EXAMPLE 3 Find side lengths
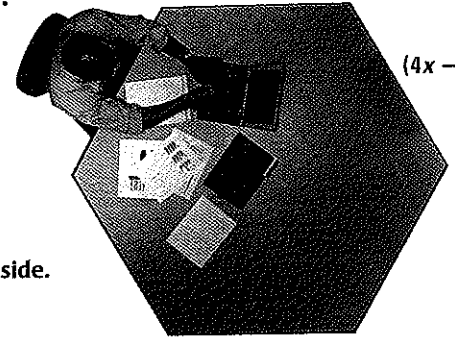
ALGEBRA A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.

$(3x + 6)$ in.

$(4x - 2)$ in.

Solution

First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

$3x + 6 = 4x - 2$ Write equation.

$6 = x - 2$ Subtract $3x$ from each side.

$8 = x$ Add 2 to each side.

Then find a side length. Evaluate one of the expressions when $x = 8$.

$3x + 6 = 3(8) + 6 = 30$

▶ The length of a side of the table is 30 inches.

✓ **GUIDED PRACTICE**   for Example 3

3. The expressions $8y°$ and $(9y - 15)°$ represent the measures of two of the angles in the table in Example 3. Find the measure of an angle.


1.6 EXERCISES

SKILL PRACTICE

1. VOCABULARY *Explain* what is meant by the term *n*-gon.

2. ★ WRITING Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? *Explain.*
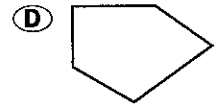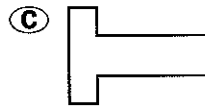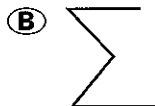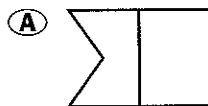
IDENTIFYING POLYGONS Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave.*
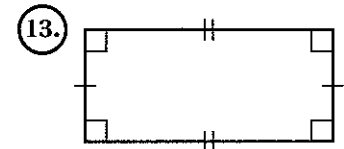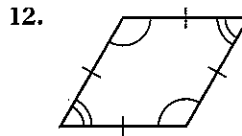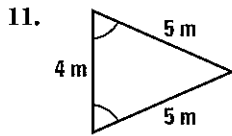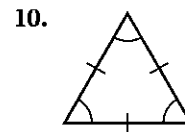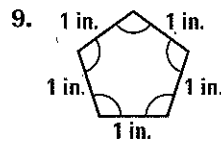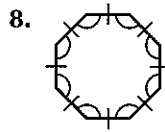
3.   **4.**   **5.**   **6.**
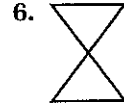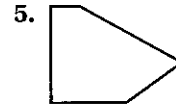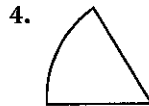
**7. ★ MULTIPLE CHOICE** Which of the figures is a concave polygon?

Ⓐ   Ⓑ   Ⓒ   Ⓓ

CLASSIFYING Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. *Explain* your reasoning.

8.

9. 1 in. 1 in.
   1 in. 1 in.
   1 in.

10.
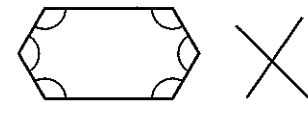
11. 5 m
    4 m
    5 m

12.

13.

14. **ERROR ANALYSIS** Two students were asked to draw a regular hexagon, as shown below. *Describe* the error made by each student.

Student A
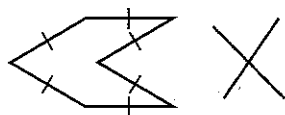
Student B

15. **ALGEBRA** The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon.

16. **ALGEBRA** The expressions $(9x + 5)°$ and $(11x - 25)°$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. **ALGEBRA** The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side.

USING PROPERTIES Tell whether the statement is *always*, *sometimes*, or *never* true.

18. A triangle is convex.

19. A decagon is regular.

20. A regular polygon is equiangular.

21. A circle is a polygon.

22. A polygon is a plane figure.

23. A concave polygon is regular.

DRAWING Draw a figure that fits the description.
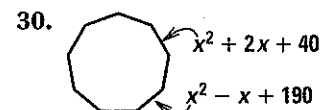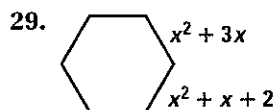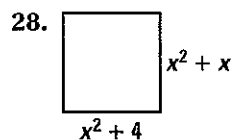
24. A triangle that is not regular

25. A concave quadrilateral

26. A pentagon that is equilateral but not equiangular

27. An octagon that is equiangular but not equilateral

ALGEBRA Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of x.

28. $x^2 + x$
    $x^2 + 4$

29. $x^2 + 3x$
    $x^2 + x + 2$

30. $x^2 + 2x + 40$
    $x^2 - x + 190$

31. **CHALLENGE** Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain* your reasoning.
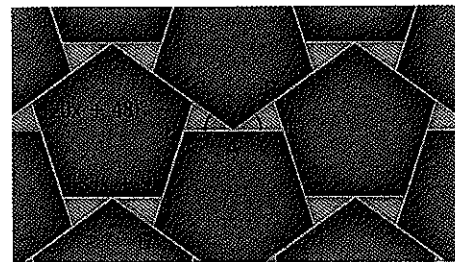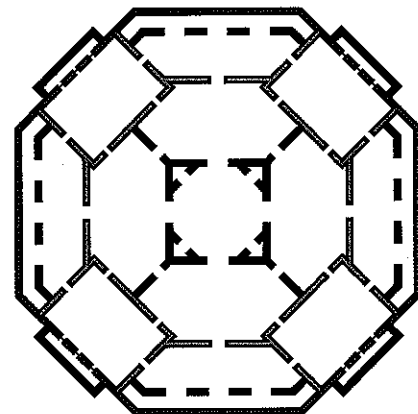

PROBLEM SOLVING

32. **ARCHITECTURE** Longwood House, shown in the photograph on page 42, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.

    a. Tell whether the red polygon in the diagram is *convex* or *concave*.

    b. Classify the red polygon and tell whether it appears to be regular.

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SIGNS Each sign suggests a polygon. Classify the polygon by the number of sides. Tell whether it appears to be *equilateral, equiangular,* or *regular.*

33. YIELD

34. RESERVED PARKING — STATE DISABLED PARKING PERMIT REQUIRED

35. STOP

36. RAIL CROSSING ROAD

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37. ★ **MULTIPLE CHOICE** Two vertices of a regular quadrilateral are $A(0, 4)$ and $B(0, -4)$. Which of the following could be the other two vertices?

    Ⓐ $C(4, 4)$ and $D(4, -4)$ Ⓑ $C(-4, 4)$ and $D(-4, -4)$

    Ⓒ $C(8, -4)$ and $D(8, 4)$ Ⓓ $C(0, 8)$ and $D(0, -8)$

38. **MULTI-STEP PROBLEM** The diagram shows the design of a lattice made in China in 1850.

    a. Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.

    b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.

○ = **WORKED-OUT SOLUTIONS** on p. WS1   ★ = **STANDARDIZED TEST PRACTICE**
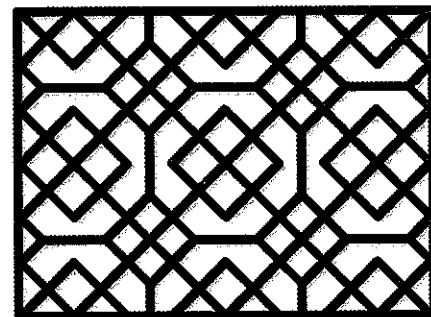
EXAMPLE 3
on p. 44
for Ex. 39

39. ★ SHORT RESPONSE The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? *Explain.*

$(3x + 12)$ mm

$(20 - 5x)$ mm

40. ★ EXTENDED RESPONSE A segment that joins two nonconsecutive vertices of a polygon is called a *diagonal*. For example, a quadrilateral has two diagonals, as shown below.
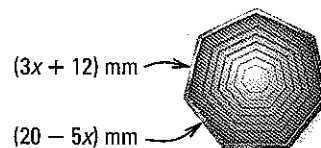
| Type of polygon	Diagram	Number of sides	Number of diagonals
Quadrilateral		4	2
Pentagon	?	?	?
Hexagon	?	?	?
Heptagon	?	?	?

a. Copy and complete the table. *Describe* any patterns you see.

b. How many diagonals does an octagon have? a nonagon? *Explain.*

c. The expression $\dfrac{n(n - 3)}{2}$ can be used to find the number of diagonals in an *n*-gon. Find the number of diagonals in a 60-gon.

41. LINE SYMMETRY A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry*. A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.

a. A regular triangle   **b.** A regular pentagon

c. A regular hexagon   **d.** A regular octagon
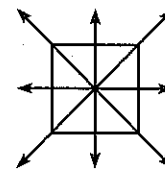
regular quadrilateral
4 lines of symmetry

42. CHALLENGE The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge.
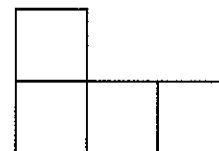
MIXED REVIEW

Solve the equation.

43. $\frac{1}{2}(35)b = 140$ *(p. 875)*

44. $x^2 = 144$ *(p. 882)*

45. $3.14r^2 = 314$ *(p. 882)*

Copy and complete the statement. *(p. 886)*

46. 500 m = __?__ cm

47. 12 mi = __?__ ft

48. 672 in. = __?__ yd

49. 1200 km = __?__ m

50. $4\frac{1}{2}$ ft = __?__ yd

51. 3800 m = __?__ km

Find the distance between the two points. *(p. 15)*

52. $D(-13, 13)$, $E(0, -12)$

53. $F(-9, -8)$, $G(-9, 7)$

54. $H(10, 5)$, $J(-2, -2)$

31. $m\angle XWZ = 35.5°$, $m\angle YWZ = 35.5°$ **33.** 38°
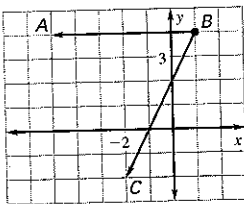**35.** 142° **37.** 53°

39. If a ray bisects $\angle AGC$, then its
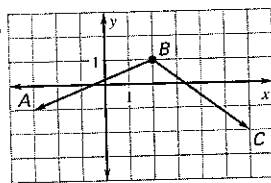endpoint must be point G. *Sample:*
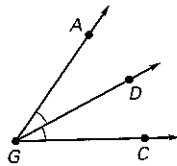
41. 80° **43.** 75°; both angle measures are 5° less.
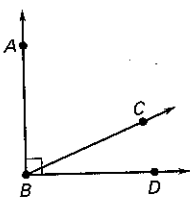
45. Acute.
Sample answer: $(-2, 0)$

47. Obtuse.
Sample answer: $(2, 0)$

1.4 Problem Solving (pp. 31–32) **51.** 32° **53. a.** 112°
b. 56° **c.** 56° **d.** 56° **55.** *Sample answer:* acute: $\angle ABG$,
obtuse: $\angle ABC$, right: $\angle DGE$, straight: $\angle DGF$
57. about 140° **59.** about 62° **61.** about 107°

1.5 Skill Practice (pp. 38–40)

1. No. *Sample answer:* Any
two angles whose angle
measures add up to 90°
are complementary, but
they do not have to have a
common vertex and side.

3. adjacent **5.** adjacent **7.** $\angle GLH$ and $\angle HLJ$, $\angle GLJ$
and $\angle JLK$ **9.** 69° **11.** 85° **13.** 25° **15.** 153° **17.** 135°, 45°
19. 54°, 36° **21.** linear pair **23.** vertical angles
25. linear pair **27.** neither **29.** The angles are
complementary so they should be equal to 90°;
$x + 3x = 90°$, $4x = 90$, $x = 22.5$. **31.** 10, 35 **33.** 55, 30
35. Never; a straight angle is 180°, and it is not possible
to have a complement of an angle that is 180°.
37. Always; the sum of complementary angles is 90°,
so each angle must be less than 90°, making them
acute. **39.** 71°, 19° **41.** 68°, 22° **43.** 58°, 122°

1.5 Problem Solving (pp. 40–41) **47.** neither
49–51. Sample answers are given. **49.** $\angle FGB$, $\angle BGC$
51. $\angle AGE$, $\angle EGD$ **53.** *Sample answer:* Subtract 90°
from $m\angle FGB$. **55. a.** $y_1 = 90 - x$, $0 < x < 90$;
$y_2 = 180 - x$, $0 < x < 180$; the measure of the
complement must be less than 90° and the measure
of its supplement must be less than 180°.

55. b.

$0 < y_1 < 90$ $0 < y_2 < 180$

1.6 Skill Practice (pp. 44–46) **1.** An n-gon is a polygon
with n sides. **3.** polygon; concave **5.** polygon;
convex **9.** Pentagon; regular; it has 5 congruent sides
and angles. **11.** Triangle; none of these; the sides and/
or the angles are not all congruent. **13.** Quadrilateral;
equiangular; it has 4 congruent angles.
15. 8 in. **17.** 3 ft **19.** sometimes **21.** never **23.** never
25. *Sample:*   **27.** *Sample:*
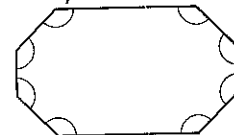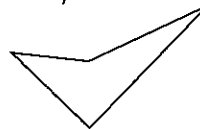
29. 1

1.6 Problem Solving (pp. 46–47) **33.** triangle; regular
35. octagon; regular **39.** 105 mm; each side of the
button is 15 millimeters long, so the perimeter of
the button is $15(7) = 105$ millimeters. **41. a.** 3 **b.** 5
c. 6 **d.** 8

1.7 Skill Practice (pp. 52–54) **1.** *Sample answer:* The
diameter is twice the radius. **3.** $(52)(9)$ must be
divided by 2; $\frac{52(9)}{2} = 234$ ft². **5.** 22.4 m, 29.4 m²
7. 180 yd, 1080 yd² **9.** 36 cm, 36 cm² **11.** 84.8 cm,
572.3 cm² **13.** 76.0 cm, 459.7 cm²

15.   59.3 cm, 280.4 cm²

18.9 cm
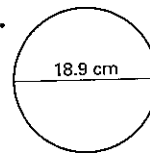
17. 12.4 **21.** 1.44 **23.** 8,000,000 **25.** 3,456 **27.** 14.5 m
29. 4.5 in. **31.** 6 in., 3 in. **33.** Octagon; dodecagon; the
square has 4 sides, so a polygon with the same side
length and twice the perimeter would have to have
$2(4) = 8$ sides, an octagon; a polygon with the same
side length and three times the perimeter would have
to have $4(3) = 12$ sides, a dodecagon. **35.** $\sqrt{346}$ in.
37. $5\sqrt{42}$ km

1.7 Problem Solving (pp. 54–56) **41.** 1350 yd²; 450 ft
43. a. 15 in. **b.** 6 in.; the spoke is 21 inches long
from the center to the tip, and it is 15 inches from
the center to the outer edge, so $21 - 15 = 6$ inches
is the length of the handle.