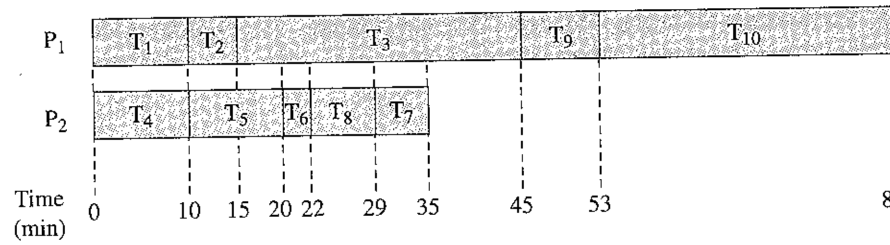


Figure 7.46

Notice that as we assign tasks according to the critical-path priority list, processor P_1 receives all the tasks in the critical path, while processor P_2 receives the remaining tasks. Because the finishing time of the project (83 minutes) is equal to the critical time, we know that this schedule is optimal.

Now that we have the complete Gantt chart and have established a schedule, we can reinterpret the results in terms of preparing the lasagna. Recall that the project was for Erica (P_1) and Enrique (P_2) to work together to make the lasagna. The optimal schedule we have designed calls for these students to perform the cooking tasks as described next.

Erica browns the meat (T_1),
adds the tomato ingredients, garlic, and basil (T_2), and
simmers the meat sauce (T_3).

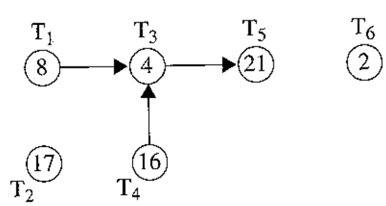
Enrique boils the water (T_4),
cooks, rinses, and drains the noodles (T_5),
beats the eggs (T_6),
slices the mozzarella cheese (T_8), and
mixes the beaten eggs with ricotta and parmesan cheeses and seasonings (T_7).

Finally,

Erica assembles the layers (T_9), and
bakes the lasagna (T_{10}).

PROBLEM SET 7.3

1. Consider the following weighted order-requirement digraph. All completion times are in minutes.



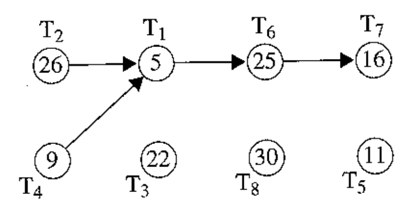
a. List all the maximal paths and their weights.

b. List all the isolated vertices and their weights.

c. Find the critical time for this project.

d. Which task would be assigned first according to an increasing-time priority list? A decreasing-time priority list? A critical-path priority list?

2. Consider the following weighted order-requirement digraph. All completion times are in minutes.



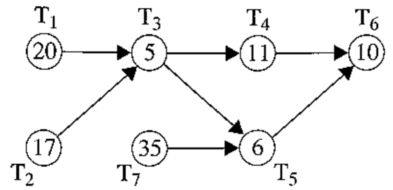
a. List all the maximal paths and their weights.

b. List all the isolated vertices and their weights.

c. Find the critical time for this project.

d. Which task would be assigned first according to an increasing-time priority list? A decreasing-time priority list? A critical-path priority list?

3. Consider the following weighted order-requirement digraph. All completion times are in minutes.



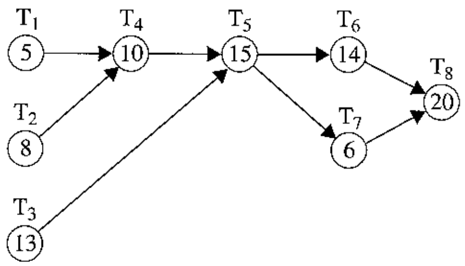
a. Find the finishing time for this project if one processor is assigned.

b. List all maximal paths and their weights.

c. What are the critical path and the critical time for this project?

d. Explain the significance of the critical time.

4. Consider the following weighted order-requirement digraph. All completion times are in minutes.



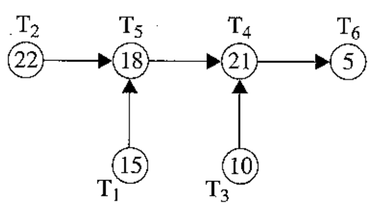
a. Find the finishing time for this project if one processor is assigned.

b. List all maximal paths and their weights.

c. What are the critical path and the critical time for this project?

d. Explain the significance of the critical time.

5. Consider the following weighted order-requirement digraph. All completion times are in minutes.



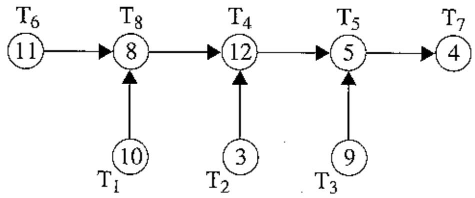
a. Find the critical path and the critical time for this project.

b. Construct the Gantt chart and find the finishing time when processor 1 is assigned the tasks in the critical path and processor 2 is assigned T_1 followed by T_3.

c. Construct the Gantt chart and find the finishing time when processor 1 is assigned the tasks in the critical path and processor 2 is assigned T_3 followed by T_1.

d. Is either of the schedules found in parts (b) or (c) optimal? Explain why or why not.

6. Consider the following weighted order-requirement digraph. All completion times are in minutes.



a. Find the critical path and the critical time for this project.

b. Construct the Gantt chart, and find the finishing time when processor 1 is assigned the tasks in the critical path and processor 2 is assigned T_2 first, T_3 second, and T_1 third.

c. Construct the Gantt chart, and find the finishing time when processor 1 is assigned the tasks in the critical path and processor 2 is assigned T_1 first, T_2 second, and T_3 third.

d. Is either of the schedules found in parts (b) or (c) optimal? Explain why or why not.

7. Consider the following table, which contains a list of all maximal paths and their weights. This project contains no isolated vertices.

Maximal Path	Weight (Time in Minutes)
$T_2 \to T_5 \to T_4 \to T_6$	$22 + 18 + 21 + 5 = 66$
$T_1 \to T_5 \to T_4 \to T_6$	$15 + 18 + 21 + 5 = 59$
$T_3 \to T_4 \to T_6$	$10 + 21 + 5 = 36$

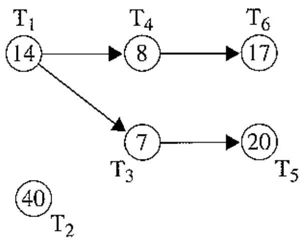
a. Explain why T_2 will be placed first in the critical-path priority list.

b. Remove T_2 and the resulting nonmaximal path from the table. Explain why T_1 is the next task placed in the critical-path priority list.

c. Remove T_1 from the table. Explain why T_5 is the next task placed in the critical-path priority list.

d. Remove T_5 and the resulting nonmaximal path from the table. In what order will the remaining tasks be placed in the critical-path priority list?

8. Consider the following weighted order-requirement digraph. All completion times are in minutes.



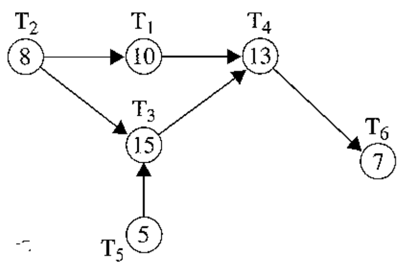
a. List all the maximal paths, isolated vertices, and their weights in a table. Explain why T_1 is first in the critical-path priority list.

b. Remove T_1 and all attached edges, draw the resulting digraph, and explain why T_2 is the next task placed in the critical-path priority list.

c. Remove T_2, draw the resulting digraph, and explain why T_3 is the next task placed in the critical-path priority list.

d. Remove T_3 and all attached edges, and draw the resulting digraph. In what order will the remaining tasks be placed in the critical-path priority list?

9. Consider the following weighted order-requirement digraph for a project. All completion times are in hours.



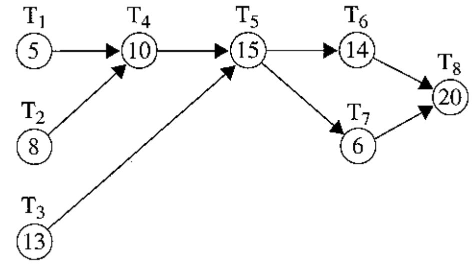
a. What is the critical time for this project?

b. Find the critical-path priority list.

c. Use the critical-path list to schedule two processors to complete this project, and construct the Gantt chart.

d. Is the schedule optimal? Explain why or why not.

10. Consider the following weighted order-requirement digraph for a project. All completion times are in weeks.



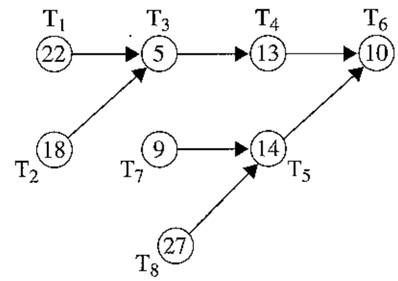
a. What is the critical time for this project?

b. Find the critical-path priority list.

c. Use the critical-path list to schedule two processors to complete this project, and construct the Gantt chart.

d. Is the schedule optimal? Explain why or why not.

11. Consider the following weighted order-requirement digraph for a project. All completion times are in minutes.

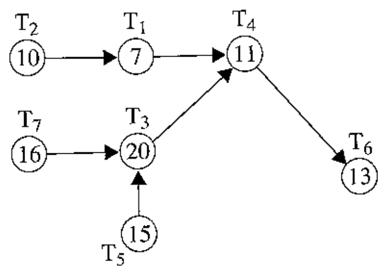


a. Find the critical path and the critical time for this project.

b. Find the critical-path priority list.

c. Use the critical-path scheduling algorithm to schedule two processors to the project. How many minutes will it take to complete the project?

12. Consider the following weighted order-requirement digraph for a project. All completion times are in minutes.



a. Find the critical path and the critical time for this project.

b. Find the critical-path priority list.

c. Use the critical-path scheduling algorithm to schedule two processors to the project. How many minutes will it take to finish the project?

13. Consider the weighted order-requirement digraph from problem 11.

a. Suppose the completion time for T_4 can be reduced from 13 minutes to 8 minutes. Will this change the schedule or the finishing time? Explain.

b. Suppose the completion time for T_8 can be reduced from 27 minutes to 20 minutes. Will this change the schedule or the finishing time? Explain.

14. Consider the weighted order-requirement digraph from problem 12.

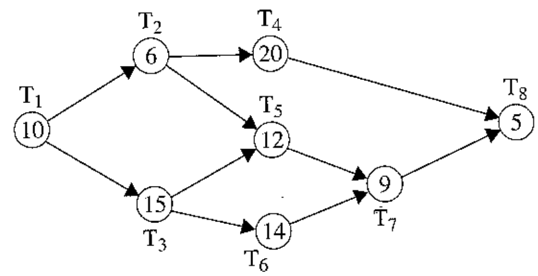
a. Suppose the completion time for T_4 can be reduced from 11 minutes to 8 minutes. Will this change the schedule or the finishing time? Explain.

b. Suppose the completion time for T_2 can be reduced from 10 minutes to 5 minutes. Will this change the schedule or the finishing time? Explain.

15. Suppose the first task in the critical-path priority list from problem 11 is delayed, causing the completion time for that task to be 10 minutes longer. How will this affect the schedule and the finishing time for the project with two processors?

16. Suppose the first task in the critical-path priority list from problem 12 is delayed, causing the completion time for that task to be 10 minutes longer. How will this affect the schedule and the finishing time for the project with two processors?

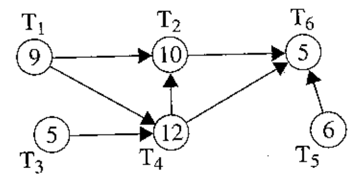
17. Consider the following weighted order-requirement digraph for a project. All completion times are in minutes.



a. Find the critical-path priority list.

b. What is the critical time for this project?

18. Consider the following weighted order-requirement digraph for a project. All completion times are in hours.



a. Find the critical-path priority list.

b. What is the critical time for this project?

19. Refer to problem 17. Use the critical-path scheduling algorithm to assign tasks to two processors. Construct the Gantt chart and determine how much idle time is in the schedule. Is the schedule optimal? Explain.

20. Refer to problem 18. Use the critical-path scheduling algorithm to assign tasks to two processors. Construct the Gantt chart and determine how much idle time is in the schedule. Is the schedule optimal? Explain.

21. Refer to problem 17. Use the critical-path scheduling algorithm to assign tasks to three processors. Construct the Gantt chart and determine how much idle time is in the schedule. Is the schedule optimal? Explain.

22. Refer to problem 18. Use the critical-path scheduling algorithm to assign tasks to three processors. Construct the Gantt chart and determine how much idle time is in the schedule. Is the schedule optimal? Explain.

23. Two people prepare to leave for a trip. They must complete the following tasks before they leave.

a. Construct the weighted order-requirement digraph for this project.

b. What is the critical time?

c. Find the critical-path priority list.

d. Using two processors, apply the critical-path scheduling algorithm and construct the Gantt chart.

e. Is the schedule determined in part (d) optimal? Explain.

Task	Description	Completion Time (Minutes)	Prerequisite Task
1	Wash and dry clothes	150	
2	Get suitcases from garage	10	
3	Pack suitcases	25	Tasks 1 and 2
4	Clean out car and trunk	30	
5	Lock up house and leave	10	Tasks 6 and 8
6	Load car	22	Tasks 3 and 4
7	Fill gas tank on the way out of town	12	Task 5
8	Mow lawn	35	

24. Two people remodel a kitchen. They must complete the following tasks.

a. Construct the weighted order-requirement digraph for this project.

b. What is the critical time?

c. Find the critical-path priority list.

d. Using two processors, apply the critical-path scheduling algorithm and construct the Gantt chart.

e. Is the schedule determined in part (d) optimal? Explain.

Task	Description	Completion Time (Hours)	Prerequisite Task
1	Remove old wall cabinets	2	
2	Remove old base cabinets	2.5	
3	Remove appliances	0.5	
4	Remove old vinyl flooring	2.75	Tasks 2 and 3
5	Install new wall cabinets	10	Task 1
6	Install new base cabinets	11	Task 4
7	Install counter tops	8	Task 6
8	Install molding and trim	2	Tasks 5 and 7
9	Paint	6	Task 8
10	Install flooring	2	Task 9
11	Install appliances	0.5	Task 10

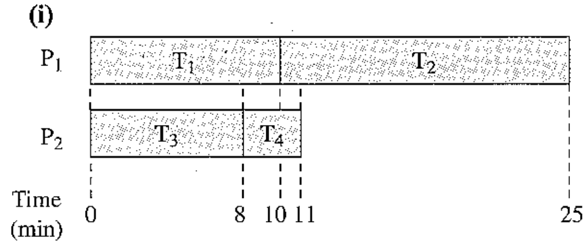
Extended Problems

Problems 25 through 29

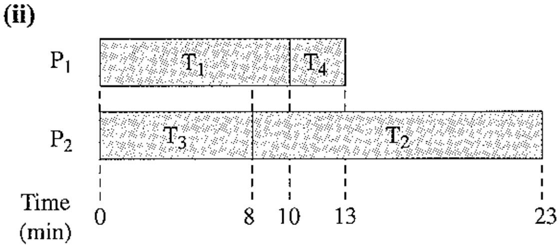
For some projects, the order-requirement digraph has no arcs; in other words, there is no precedence relation between the tasks. We call the tasks independent in such cases. Tasks may be assigned in any order, and the schedule includes no idle time because no task is delayed while a processor finishes a different task.

25. Consider the independent tasks T_1 (10 minutes), T_2 (15 minutes), T_3 (8 minutes), T_4 (3 minutes), and the following Gantt charts.

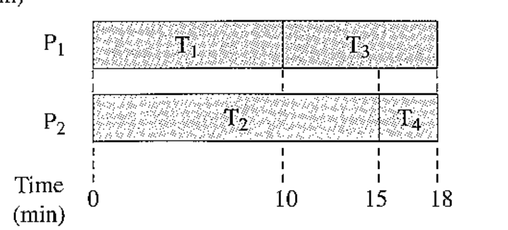
(i)



(ii)



(iii)



a. What is the finishing time in each case?

b. Is any of the three schedules optimal? Explain how you know.

c. If three processors were available, what would be an optimal schedule? Explain how you know the schedule is optimal.

d. Would it be effective to use four processors? Explain why or why not.