x + \frac{1}{2}x^4 - \frac{3}{4}x^3 + 10$ ;  $x = -4$       14.  $g(x) = 4x^5 + 6x^3 + x^2 - 10x + 5$ ;  $x = -2$

**EXAMPLE 3**  
on p. 338  
for Exs. 15–23

**SYNTHETIC SUBSTITUTION** Use synthetic substitution to evaluate the polynomial function for the given value of  $x$ .

15.  $f(x) = 5x^3 - 2x^2 - 8x + 16$ ;  $x = 3$       16.  $f(x) = 8x^4 + 12x^3 + 6x^2 - 5x + 9$ ;  $x = -2$
17.  $g(x) = x^3 + 8x^2 - 7x + 35$ ;  $x = -6$       18.  $h(x) = -8x^3 + 14x - 35$ ;  $x = 4$
19.  $f(x) = -2x^4 + 3x^3 - 8x + 13$ ;  $x = 2$       20.  $g(x) = 6x^5 + 10x^3 - 27$ ;  $x = -3$
21.  $h(x) = -7x^3 + 11x^2 + 4x$ ;  $x = 3$       22.  $f(x) = x^4 + 3x - 20$ ;  $x = 4$

23. **ERROR ANALYSIS** Describe and correct the error in evaluating the polynomial function  $f(x) = -4x^4 + 9x^2 - 21x + 7$  when  $x = -2$ .

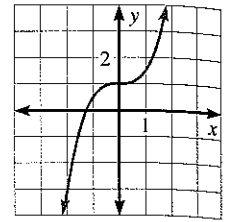
-2	-4	9	-21	7
		8	-34	110
		-4	17	-55
				117



**EXAMPLE 4**

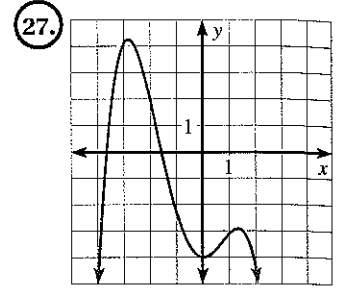
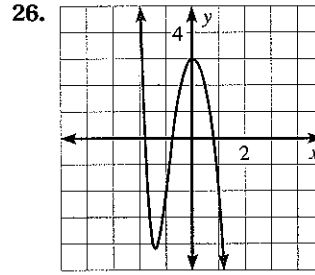
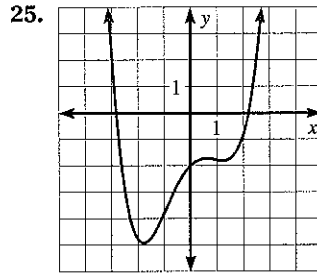
on p. 339  
for Exs. 24–27

24. ★ **MULTIPLE CHOICE** The graph of a polynomial function is shown. What is true about the function's degree and leading coefficient?



- (A) The degree is odd and the leading coefficient is positive.  
(B) The degree is odd and the leading coefficient is negative.  
(C) The degree is even and the leading coefficient is positive.  
(D) The degree is even and the leading coefficient is negative.

**USING END BEHAVIOR** Describe the degree and leading coefficient of the polynomial function whose graph is shown.



**DESCRIBING END BEHAVIOR** Describe the end behavior of the graph of the polynomial function by completing these statements:  $f(x) \rightarrow \underline{\quad? \quad}$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow \underline{\quad? \quad}$  as  $x \rightarrow +\infty$ .

28.  $f(x) = 10x^4$                       29.  $f(x) = -x^6 + 4x^3 - 3x$                       30.  $f(x) = -2x^3 + 7x - 4$   
31.  $f(x) = x^7 + 3x^4 - x^2$                       32.  $f(x) = 3x^{10} - 16x$                       33.  $f(x) = -6x^5 + 14x^2 + 20$   
34.  $f(x) = 0.2x^3 - x + 45$                       35.  $f(x) = 5x^8 + 8x^7$                       36.  $f(x) = -x^{273} + 500x^{271}$

37. ★ **OPEN-ENDED MATH** Write a polynomial function  $f$  of degree 5 such that the end behavior of the graph of  $f$  is given by  $f(x) \rightarrow +\infty$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow -\infty$  as  $x \rightarrow +\infty$ . Then graph the function to verify your answer.

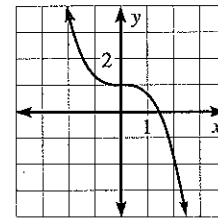
**EXAMPLE 5**

on p. 340  
for Exs. 38–50

**GRAPHING POLYNOMIALS** Graph the polynomial function.

38.  $f(x) = x^3$                       39.  $f(x) = -x^4$                       40.  $f(x) = x^5 + 3$   
41.  $f(x) = x^4 - 2$                       42.  $f(x) = -x^3 + 5$                       43.  $f(x) = x^3 - 5x$   
44.  $f(x) = -x^4 + 8x$                       45.  $f(x) = x^5 + x$                       46.  $f(x) = -x^3 + 3x^2 - 2x + 5$   
47.  $f(x) = x^5 + x^2 - 4$                       48.  $f(x) = x^4 - 5x^2 + 6$                       49.  $f(x) = -x^4 + 3x^3 - x + 1$

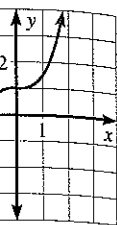
50. ★ **MULTIPLE CHOICE** Which function is represented by the graph shown?



- (A)  $f(x) = \frac{1}{3}x^3 + 1$     (B)  $f(x) = -\frac{1}{3}x^3 + 1$   
(C)  $f(x) = \frac{1}{3}x^3 - 1$     (D)  $f(x) = -\frac{1}{3}x^3 - 1$

51. **VISUAL THINKING** Suppose  $f(x) \rightarrow +\infty$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow -\infty$  as  $x \rightarrow +\infty$ . Describe the end behavior of  $g(x) = -f(x)$ .

52. ★ **SHORT RESPONSE** A cubic polynomial function  $f$  has leading coefficient 2 and constant term  $-5$ . If  $f(1) = 0$  and  $f(2) = 3$ , what is  $f(-5)$ ? Explain how you found your answer.



$$+ 7x - 4$$

$$+ 14x^2 + 20$$

$$+ 500x^{271}$$

$$3x^2 - 2x + 5$$

$$3x^3 - x + 1$$

53. **CHALLENGE** Let  $f(x) = x^3$  and  $g(x) = x^3 - 2x^2 + 4x$ .

- Copy and complete the table.
- Use the numbers in the table to complete this statement: As  $x \rightarrow +\infty$ ,  $\frac{f(x)}{g(x)} \rightarrow ?$ .
- Explain how the result from part (b) shows that the functions  $f$  and  $g$  have the same end behavior as  $x \rightarrow +\infty$ .

$x$	$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
10	?	?	?
20	?	?	?
50	?	?	?
100	?	?	?
200	?	?	?

## PROBLEM SOLVING

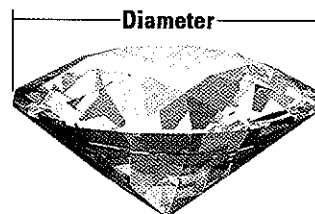
**EXAMPLE 6**  
on p. 340  
for Exs. 54–59

54. **DIAMONDS** The weight of an ideal round-cut diamond can be modeled by

$$w = 0.0071d^3 - 0.090d^2 + 0.48d$$

where  $w$  is the diamond's weight (in carats) and  $d$  is its diameter (in millimeters). According to the model, what is the weight of a diamond with a diameter of 15 millimeters?

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55. **SKATEBOARDING** From 1992 to 2003, the number of people in the United States who participated in skateboarding can be modeled by

$$S = -0.0076t^4 + 0.14t^3 - 0.62t^2 + 0.52t + 5.5$$

where  $S$  is the number of participants (in millions) and  $t$  is the number of years since 1992. Graph the model. Then use the graph to estimate the first year that the number of skateboarding participants was greater than 8 million.

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56. **MULTIPLE REPRESENTATIONS** From 1987 to 2003, the number of indoor movie screens  $M$  in the United States can be modeled by

$$M = -11.0t^3 + 267t^2 - 592t + 21,600$$

where  $t$  is the number of years since 1987.

- Classifying a Function** State the degree and type of the function.
- Making a Table** Make a table of values for the function.
- Sketching a Graph** Use your table to graph the function.

57. **SNOWBOARDING** From 1992 to 2003, the number of people in the United States who participated in snowboarding can be modeled by

$$S = 0.0013t^4 - 0.021t^3 + 0.084t^2 + 0.037t + 1.2$$

where  $S$  is the number of participants (in millions) and  $t$  is the number of years since 1992. Graph the model. Use the graph to estimate the first year that the number of snowboarding participants was greater than 2 million.



58. **MULTI-STEP PROBLEM** From 1980 to 2002, the number of quarterly periodicals  $P$  published in the United States can be modeled by

$$P = 0.138t^4 - 6.24t^3 + 86.8t^2 - 239t + 1450$$

where  $t$  is the number of years since 1980.

- Describe the end behavior of the graph of the model.
- Graph the model on the domain  $0 \leq t \leq 22$ .
- Use the model to predict the number of quarterly periodicals in the year 2010. Is it appropriate to use the model to make this prediction? *Explain.*

59. **★ EXTENDED RESPONSE** The weights of Sarus crane chicks  $S$  and hooded crane chicks  $H$  (both in grams) during the 10 days following hatching can be modeled by the functions

$$S = -0.122t^3 + 3.49t^2 - 14.6t + 136$$

$$H = -0.115t^3 + 3.71t^2 - 20.6t + 124$$

where  $t$  is the number of days after hatching.

- Calculate** According to the models, what is the difference in weight between 5-day-old Sarus crane chicks and hooded crane chicks?
- Graph** Sketch the graphs of the two models.
- Apply** A biologist finds that the weight of a crane chick 3 days after hatching is 130 grams. What species of crane is the chick more likely to be? *Explain* how you found your answer.

60. **CHALLENGE** The weight  $y$  (in pounds) of a rainbow trout can be modeled by  $y = 0.000304x^3$  where  $x$  is the length of the trout (in inches).

- Write a function that relates the weight  $y$  and length  $x$  of a rainbow trout if  $y$  is measured in kilograms and  $x$  is measured in centimeters. Use the fact that 1 kilogram  $\approx$  2.20 pounds and 1 centimeter  $\approx$  0.394 inch.
- Graph the original function and the function from part (a) in the same coordinate plane. What type of transformation can you apply to the graph of  $y = 0.000304x^3$  to produce the graph from part (a)?

## MIXED REVIEW

Solve the equation or inequality.

61.  $2b + 11 = 15 - 6b$  (p. 18)      62.  $2.7n + 4.3 = 12.94$  (p. 18)      63.  $-7 < 6y - 1 < 5$  (p. 41)  
 64.  $x^2 - 14x + 48 = 0$  (p. 252)      65.  $-24q^2 - 90q = 21$  (p. 259)      66.  $z^2 + 5z < 36$  (p. 300)

The variables  $x$  and  $y$  vary directly. Write an equation that relates  $x$  and  $y$ . Then find the value of  $x$  when  $y = -3$ . (p. 107)

67.  $x = 4, y = 12$       68.  $x = 3, y = -21$       69.  $x = 10, y = -4$   
 70.  $x = 0.8, y = 0.2$       71.  $x = -0.45, y = -0.35$       72.  $x = -6.5, y = 3.9$

Write the quadratic function in standard form. (p. 245)

73.  $y = (x + 3)(x - 7)$       74.  $y = 8(x - 4)(x + 2)$       75.  $y = -3(x - 5)^2 - 25$   
 76.  $y = 2.5(x - 6)^2 + 9.3$       77.  $y = \frac{1}{2}(x - 4)^2$       78.  $y = -\frac{5}{3}(x + 4)(x + 9)$

### PREVIEW

Prepare for  
Lesson 5.3  
in Exs. 73–78.

# 5.3 EXERCISES

**HOMEWORK KEY**

○ = WORKED-OUT SOLUTIONS  
on p. WS10 for Exs. 11, 21, and 61  
★ = STANDARDIZED TEST PRACTICE  
Exs. 2, 15, 47, 56, and 63

## SKILL PRACTICE

- VOCABULARY** When you add or subtract polynomials, you add or subtract the coefficients of   ?
- ★ **WRITING** Explain how a polynomial subtraction problem is equivalent to a polynomial addition problem.

**EXAMPLES 1 and 2**  
on p. 346  
for Exs. 3–15

### ADDING AND SUBTRACTING POLYNOMIALS Find the sum or difference.

- |   |   |
|---|---|
| 3. $(3x^2 - 5) + (7x^2 - 3)$                          | 4. $(x^2 - 3x + 5) - (-4x^2 + 8x + 9)$            |
| 5. $(4y^2 + 9y - 5) - (4y^2 - 5y + 3)$                | 6. $(z^2 + 5z - 7) + (5z^2 - 11z - 6)$            |
| 7. $(3s^3 + s) + (4s^3 - 2s^2 + 7s + 10)$             | 8. $(2a^2 - 8) - (a^3 + 4a^2 - 12a + 4)$          |
| 9. $(5c^2 + 7c + 1) + (2c^3 - 6c + 8)$                | 10. $(4t^3 - 11t^2 + 4t) - (-7t^2 - 5t + 8)$      |
| 11. $(5b - 6b^3 + 2b^4) - (9b^3 + 4b^4 - 7)$          | 12. $(3y^2 - 6y^4 + 5 - 6y) + (5y^4 - 6y^3 + 4y)$ |
| 13. $(x^4 - x^3 + x^2 - x + 1) + (x + x^4 - 1 - x^2)$ | 14. $(8v^4 - 2v^2 + v - 4) - (3v^3 - 12v^2 + 8v)$ |
15. ★ **MULTIPLE CHOICE** What is the result when  $2x^4 - 8x^2 - x + 10$  is subtracted from  $8x^4 - 4x^3 - x + 2$ ?
- |                                |                              |
|--------------------------------|------------------------------|
| (A) $-6x^4 + 4x^3 - 8x^2 + 8$  | (B) $6x^4 - 4x^3 + 8x^2 - 8$ |
| (C) $10x^4 - 8x^3 - 4x^2 + 12$ | (D) $6x^4 + 4x^3 - 2x - 8$   |

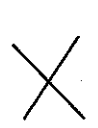
**EXAMPLE 3**  
on p. 347  
for Exs. 16–25

### MULTIPLYING POLYNOMIALS Find the product of the polynomials.


- |                                      |                                       |
|--------------------------------------|---------------------------------------|
| 16. $x(2x^2 - 5x + 7)$               | 17. $5x^2(6x + 2)$                    |
| 18. $(y - 7)(y + 6)$                 | 19. $(3z + 1)(z - 3)$                 |
| 20. $(w + 4)(w^2 + 6w - 11)$         | 21. $(2a - 3)(a^2 - 10a - 2)$         |
| 22. $(5c^2 - 4)(2c^2 + c - 3)$       | 23. $(-x^2 + 4x + 1)(x^2 - 8x + 3)$   |
| 24. $(-d^2 + 4d + 3)(3d^2 - 7d + 6)$ | 25. $(3y^2 + 6y - 1)(4y^2 - 11y - 5)$ |

### ERROR ANALYSIS Describe and correct the error in simplifying the expression.

26.

$$\begin{aligned} &(x^2 - 3x + 4) - (x^3 + 7x - 2) \\ &= x^2 - 3x + 4 - x^3 + 7x - 2 \\ &= -x^3 + x^2 + 4x + 2 \end{aligned}$$


27.

$$\begin{aligned} (2x - 7)^3 &= (2x)^3 - 7^3 \\ &= 8x^3 - 343 \end{aligned}$$


**EXAMPLE 4**  
on p. 347  
for Exs. 28–37

### MULTIPLYING THREE BINOMIALS Find the product of the binomials.

- |                                  |                                |
|----------------------------------|--------------------------------|
| 28. $(x + 4)(x - 6)(x - 5)$      | 29. $(x + 1)(x - 7)(x + 3)$    |
| 30. $(z - 4)(-z + 2)(z + 8)$     | 31. $(a - 6)(2a + 5)(a + 1)$   |
| 32. $(3p + 1)(p + 3)(p + 1)$     | 33. $(b - 2)(2b - 1)(-b + 1)$  |
| 34. $(2s + 1)(3s - 2)(4s - 3)$   | 35. $(w - 6)(4w - 1)(-3w + 5)$ |
| 36. $(4x - 1)(-2x - 7)(-5x - 4)$ | 37. $(3q - 8)(-9q + 2)(q - 2)$ |



**EXAMPLE 5**

on p. 348  
for Exs. 38–47

**SPECIAL PRODUCTS Find the product.**

38.  $(x + 5)(x - 5)$

39.  $(w - 9)^2$

40.  $(y + 4)^3$

41.  $(2c + 5)^2$

42.  $(3t - 4)^3$

43.  $(5p - 3)(5p + 3)$

44.  $(7x - y)^3$

45.  $(2a + 9b)(2a - 9b)$

46.  $(3z + 7y)^3$

47. **★ MULTIPLE CHOICE** Which expression is equivalent to  $(3x - 2y)^2$ ?

Ⓐ  $9x^2 - 4y^2$

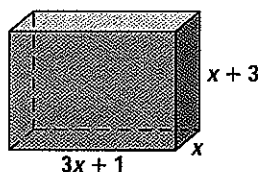
Ⓑ  $9x^2 + 4y^2$

Ⓒ  $9x^2 + 12xy + 4y^2$

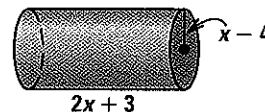
Ⓓ  $9x^2 - 12xy + 4y^2$

 **GEOMETRY** Write the figure's volume as a polynomial in standard form.

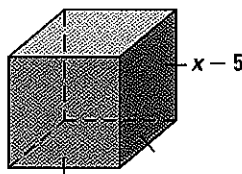
48.  $V = \ell wh$



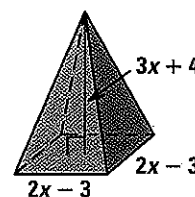
49.  $V = \pi r^2 h$



50.  $V = s^3$



51.  $V = \frac{1}{3}Bh$

**SPECIAL PRODUCTS Verify the special product pattern by multiplying.**

52.  $(a + b)(a - b) = a^2 - b^2$

53.  $(a + b)^2 = a^2 + 2ab + b^2$

54.  $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

55.  $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

56. **★ EXTENDED RESPONSE** Let  $p(x) = x^4 - 7x + 14$  and  $q(x) = x^2 - 5$ .

- What is the degree of the polynomial  $p(x) + q(x)$ ?
- What is the degree of the polynomial  $p(x) - q(x)$ ?
- What is the degree of the polynomial  $p(x) \cdot q(x)$ ?
- In general, if  $p(x)$  and  $q(x)$  are polynomials such that  $p(x)$  has degree  $m$ ,  $q(x)$  has degree  $n$ , and  $m > n$ , what are the degrees of  $p(x) + q(x)$ ,  $p(x) - q(x)$ , and  $p(x) \cdot q(x)$ ?

57. **FINDING A PATTERN** Look at the following polynomial factorizations.

$$x^2 - 1 = (x - 1)(x + 1)$$

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

$$x^4 - 1 = (x - 1)(x^3 + x^2 + x + 1)$$

- Factor  $x^5 - 1$  and  $x^6 - 1$  into the product of  $x - 1$  and another polynomial. Check your answers by multiplying.
- In general, how can  $x^n - 1$  be factored? Show that this factorization works by multiplying the factors.

58. **CHALLENGE** Suppose  $f(x) = (x + a)(x + b)(x + c)(x + d)$ . If  $f(x)$  is written in standard form, show that the coefficient of  $x^3$  is the sum of  $a$ ,  $b$ ,  $c$ , and  $d$ , and the constant term is the product of  $a$ ,  $b$ ,  $c$ , and  $d$ .


## PROBLEM SOLVING

**EXAMPLE 6**  
on p. 348  
for Exs. 59–61

- 59. HIGHER EDUCATION** Since 1970, the number (in thousands) of males  $M$  and females  $F$  attending institutes of higher education can be modeled by

$$M = 0.091t^3 - 4.8t^2 + 110t + 5000 \quad \text{and} \quad F = 0.19t^3 - 12t^2 + 350t + 3600$$


where  $t$  is the number of years since 1970. Write a model for the total number of people attending institutes of higher education.

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- 60. ELECTRONICS** From 1999 to 2004, the number of DVD players  $D$  (in millions) sold in the United States and the average price per DVD player  $P$  (in dollars) can be modeled by

$$D = 4.11t + 4.44 \quad \text{and} \quad P = 6.82t^2 - 61.7t + 265$$

where  $t$  is the number of years since 1999. Write a model for the total revenue  $R$  from DVD sales. According to the model, what was the total revenue in 2002?

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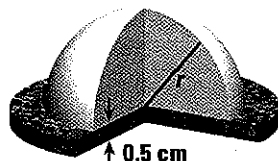
- 61. BICYCLING** The equation  $P = 0.00267sF$  gives the power  $P$  (in horsepower) needed to keep a certain bicycle moving at speed  $s$  (in miles per hour), where  $F$  is the force (in pounds) of road and air resistance. On level ground, the equation

$$F = 0.0116s^2 + 0.789$$

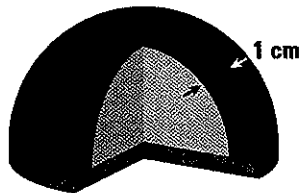
models the force  $F$ . Write a model (in terms of  $s$  only) for the power needed to keep the bicycle moving at speed  $s$  on level ground. How much power is needed to keep the bicycle moving at 10 miles per hour?

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- 62. MULTI-STEP PROBLEM** A dessert is made by taking a hemispherical mound of marshmallow on a 0.5 centimeter thick cookie and covering it with a chocolate shell 1 centimeter thick. Use the diagrams to write two polynomial functions in standard form:  $M(r)$  for the combined volume of the marshmallow plus cookie, and  $D(r)$  for the volume of the entire dessert. Then use  $M(r)$  and  $D(r)$  to write a function  $C(r)$  for the volume of the chocolate.



Marshmallow on cookie



Chocolate layer added

- 63. ★ SHORT RESPONSE** From 1997 to 2002, the number of NCAA lacrosse teams for men  $L_m$  and women  $L_w$ , as well as the average size of a men's team  $S_m$  and a women's team  $S_w$ , can be modeled by

$$L_m = 5.57t + 182 \quad \text{and} \quad S_m = -0.127t^3 + 0.822t^2 - 1.02t + 31.5$$

$$L_w = 12.2t + 185 \quad \text{and} \quad S_w = -0.0662t^3 + 0.437t^2 - 0.725t + 22.3$$

where  $t$  is the number of years since 1997. Write a model for the *total* number of people  $N$  on NCAA lacrosse teams. *Explain* how you obtained your model.

64. **CHALLENGE** From 1970 to 2002, the circulation  $C$  (in millions) of Sunday newspapers in the United States can be modeled by

$$C = -0.00105t^3 + 0.0281t^2 + 0.465t + 48.8$$

where  $t$  is the number of years since 1970. Rewrite  $C$  as a function of  $s$ , where  $s$  is the number of years since 1975.

## MIXED REVIEW

### PREVIEW

Prepare for  
Lesson 5.4 in  
Exs. 65–72.

Solve the equation.

65.  $2x - 7 = 11$  (p. 18)

66.  $10 - 3x = 25$  (p. 18)

67.  $4t - 7 = 2t$  (p. 18)

68.  $y^2 - 2y - 48 = 0$  (p. 252)

69.  $w^2 - 15w + 54 = 0$  (p. 252)

70.  $x^2 + 9x + 14 = 0$  (p. 252)

71.  $4z^2 + 21z - 18 = 0$  (p. 259)

72.  $9a^2 - 30a + 25 = 0$  (p. 259)

Solve the system of equations. (p. 178)

73.  $x + y - 2z = -4$

74.  $x - 2y + z = -13$

75.  $3x - y - 2z = 20$

$3x - y + z = 22$

$-x + 4y + z = 35$

$-x + 3y - z = -16$

$-x + 2y + 3z = -9$

$3x + 2y + 4z = 28$

$-2x - y + 3z = -5$

Evaluate the determinant of the matrix. (p. 203)

76.  $\begin{bmatrix} 3 & -4 \\ 3 & 1 \end{bmatrix}$

77.  $\begin{bmatrix} 5 & 7 \\ -4 & 9 \end{bmatrix}$

78.  $\begin{bmatrix} -1 & 8 & 0 \\ 3 & 4 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

79.  $\begin{bmatrix} 2 & 3 & -4 \\ -6 & 1 & 5 \\ -3 & -1 & -2 \end{bmatrix}$

## QUIZ for Lessons 5.1–5.3

Evaluate the expression. (p. 330)

1.  $3^5 \cdot 3^{-1}$

2.  $(2^4)^2$

3.  $\left(\frac{2}{3^{-2}}\right)^2$

4.  $\left(\frac{3}{5}\right)^{-2}$

Simplify the expression. (p. 330)

5.  $(x^4y^{-2})(x^{-3}y^8)$

6.  $(a^2b^{-5})^{-3}$

7.  $\frac{x^3y^7}{x^{-4}y^0}$

8.  $\frac{c^3d^{-2}}{c^5d^{-1}}$

Graph the polynomial function. (p. 337)

9.  $g(x) = 2x^3 - 3x + 1$

10.  $h(x) = x^4 - 4x + 2$

11.  $f(x) = -2x^3 + x^2 - 5$

Perform the indicated operation. (p. 346)

12.  $(x^3 + x^2 - 6) - (2x^2 + 4x - 8)$

13.  $(-3x^2 + 4x - 10) + (x^2 - 9x + 15)$

14.  $(x - 5)(x^2 - 5x + 7)$

15.  $(x + 3)(x - 6)(3x - 1)$

16. **NATIONAL DEBT** On July 21, 2004, the national debt of the United States was about \$7,282,000,000,000. The population of the United States at that time was about 294,000,000. Suppose the national debt was divided evenly among everyone in the United States. How much would each person owe? (p. 330)