T_9. Beginning at each possible source and ending at the sink, we find that the maximal paths in the figure are

$$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8a} \to T_{10},$$
$$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8b} \to T_{10},$$
$$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8c} \to T_{10},$$
$$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8d} \to T_{10},$$
$$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8e} \to T_{10},$$
$$T_{7a} \to T_{8a} \to T_{10},$$
$$T_{7b} \to T_{8b} \to T_{10},$$
$$T_9 \to T_{10}$$

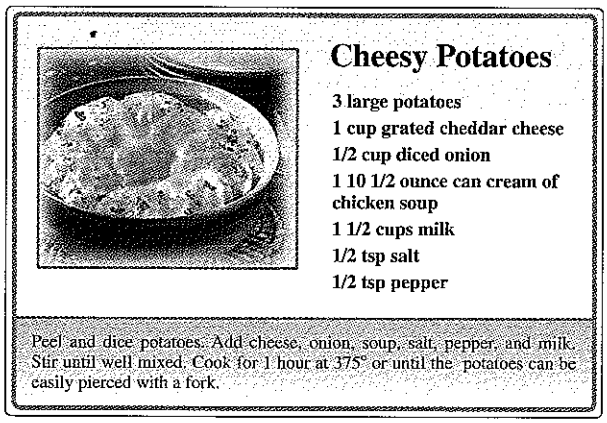
We compute the weight of each maximal path, as shown in Figure 7.14, and note which path has the largest weight.

Maximal Path	Weight of Path
$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8a} \to T_{10}$	$8 + 12 + 10 + 6 + 6 + 6 + 1 + 3 = 52$
$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8b} \to T_{10}$	$8 + 12 + 10 + 6 + 6 + 6 + 1 + 3 = 52$
$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8c} \to T_{10}$	$8 + 12 + 10 + 6 + 6 + 6 + 1 + 3 = 52$
$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8d} \to T_{10}$	$8 + 12 + 10 + 6 + 6 + 6 + 1 + 3 = 52$
$T_1 \to T_2 \to T_3 \to T_4 \to T_5 \to T_6 \to T_{8e} \to T_{10}$	$8 + 12 + 10 + 6 + 6 + 6 + 1 + 3 = 52$
$T_{7a} \to T_{8a} \to T_{10}$	$10 + 1 + 3 = 14$
$T_{7b} \to T_{8b} \to T_{10}$	$10 + 1 + 3 = 14$
$T_9 \to T_{10}$	$5 + 3 = 8$

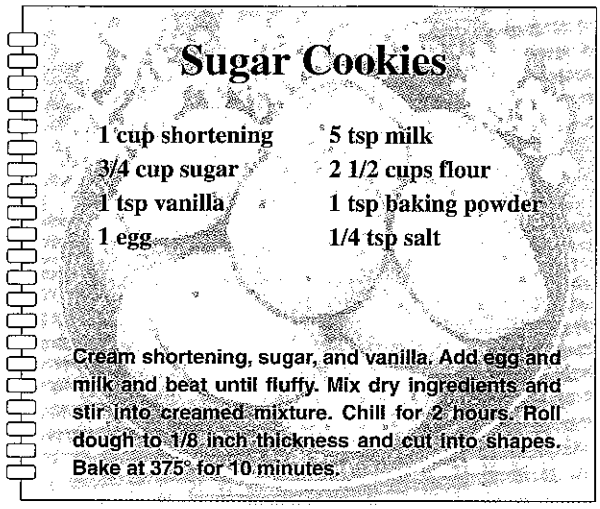
The largest weight of a maximal path is 52, so 52 hours is the critical time for the model railroad project. Thus, it will take at least 52 hours to complete the project.

PROBLEM SET 7.1

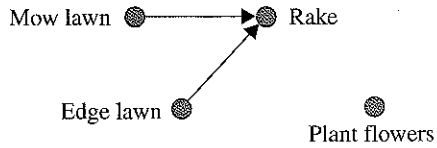
1. The following recipe will be used to make a potato dish. Divide this project into tasks.



2. The following recipe will be used to make cookies. Divide this project into tasks.

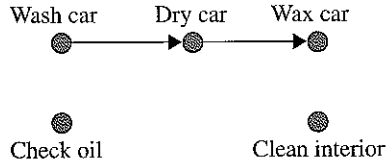


3. Consider the following digraph.



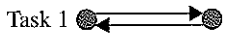
   a. How many vertices are there?
   b. Is this an order-requirement digraph? Explain.
   c. Are there any isolated vertices? Identify them.

4. Consider the following digraph.



   a. How many vertices are there?
   b. Is this an order-requirement digraph? Explain.
   c. Are there any isolated vertices? Identify them.

5. Explain why the following order-requirement digraph does not make sense.



6. Explain what is wrong with the following order-requirement digraph.

7. The tasks for the project of getting ready for bed are wash face, brush teeth, floss teeth, put on pajamas, set alarm, and climb into bed.
   a. Use a digraph to represent the precedence relations for getting ready for bed. Be sure to define the labels you use to represent the tasks involved.
   b. How many vertices are there and which vertices, if any, are isolated?

8. The tasks for the project of washing clothes are sort clothes, open the lid of the washer, turn on the washer, add soap, add clothes, and close the lid.
   a. Use a digraph to represent the precedence relations for washing clothes. Be sure to define the labels you use to represent the tasks involved.
   b. How many vertices are there and which vertices, if any, are isolated?

9. Construct an order-requirement digraph for the project of planting a tree. The tasks are find a location, dig a hole, insert tree, fill hole, fertilize, and water. Be sure to define the labels you use to represent the tasks involved.

10. Construct the order-requirement digraph for the project of leaving for work. The tasks are unlock the car door, open the door, sit in the car, start the car, release the parking brake, put the car in gear, and drive. Be sure to define the labels you use to represent the tasks involved.

11. Construct the order-requirement digraph for the project of making a pizza. The tasks are grease pan, roll out the dough, put dough on the pan, add sauce, add cheese, add toppings, turn on the oven, put pizza in oven, bake, and remove from oven. Be sure to define the labels you use to represent the tasks involved.

12. Construct an order-requirement digraph for the project of making a peanut butter and jelly sandwich. The tasks are remove two slices of bread from the package, open the peanut butter, open the jelly, get a knife from the drawer, spread the peanut butter, spread the jelly, and assemble the sandwich. Be sure to define the labels you use to represent the tasks involved.

13. Consider the tasks and the precedence relations listed in the following table.

Task	A	B	C	D	E	F	G	H	I
Prerequisite Task	D		A			A, G	B, E	G	H, C

   a. Construct an order-requirement digraph to represent the tasks and precedence relations.
   b. Which vertices appear only at the beginning of an arc?
   c. Which vertices appear only at the end of an arc?
   d. Which vertices, if any, are isolated?

14. Consider the tasks and the precedence relations listed in the following table.

Task	A	B	C	D	E	F	G	H	I
Prerequisite Task	C, F	A		A, H	B		H, E		D

a. Construct an order-requirement digraph to represent the tasks and precedence relations.

b. Which vertices appear only at the beginning of an arc?

c. Which vertices appear only at the end of an arc?

d. Which vertices, if any, are isolated?

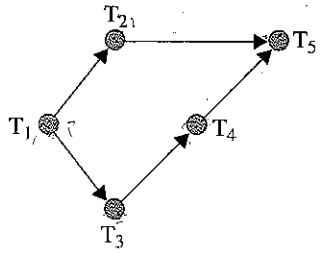
15. College courses often have prerequisites. Consider the following engineering courses offered at a community college and their prerequisites. Construct an order-requirement digraph that represents the order relationship between the courses.

Course	Title	Prerequisite
Engr 201	Electrical Fundamentals	
Engr 202	Electrical Fundamentals II	Engr 201
Engr 203	Electrical Fundamentals III	Engr 202
Engr 211	Statics	
Engr 212	Dynamics	Engr 211
Engr 213	Strengths of Materials	Engr 211
Engr 271	Digital Logic Design	Engr 201

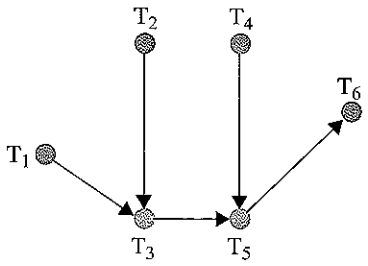
16. College courses often have prerequisites. Consider the following mathematics courses offered at a community college and their prerequisites. Construct an order-requirement digraph that represents the order relationship between the courses.

Course	Title	Prerequisite
Math 20	Basic Mathematics	
Math 60	Introduction to Algebra	Math 20
Math 65	Elementary Algebra	Math 60
Math 95	Intermediate Algebra	Math 65
Math 97	Practical Geometry	Math 95
Math 111	College Algebra	Math 97
Math 112	Trigonometry	Math 111
Math 241	Calculus for Social Sciences	Math 111
Math 243	Introduction to Statistics	Math 111
Math 245	Math for Sciences	Math 111
Math 251	Differential Calculus	Math 112

17. List all the paths in the following order-requirement digraph.



18. List all the paths in the following order-requirement digraph.



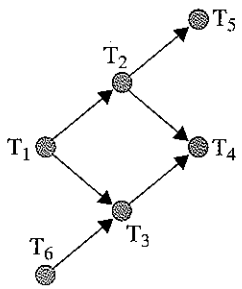
19. For the following paths, construct an order-requirement digraph that contains them and then list all the maximal paths.

$$T_1 \to T_3, \quad T_3 \to T_4, \quad T_2 \to T_3, \text{ and } T_1 \to T_5 \to T_6$$

20. For the following paths, construct an order-requirement digraph that contains them and then list all the maximal paths.

$$T_1 \to T_2, \quad T_2 \to T_4, \quad T_6 \to T_3, \quad T_2 \to T_3, \text{ and } T_1 \to T_5 \to T_6 \to T_7$$

21. Consider the following order-requirement digraph.



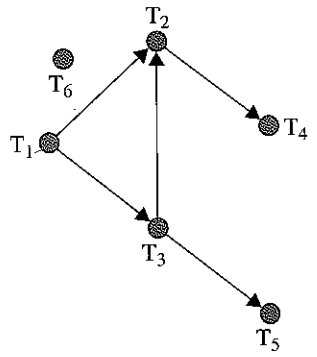
a. List all the sinks that are not isolated vertices.

b. List all the sources that are not isolated vertices.

c. List all the paths.

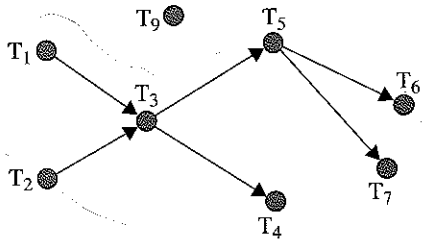
d. Which paths from part (c) are maximal paths?

22. Consider the following order-requirement digraph.



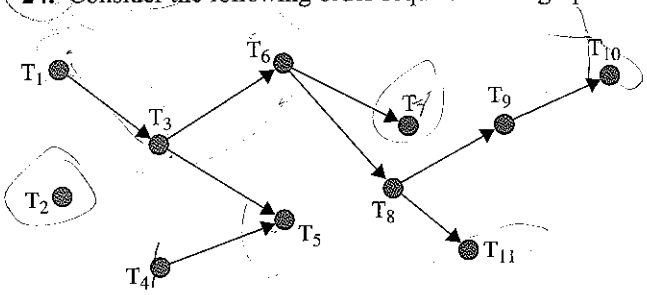
a. List all the sinks that are not isolated vertices.
b. List all the sources that are not isolated vertices.
c. List all the paths.
d. Which paths from part (c) are maximal paths?

23. Consider the following order-requirement digraph.



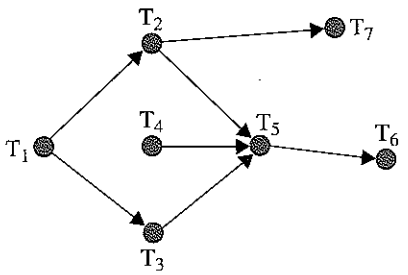
a. List all the sinks that are not isolated vertices.
b. List all the sources that are not isolated vertices.
c. List all the maximal paths.

24. Consider the following order-requirement digraph.



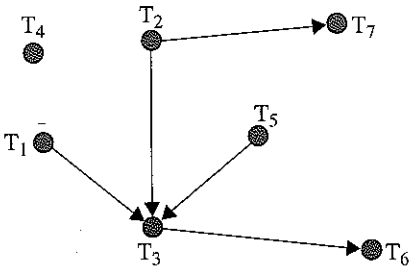
a. List all the sinks that are not isolated vertices.
b. List all the sources that are not isolated vertices.
c. List all the maximal paths.

25. Consider the following order-requirement digraph.



a. List all the sinks that are not isolated vertices.
b. List all the sources that are not isolated vertices.
c. List all the maximal paths.

26. Consider the following order-requirement digraph.



a. List all the sinks that are not isolated vertices.
b. List all the sources that are not isolated vertices.
c. List all the maximal paths.

27. Suppose the tasks involved in making a bed have the following completion times.

Task Number	Task	Completion Time
T_1	Put on fitted bottom sheet	120 seconds
T_2	Put on flat top sheet	120 seconds
T_3	Put on blanket	60 seconds
T_4	Tuck in sheets and blanket	60 seconds
T_5	Put on bedspread	60 seconds
T_6	Put pillowcases on pillows	60 seconds
T_7	Put pillows on bed	20 seconds

a. Construct a weighted order-requirement digraph to represent making the bed.
b. If there is one processor to perform all of the required tasks, what is the shortest possible finishing time?

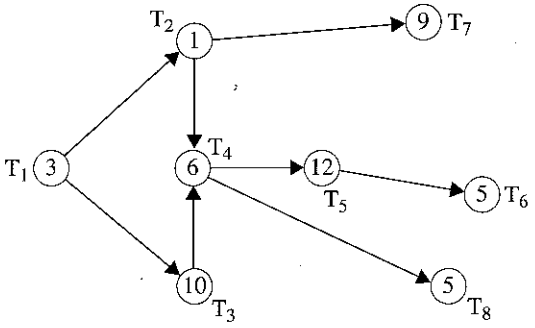
28. Suppose the tasks involved in making breakfast have the following completion times.

Task Number	Task	Completion Time
T_1	Warm up griddle	5 minutes
T_2	Mix the pancake batter	6 minutes
T_3	Cook pancakes	10 minutes
T_4	Warm up skillet	10 minutes
T_5	Fry bacon	4 minutes
T_6	Set table	2 minutes
T_7	Serve food	1 minute
T_8	Pour juice	1 minute

a. Construct a weighted order-requirement digraph to represent making breakfast.

b. If there is one processor to perform all of the required tasks, what is the shortest possible finishing time?

29. Consider the following weighted order-requirement digraph. All times are in hours.



a. Find the weight of the path $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_8$.

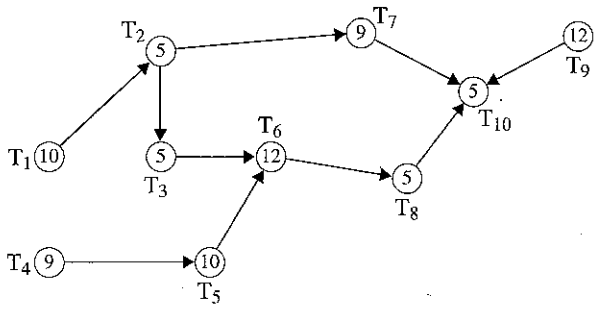
b. If there is exactly one processor, what is the finishing time?

c. List all maximal paths and their weights.

d. Find the critical path for this project.

e. What is the critical time for this project?

30. Consider the following weighted order-requirement digraph. All times are in seconds.



a. Find the weight of the path $T_1 \rightarrow T_2 \rightarrow T_7 \rightarrow T_{10}$.

b. If there is exactly one processor, what is the finishing time?

c. List all maximal paths and their weights.

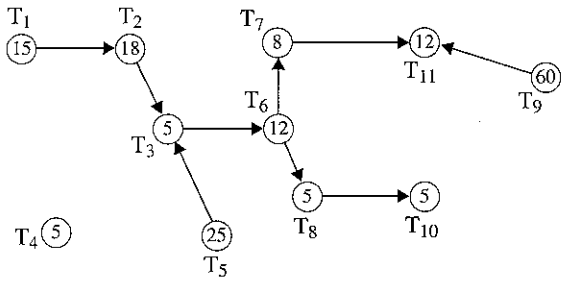
d. Find the critical path for this project.

e. What is the critical time for this project?

31. For the project of making the bed in problem 27, find the critical time.

32. For the project of making breakfast in problem 28, find the critical time.

33. Consider the following weighted order-requirement digraph. All times are in days.

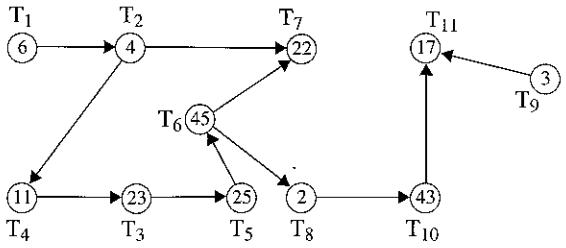


a. List all maximal paths and their weights.

b. Find the finishing time.

c. Find the critical path and the critical time for this project.

34. Consider the following weighted order-requirement digraph. All times are in minutes.



a. List all maximal paths and their weights.

b. Find the finishing time.

c. Find the critical path and the critical time for this project.

35. A certain project has four tasks and a critical time of 78 minutes. For each of the following conditions, construct a weighted order-requirement digraph to represent the project.

a. The critical path is $T_1 \rightarrow T_4$.

b. The critical path is $T_1 \rightarrow T_3 \rightarrow T_4$.

c. The critical path is $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4$.

36. A certain project has five tasks and a critical time of 58 minutes. For each of the following conditions, construct a weighted order-requirement digraph to represent the project.

a. The critical path is $T_1 \rightarrow T_3 \rightarrow T_4$.

b. The critical path is $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_5$.

c. The critical path is $T_2 \rightarrow T_3 \rightarrow T_4 \rightarrow T_5$.

37. Suppose that one person will take on the project of updating a living room. The associated tasks, precedence relations, and completion times are listed in the following table. Construct a weighted order-requirement digraph, calculate the finishing time, and find the critical time.

Task	Description	Completion Time in Minutes	Prerequisite Task
T_1	Remove furniture	45	
T_2	Take down curtains	10	
T_3	Remove curtain rods and hardware	15	T_2
T_4	Remove all pictures and nails from walls	10	
T_5	Patch holes	30	T_3, T_4
T_6	Remove switch plates	10	
T_7	Tape room	60	T_1, T_5, T_6
T_8	Paint ceiling	70	T_7
T_9	Paint walls	180	T_8
T_{10}	Let ceiling and walls dry	120	T_9
T_{11}	Apply second coat to walls	180	T_{10}
T_{12}	Let walls dry	120	T_{11}
T_{13}	Remove tape	20	T_{12}
T_{14}	Install switch plates	20	T_{12}
T_{15}	Install new window blinds	60	T_{13}
T_{16}	Move in furniture	45	T_{13}

38. Suppose that one person will take on the project of constructing a fence. The associated tasks, precedence relations, and completion times are listed in the following table. Construct a weighted order-requirement digraph, calculate the finishing time, and find the critical time.

Task	Description	Completion Time in Hours	Prerequisite Task
T_1	Clear debris	2	
T_2	Measure yard	0.5	
T_3	Mark post locations	1	T_2
T_4	Make a list of needed supplies	0.5	T_2
T_5	Buy supplies	2	T_4
T_6	Dig post holes	3	T_3, T_1
T_7	Mix concrete	0.5	T_6, T_5
T_8	Set posts	2	T_7
T_9	Let concrete dry	24	T_8
T_{10}	Attach brackets to posts	0.75	T_9
T_{11}	Attach 2-by-4s between posts	3	T_{10}
T_{12}	Nail up boards	6	T_{11}
T_{13}	Stain fence	24	T_{12}, T_{15}
T_{14}	Construct gate	2	T_5
T_{15}	Install gate	1	T_{14}

Extended Problems

39. When did the idea of using systematic project management tools, such as digraphs, first originate? For what purpose was it first used? Research the origins of scheduling and critical paths. For information on the Internet, search keywords "origins of project management." Summarize your findings in a report.

40. Decisions about care provided to patients at UCLA School of Medicine have been based on critical-path methods. The academic institution had to make cost-cutting changes to compete with other hospitals. For one procedure called a "radical prostatectomy," they devised "critical-care pathways" to efficiently and effectively schedule medicines, patient education, treatment, nutrition, and activity. The results of implementing critical-care pathways were dramatic. Read about how scheduling methods were applied at the UCLA School of Medicine in *Oncology News International*, Vol. 5, No. 5 (May 1996). Summarize your findings in a report.

41. President John F. Kennedy spoke these words to Congress on May 25, 1961, as a challenge to get Americans to the moon within 10 years.

> I believe that this nation should commit itself to achieving the goal, before this decade is out, of landing a man on the Moon and returning him safely to the Earth. No single space program in this period will be more impressive to mankind, or more important in the long-range exploration of space; and none will be so difficult or expensive to accomplish.

After only a little more than eight years, on July 20, 1969, Apollo 11 landed on the moon. The planning, coordinating, construction, and management efforts required to complete such an amazing project in such a short time were accomplished, in part, because of the use of critical paths. Research the Apollo Project to find out how critical paths helped land men on the moon. Summarize your findings in a report.

42. Think of a project you completed recently. For example, you may have cleaned a room, prepared a meal, built a doghouse, landscaped a yard, or repaired a car. For a project of your choice, create the task list and assign reasonable completion times for each task. Construct a weighted digraph for the project, and find the critical path and critical time. What is the finishing time if you are the lone processor?