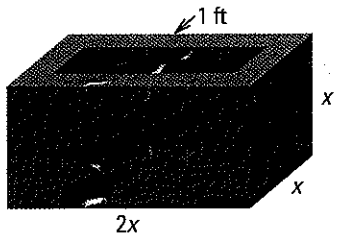
EXAMPLE 6 Solve a polynomial equation

CITY PARK You are designing a marble basin that will hold a fountain for a city park. The basin's sides and bottom should be 1 foot thick. Its outer length should be twice its outer width and outer height.

What should the outer dimensions of the basin be if it is to hold 36 cubic feet of water?



ANOTHER WAY
For alternative methods to solving the problem in Example 6, turn to page 360 for the **Problem Solving Workshop**.

Solution

Volume (cubic feet)	=	Interior length (feet)	•	Interior width (feet)	•	Interior height (feet)
⇩		⇩		⇩		⇩
36	=	$(2x - 2)$	•	$(x - 2)$	•	$(x - 1)$

$36 = (2x - 2)(x - 2)(x - 1)$ **Write equation.**

$0 = 2x^3 - 8x^2 + 10x - 40$ **Write in standard form.**

$0 = 2x^2(x - 4) + 10(x - 4)$ **Factor by grouping.**

$0 = (2x^2 + 10)(x - 4)$ **Distributive property**

▶ The only real solution is $x = 4$. The basin is 8 ft long, 4 ft wide, and 4 ft high.

✓ GUIDED PRACTICE for Example 6

11. **WHAT IF?** In Example 6, what should the basin's dimensions be if it is to hold 40 cubic feet of water and have outer length $6x$, width $3x$, and height x?

5.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS10 for Exs. 7, 23, and 61
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 41, 63, and 64

SKILL PRACTICE

1. **VOCABULARY** The expression $8x^6 + 10x^3 - 3$ is in _?_ form because it can be written as $2u^2 + 5u - 3$ where $u = 2x^3$.

2. ★ **WRITING** What condition must the factorization of a polynomial satisfy in order for the polynomial to be factored completely?

EXAMPLE 1
on p. 353
for Exs. 3–9

MONOMIAL FACTORS **Factor the polynomial completely.**

3. $14x^2 - 21x$
4. $30b^3 - 54b^2$
5. $c^3 + 9c^2 + 18c$
6. $z^3 - 6z^2 - 72z$
7. $3y^5 - 48y^3$
8. $54m^5 + 18m^4 + 9m^3$

9. ★ **MULTIPLE CHOICE** What is the complete factorization of $2x^7 - 32x^3$?

Ⓐ $2x^3(x + 2)(x - 2)(x^2 + 4)$
Ⓑ $2x^3(x^2 + 2)(x^2 - 2)$
Ⓒ $2x^3(x^2 + 4)^2$
Ⓓ $2x^3(x + 2)^2(x - 2)^2$

EXAMPLE 2 on p. 354 for Exs. 10–17

SUM OR DIFFERENCE OF CUBES Factor the polynomial completely.

10. $x^3 + 8$ **11.** $y^3 - 64$ **12.** $27m^3 + 1$ **13.** $125n^3 + 216$

14. $27a^3 - 1000$ **15.** $8c^3 + 343$ **16.** $192w^3 - 3$ **17.** $-5z^3 + 320$

EXAMPLE 3 on p. 354 for Exs. 18–23

FACTORING BY GROUPING Factor the polynomial completely.

18. $x^3 + x^2 + x + 1$ **19.** $y^3 - 7y^2 + 4y - 28$ **20.** $n^3 + 5n^2 - 9n - 45$

21. $3m^3 - m^2 + 9m - 3$ **22.** $25s^3 - 100s^2 - s + 4$ **23.** $4c^3 + 8c^2 - 9c - 18$

EXAMPLE 4 on p. 355 for Exs. 24–29

QUADRATIC FORM Factor the polynomial completely.

24. $x^4 - 25$ **25.** $a^4 + 7a^2 + 6$ **26.** $3s^4 - s^2 - 24$

27. $32z^5 - 2z$ **28.** $36m^6 + 12m^4 + m^2$ **29.** $15x^5 - 72x^3 - 108x$

EXAMPLE 5 on p. 355 for Exs. 30–41

ERROR ANALYSIS *Describe* and correct the error in finding all real-number solutions.

30.

$$8x^3 - 27 = 0$$
$$(2x + 3)(4x^2 + 6x + 9) = 0$$
$$x = -\frac{3}{2}$$

31.

$$3x^3 - 48x = 0$$
$$3x(x^2 - 16) = 0$$
$$x^2 - 16 = 0$$
$$x = -4 \text{ or } x = 4$$

SOLVING EQUATIONS Find the real-number solutions of the equation.

32. $y^3 - 5y^2 = 0$ **33.** $18s^3 = 50s$ **34.** $g^3 + 3g^2 - g - 3 = 0$

35. $m^3 + 6m^2 - 4m - 24 = 0$ **36.** $4w^4 + 40w^2 - 44 = 0$ **37.** $4z^5 = 84z^3$

38. $5b^3 + 15b^2 + 12b = -36$ **39.** $x^6 - 4x^4 - 9x^2 + 36 = 0$ **40.** $48p^5 = 27p^3$

41. ★ **MULTIPLE CHOICE** What are the real-number solutions of the equation $3x^4 - 27x^2 + 9x = x^3$?

(A) $-1, 0, 3$ (B) $-3, 0, 3$ (C) $-3, 0, \frac{1}{3}, 3$ (D) $-3, -\frac{1}{3}, 0, 3$

CHOOSING A METHOD Factor the polynomial completely using any method.

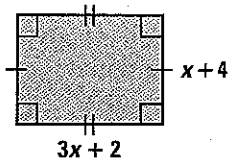
42. $16x^3 - 44x^2 - 42x$ **43.** $n^4 - 4n^2 - 60$ **44.** $-4b^4 - 500b$

45. $36a^3 - 15a^2 + 84a - 35$ **46.** $18c^4 + 57c^3 - 10c^2$ **47.** $2d^4 - 13d^2 - 45$

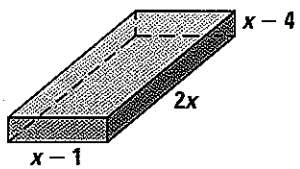
48. $32x^5 - 108x^2$ **49.** $8y^6 - 38y^4 - 10y^2$ **50.** $z^5 - 3z^4 - 16z + 48$

GEOMETRY Find the possible value(s) of x.

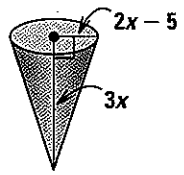
51. Area = 48



52. Volume = 40



53. Volume = 125π



CHOOSING A METHOD Factor the polynomial completely using any method.

54. $x^3y^6 - 27$ **55.** $7ac^2 + bc^2 - 7ad^2 - bd^2$ **56.** $x^{2n} - 2x^n + 1$

57. CHALLENGE Factor $a^5b^2 - a^2b^4 + 2a^4b - 2ab^3 + a^3 - b^2$ completely.

PROBLEM SOLVING

EXAMPLE 6 on p. 356 for Exs. 58–63

58. ARCHAEOLOGY At the ruins of Caesarea, archaeologists discovered a huge hydraulic concrete block with a volume of 945 cubic meters. The block's dimensions are x meters high by $12x - 15$ meters long by $12x - 21$ meters wide. What is the height of the block?

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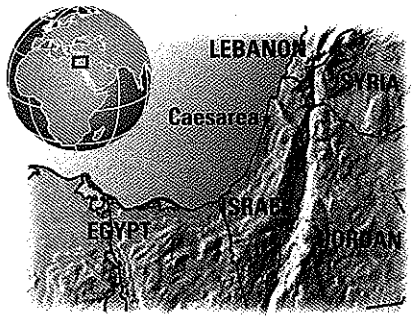

59. CHOCOLATE MOLD You are designing a chocolate mold shaped like a hollow rectangular prism for a candy manufacturer. The mold must have a thickness of 1 centimeter in all dimensions. The mold's outer dimensions should also be in the ratio 1:3:6. What should the outer dimensions of the mold be if it is to hold 112 cubic centimeters of chocolate?

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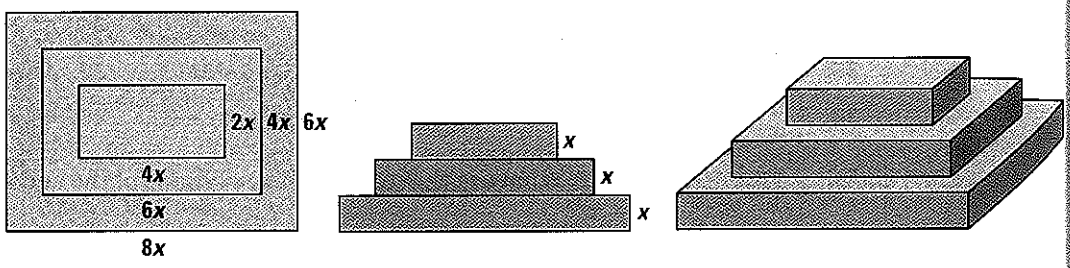
60. MULTI-STEP PROBLEM A production crew is assembling a three-level platform inside a stadium for a performance. The platform has the dimensions shown in the diagrams, and has a total volume of 1250 cubic feet.



a. **Write Expressions** What is the volume, in terms of x, of each of the three levels of the platform?

b. **Write an Equation** Use what you know about the total volume to write an equation involving x.

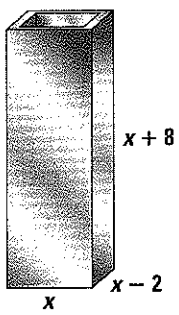
c. **Solve** Solve the equation from part (b). Use your solution to calculate the dimensions of each of the three levels of the platform.

61. SCULPTURE Suppose you have 250 cubic inches of clay with which to make a sculpture shaped as a rectangular prism. You want the height and width each to be 5 inches less than the length. What should the dimensions of the prism be?

62. MANUFACTURING A manufacturer wants to build a rectangular stainless steel tank with a holding capacity of 670 gallons, or about 89.58 cubic feet. The tank's walls will be one half inch thick, and about 6.42 cubic feet of steel will be used for the tank. The manufacturer wants the outer dimensions of the tank to be related as follows:

- The width should be 2 feet less than the length.
- The height should be 8 feet more than the length.

What should the outer dimensions of the tank be?



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

63. ★ **SHORT RESPONSE** A platform shaped like a rectangular prism has dimensions $x - 2$ feet by $3 - 2x$ feet by $3x + 4$ feet. *Explain* why the volume of the platform cannot be $\frac{7}{3}$ cubic feet.

64. ★ **EXTENDED RESPONSE** In 2000 B.C., the Babylonians solved polynomial equations using tables of values. One such table gave values of $y^3 + y^2$. To be able to use this table, the Babylonians sometimes had to manipulate the equation, as shown below.

$ax^3 + bx^2 = c$ **Original equation**

$\frac{a^3x^3}{b^3} + \frac{a^2x^2}{b^2} = \frac{a^2c}{b^3}$ **Multiply each side by $\frac{a^2}{b^3}$.**

$\left(\frac{ax}{b}\right)^3 + \left(\frac{ax}{b}\right)^2 = \frac{a^2c}{b^3}$ **Rewrite cubes and squares.**

They then found $\frac{a^2c}{b^3}$ in the $y^3 + y^2$ column of the table. Because the corresponding y-value was $y = \frac{ax}{b}$, they could conclude that $x = \frac{by}{a}$.

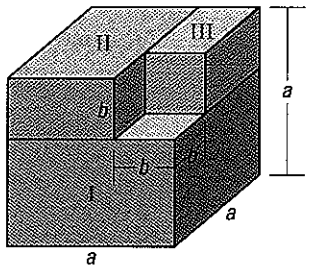
a. Calculate $y^3 + y^2$ for $y = 1, 2, 3, \ldots, 10$. Record the values in a table.

b. Use your table and the method described above to solve $x^3 + 2x^2 = 96$.

c. Use your table and the method described above to solve $3x^3 + 2x^2 = 512$.

d. How can you modify the method described above for equations of the form $ax^4 + bx^3 = c$?

65. **CHALLENGE** Use the diagram to complete parts (a)–(c).



a. *Explain* why $a^3 - b^3$ is equal to the sum of the volumes of solid I, solid II, and solid III.

b. Write an algebraic expression for the volume of each of the three solids. Leave your expressions in factored form.

c. Use the results from parts (a) and (b) to derive the factoring pattern for $a^3 - b^3$ given on page 354.

MIXED REVIEW

Graph the function.

66. $f(x) = -2|x - 3| + 5$ *(p. 123)*

67. $y = \frac{1}{2}x^2 + 4x + 5$ *(p. 236)*

68. $y = 3(x + 4)^2 + 7$ *(p. 245)*

69. $f(x) = x^3 - 2x - 5$ *(p. 337)*

Graph the inequality in a coordinate plane. *(p. 132)*

70. $y \leq 2x - 3$

71. $y > -5 - x$

72. $y < 0.5x + 5$

73. $4x + 12y \leq 4$

74. $9x - 9y \geq 27$

75. $\frac{2}{5}x + \frac{5}{2}y > 5$

PREVIEW
Prepare for Lesson 5.5 in Exs. 76–79.

Use synthetic substitution to evaluate the polynomial function for the given value of x. *(p. 337)*

76. $f(x) = 5x^4 - 3x^3 + 4x^2 - x + 10; x = 2$

77. $f(x) = -3x^5 + x^3 - 6x^2 + 2x + 4; x = -3$

78. $f(x) = 5x^5 - 4x^3 + 12x^2 + 20; x = -2$

79. $f(x) = -6x^4 + 9x - 15; x = 4$

5.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS11 for Exs. 7, 21, and 47

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 38, 39, 40, and 50

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If a polynomial function has integer coefficients, then every rational zero of the function has the form $\frac{p}{q}$, where p is a factor of the __?__ and q is a factor of the __?__.

2. ★ **WRITING** *Describe* a method you can use to shorten the list of possible rational zeros when using the rational zero theorem.

EXAMPLE 1 on p. 370 for Exs. 3–10

LISTING RATIONAL ZEROS **List the possible rational zeros of the function using the rational zero theorem.**

3. $f(x) = x^3 - 3x + 28$
4. $g(x) = x^3 - 4x^2 + x - 10$
5. $f(x) = 2x^4 + 6x^3 - 7x + 9$
6. $h(x) = 2x^3 + x^2 - x - 18$
7. $g(x) = 4x^5 + 3x^3 - 2x - 14$
8. $f(x) = 3x^4 + 5x^3 - 3x + 42$
9. $h(x) = 8x^4 + 4x^3 - 10x + 15$
10. $h(x) = 6x^3 - 3x^2 + 12$

EXAMPLE 2 on p. 371 for Exs. 11–18

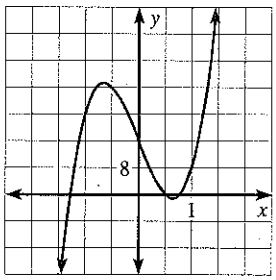
FINDING REAL ZEROS **Find all real zeros of the function.**

11. $f(x) = x^3 - 12x^2 + 35x - 24$
12. $f(x) = x^3 - 5x^2 - 22x + 56$
13. $g(x) = x^3 - 31x - 30$
14. $h(x) = x^3 + 8x^2 - 9x - 72$
15. $h(x) = x^4 + 7x^3 + 26x^2 + 44x + 24$
16. $f(x) = x^4 - 2x^3 - 9x^2 + 10x - 24$
17. $f(x) = x^4 + 2x^3 - 9x^2 - 2x + 8$
18. $g(x) = x^4 - 16x^2 - 40x - 25$

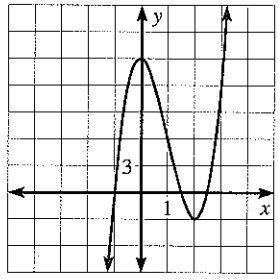
EXAMPLE 3 on p. 372 for Exs. 19–35

ELIMINATING POSSIBLE ZEROS **Use the graph to shorten the list of possible rational zeros of the function. Then find all real zeros of the function.**

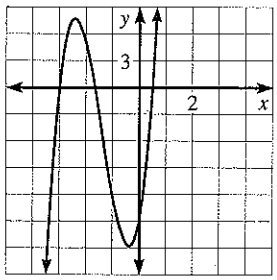
19. $f(x) = 4x^3 - 20x + 16$



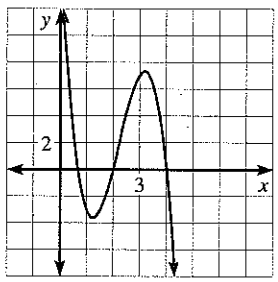
20. $f(x) = 4x^3 - 12x^2 - x + 15$



21. $f(x) = 6x^3 + 25x^2 + 16x - 15$



22. $f(x) = -3x^3 + 20x^2 - 36x + 16$



23. ★ **MULTIPLE CHOICE** According to the rational zero theorem, which is *not* a possible zero of the function $f(x) = 2x^4 - 5x^3 + 10x^2 - 9$?

Ⓐ -9 Ⓑ $-\frac{1}{2}$ Ⓒ $\frac{5}{2}$ Ⓓ 3

FINDING REAL ZEROS **Find all real zeros of the function.**

24. $f(x) = 2x^3 + 2x^2 - 8x - 8$

25. $g(x) = 2x^3 - 7x^2 + 9$

26. $h(x) = 2x^3 - 3x^2 - 14x + 15$

27. $f(x) = 3x^3 + 4x^2 - 35x - 12$

28. $f(x) = 3x^3 + 19x^2 + 4x - 12$

29. $g(x) = 2x^3 + 5x^2 - 11x - 14$

30. $g(x) = 2x^4 + 9x^3 + 5x^2 + 3x - 4$

31. $h(x) = 2x^4 - x^3 - 7x^2 + 4x - 4$

32. $h(x) = 3x^4 - 6x^3 - 32x^2 + 35x - 12$

33. $f(x) = 2x^4 - 9x^3 + 37x - 30$

34. $f(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$

35. $h(x) = 2x^5 + 5x^4 - 3x^3 - 2x^2 - 5x + 3$

ERROR ANALYSIS ***Describe*** **and correct the error in listing the possible rational zeros of the function.**

36.

$f(x) = x^3 + 7x^2 + 2x + 14$

Possible zeros:

1, 2, 7, 14 ✗

37.

$f(x) = 6x^3 - 3x^2 + 12x + 5$

Possible zeros:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}$ ✗

38. ★ **OPEN-ENDED MATH** Write a polynomial function f that has a leading coefficient of 4 and has 12 possible rational zeros according to the rational zero theorem.

39. ★ **MULTIPLE CHOICE** Which of the following is *not* a zero of the function $f(x) = 40x^5 - 42x^4 - 107x^3 + 107x^2 + 33x - 36$?

Ⓐ $-\frac{3}{2}$ Ⓑ $-\frac{3}{8}$ Ⓒ $\frac{3}{4}$ Ⓓ $\frac{4}{5}$

40. ★ **SHORT RESPONSE** Let a_n be the leading coefficient of a polynomial function f and a_0 be the constant term. If a_n has r factors and a_0 has s factors, what is the largest number of possible rational zeros of f that can be generated by the rational zero theorem? *Explain* your reasoning.

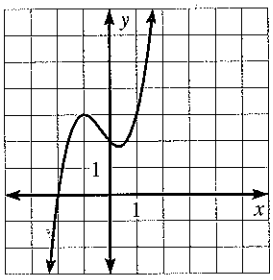
MATCHING **Find all real zeros of the function. Then match each function with its graph.**

41. $f(x) = x^3 - 2x^2 - x + 2$

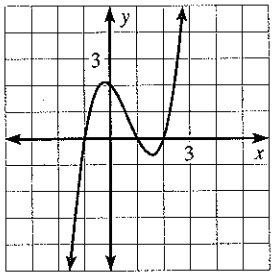
42. $g(x) = x^3 - 3x^2 + 2$

43. $h(x) = x^3 + x^2 - x + 2$

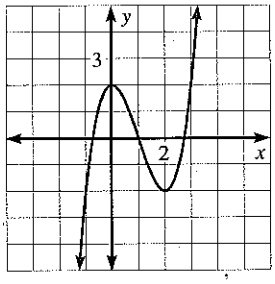
A.



B.



C.



44. **CHALLENGE** Is it possible for a cubic function to have more than three real zeros? Is it possible for a cubic function to have no real zeros? *Explain.*

PROBLEM SOLVING

EXAMPLE 4 on p. 373 for Exs. 45–48

45. MANUFACTURING At a factory, molten glass is poured into molds to make paperweights. Each mold is a rectangular prism with a height 4 inches greater than the length of each side of its square base. Each mold holds 63 cubic inches of molten glass. What are the dimensions of the mold?

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46. SWIMMING POOL You are designing a rectangular swimming pool that is to be set into the ground. The width of the pool is 5 feet more than the depth, and the length is 35 feet more than the depth. The pool holds 2000 cubic feet of water. What are the dimensions of the pool?

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GEOMETRY In Exercises 47 and 48, write a polynomial equation to model the situation. Then list the possible rational solutions of the equation.

47. A rectangular prism has edges of lengths x, $x - 1$, and $x - 2$ and a volume of 24.

48. A pyramid has a square base with sides of length x, a height of $2x - 5$, and a volume of 3.

49. MULTI-STEP PROBLEM From 1994 to 2003, the amount of athletic equipment E (in millions of dollars) sold domestically can be modeled by

$$E(t) = -10t^3 + 140t^2 - 20t + 18{,}150$$

where t is the number of years since 1994. Use the following steps to find the year when about \$20,300,000,000 of athletic equipment was sold.

a. Write a polynomial equation that can be used to find the answer.

b. List the possible whole-number solutions of the equation in part (a) that are less than 10.

c. Use synthetic division to determine which of the possible solutions in part (b) is an actual solution. Then calculate the year which corresponds to the solution.

50. ★ EXTENDED RESPONSE Since 1990, the number of U.S. travelers to foreign countries F (in thousands) can be modeled by

$$F(t) = 12t^4 - 264t^3 + 2028t^2 - 3924t + 43{,}916$$

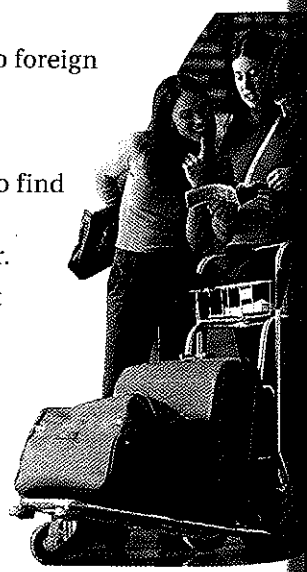
where t is the number of years since 1990. Use the following steps to find the year when there were about 56,300,000 travelers.

a. Write a polynomial equation that can be used to find the answer.

b. List the possible whole-number solutions of the equation in part (a) that are less than or equal to 10.

c. Use synthetic division to determine which of the possible solutions in part (b) is an actual solution.

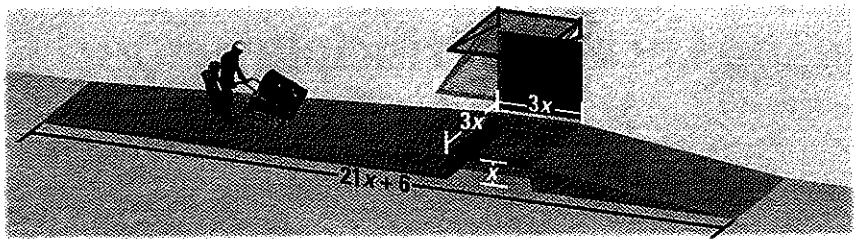
d. Graph the function $F(t)$ and explain why there are no other reasonable solutions. Then calculate the year which corresponds to the solution.



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

51. CHALLENGE You are building a pair of ramps for a loading platform. The left ramp is twice as long as the right ramp. If 150 cubic feet of concrete are used to build the two ramps, what are the dimensions of each ramp?



MIXED REVIEW

Solve the equation.

52. $4x - 6 = 18$ *(p. 18)*

53. $3y + 7 = -14$ *(p. 18)*

54. $|2p + 5| = 15$ *(p. 51)*

55. $49z^2 - 14z + 1 = 0$ *(p. 259)*

56. $8x^2 - 30x + 7 = 0$ *(p. 259)*

57. $-3(q + 2)^2 = -18$ *(p. 266)*

Solve the matrix equation. *(p. 210)*

58. $\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix} X = \begin{bmatrix} -3 & 5 \\ 6 & -1 \end{bmatrix}$

59. $\begin{bmatrix} -2 & 1 \\ 4 & 0 \end{bmatrix} X = \begin{bmatrix} 6 & 0 \\ -1 & 10 \end{bmatrix}$

60. $\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} X = \begin{bmatrix} -3 & 1 & 2 \\ 0 & -4 & -1 \end{bmatrix}$

61. $\begin{bmatrix} 2 & -8 \\ 3 & -7 \end{bmatrix} X = \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & -3 \end{bmatrix}$

PREVIEW
Prepare for Lesson 5.7 in Exs. 62–67.

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation. *(p. 292)*

62. $x^2 - 4x + 11 = 0$

63. $s^2 - 14s + 49 = 0$

64. $3t^2 - 8t - 5 = 0$

65. $-2y^2 - 5y - 3 = 0$

66. $81p^2 + 18p + 1 = 0$

67. $7r^2 + 5 = 0$

QUIZ *for Lessons 5.4–5.6*

Factor the polynomial completely. *(p. 353)*

1. $2x^3 - 54$

2. $x^3 - 3x^2 + 2x - 6$

3. $x^3 + x^2 + x + 1$

4. $6x^5 - 150x$

5. $3x^4 - 24x^2 + 48$

6. $2x^3 - 3x^2 - 12x + 18$

Divide using polynomial long division or synthetic division. *(p. 362)*

7. $(x^4 + x^3 - 8x^2 + 5x + 5) \div (x^2 + 5x - 2)$

8. $(4x^3 + 27x^2 + 3x + 64) \div (x + 7)$

Find all real zeros of the function. *(p. 370)*

9. $f(x) = 2x^3 - 19x^2 + 50x + 30$

10. $f(x) = x^3 - 4x^2 - 25x - 56$

11. $f(x) = x^4 + 4x^3 - 13x^2 - 4x + 12$

12. $f(x) = 4x^4 - 5x^2 + 42x - 20$

13. LANDSCAPING You are a landscape artist designing a square patio that is to be made from 128 cubic feet of concrete. The thickness of the patio is 15.5 feet less than each side length. What are the dimensions of the patio? *(p. 370)*