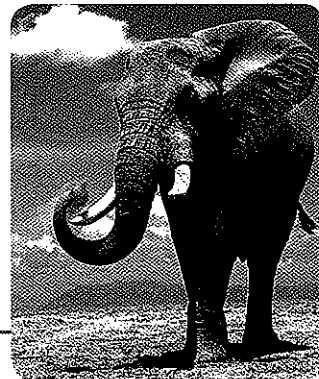


# 1.7 Solve Absolute Value Equations and Inequalities



**Before**

You solved linear equations and inequalities.

**Now**

You will solve absolute value equations and inequalities.

**Why?**

So you can describe hearing ranges of animals, as in Ex. 81.

## Key Vocabulary

- absolute value
- extraneous solution

Recall that the **absolute value** of a number  $x$ , written  $|x|$ , is the distance the number is from 0 on a number line. This understanding of absolute value can be extended to apply to simple absolute value equations.

$$|x| = \begin{cases} x, & \text{if } x \text{ is positive} \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x \text{ is negative} \end{cases}$$

### KEY CONCEPT

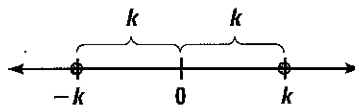
*For Your Notebook*

#### Interpreting Absolute Value Equations

**Equation**  $|x| = |x - 0| = k$

**Meaning** The distance between  $x$  and 0 is  $k$ .

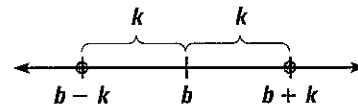
**Graph**



**Solutions**  $x - 0 = -k$  or  $x - 0 = k$   
 $x = -k$  or  $x = k$

$|x - b| = k$

The distance between  $x$  and  $b$  is  $k$ .



$x - b = -k$  or  $x - b = k$   
 $x = b - k$  or  $x = b + k$

### EXAMPLE 1 Solve a simple absolute value equation

Solve  $|x - 5| = 7$ . Graph the solution.

**Solution**

$|x - 5| = 7$

Write original equation.

$x - 5 = -7$  or  $x - 5 = 7$

Write equivalent equations.

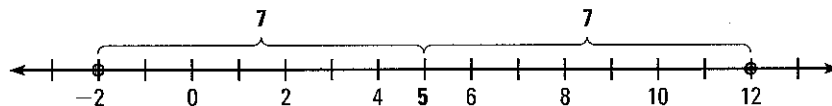
$x = 5 - 7$  or  $x = 5 + 7$

Solve for  $x$ .

$x = -2$  or  $x = 12$

Simplify.

► The solutions are  $-2$  and  $12$ . These are the values of  $x$  that are 7 units away from 5 on a number line. The graph is shown below.



**GUIDED PRACTICE** for Examples 5 and 6

Solve the inequality. Then graph the solution.

10.  $|x + 2| < 6$

11.  $|2x + 1| \leq 9$

12.  $|7 - x| \leq 4$

13. **GYMNASTICS** For Example 6, write an absolute value inequality describing the *unacceptable* mat thicknesses.**1.7 EXERCISES****HOMework KEY**○ = **WORKED-OUT SOLUTIONS**  
on p. WS2 for Exs. 21, 47, and 77★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 33, 40, 63, and 64◆ = **MULTIPLE REPRESENTATIONS**  
Ex. 78**SKILL PRACTICE**

1. **VOCABULARY** What is an extraneous solution of an equation?
2. ★ **WRITING** The absolute value of a number cannot be negative. How, then, can the absolute value of  $x$  be  $-x$  for certain values of  $x$ ?

**CHECKING SOLUTIONS** Decide whether the given number is a solution of the equation.

3.  $|b - 1| = 14; -13$

4.  $|d + 6| = 10; -4$

5.  $|32 - 6f| = 20; -2$

6.  $|2m + 6| = 10; -8$

7.  $|3n - 7| = 4; 1$

8.  $|17 - 8r| = 15; 4$

**EXAMPLE 1**on p. 51  
for Exs. 9–20**SOLVING EQUATIONS** Solve the equation. Graph the solution.

9.  $|x| = 9$

10.  $|y| = -5$

11.  $|z| = 0$

12.  $|f - 5| = 3$

13.  $|g - 2| = 7$

14.  $|h - 4| = 4$

15.  $|k + 3| = 6$

16.  $|m + 5| = 1$

17.  $|n + 9| = 10$

18.  $|6 - p| = 4$

19.  $|5 - q| = 7$

20.  $|-4 - r| = 4$

**EXAMPLE 2**on p. 52  
for Exs. 21–32**SOLVING EQUATIONS** Solve the equation.

21.  $|2d - 5| = 13$

22.  $|3g + 14| = 7$

23.  $|7h - 10| = 4$

24.  $|3p - 6| = 21$

25.  $|2q + 3| = 11$

26.  $|4r + 7| = 43$

27.  $|5 + 2j| = 9$

28.  $|6 - 3k| = 21$

29.  $|20 - 9m| = 7$

30.  $|\frac{1}{4}x - 3| = 10$

31.  $|\frac{1}{2}y + 4| = 6$

32.  $|\frac{2}{3}z - 6| = 12$

33. ★ **SHORT RESPONSE** The equation  $|5x - 10| = 45$  in Example 2 has two solutions. Does the equation  $|5x - 10| = -45$  also have two solutions? *Explain.***EXAMPLE 3**on p. 52  
for Exs. 34–42**EXTRANEIOUS SOLUTIONS** Solve the equation. Check for extraneous solutions.

34.  $|3x - 4| = x$

35.  $|x + 24| = -7x$

36.  $|8x - 1| = 6x$

37.  $|4x + 5| = 2x + 4$


38.  $|9 - 2x| = 10 + 3x$


39.  $|8 + 5x| = 7 - x$

40. ★ **MULTIPLE CHOICE** What is (are) the solution(s) of  $|3x + 7| = 5x$ ?

- (A)  $-4, -\frac{2}{3}$       (B)  $-\frac{7}{8}, \frac{7}{2}$       (C)  $\frac{7}{8}, \frac{7}{2}$       (D)  $\frac{7}{2}$

**ERROR ANALYSIS** Describe and correct the error in solving the equation.

41.  $|5x - 9| = x + 3$   
 $5x - 9 = x + 3$  or  $5x - 9 = -x + 3$   
 $4x - 9 = 3$  or  $6x - 9 = 3$   
 $4x = 12$  or  $6x = 12$   
 $x = 3$  or  $x = 2$   
 The solutions are 3 and 2. 

42.  $|n - 7| = 3n - 1$   
 $n - 7 = 3n - 1$  or  $n - 7 = -3n + 1$   
 $-7 = 2n - 1$  or  $4n - 7 = 1$   
 $-6 = 2n$  or  $4n = 8$   
 $-3 = n$  or  $n = 2$   
 The solutions are -3 and 2. 

**EXAMPLES**  
**4 and 5**  
 on pp. 53-54  
 for Exs. 43-63

**SOLVING INEQUALITIES** Solve the inequality. Then graph the solution.

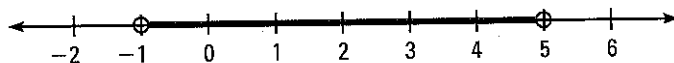
43.  $|j| \leq 5$       44.  $|k| > 4$       45.  $|m - 2| < 7$       46.  $|n - 11| \geq 1$   
 47.  $|d + 4| \geq 3$       48.  $|f + 6| < 2$       49.  $|g - 1| > 0$       50.  $|h + 10| \leq 10$   
 51.  $|3w - 15| < 30$       52.  $|2x + 6| \geq 10$       53.  $|4y - 9| \leq 7$       54.  $|5z + 1| > 14$   
 55.  $|16 - p| > 3$       56.  $|24 - q| \leq 11$       57.  $|7 - 2r| < 19$       58.  $|19 - 5t| > 7$   
 59.  $|\frac{1}{2}x - 10| \leq 4$       60.  $|\frac{1}{3}m - 15| < 6$       61.  $|\frac{1}{7}y + 2| - 5 > 3$       62.  $|\frac{2}{5}n - 8| + 4 \geq 12$

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63. ★ **MULTIPLE CHOICE** What is the solution of  $|6x - 9| \geq 33$ ?

- (A)  $-4 \leq x \leq 7$       (B)  $-7 \leq x \leq 4$   
 (C)  $x \leq -4$  or  $x \geq 7$       (D)  $x \leq -7$  or  $x \geq 4$

64. ★ **MULTIPLE CHOICE** Which absolute value inequality represents the graph shown below?



- (A)  $-1 < |x| < 5$       (B)  $|x + 2| < 3$       (C)  $|x - 2| < 3$       (D)  $|x - 2| < 5$

65. **REASONING** For the equation  $|ax + b| = c$  (where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ ), describe the value(s) of  $c$  that yield two solutions, one solution, and no solution.

**SOLVING INEQUALITIES** Solve the inequality. Then graph the solution.

66.  $|x + 1| \geq -16$       67.  $|2x - 1| < -25$       68.  $|7x + 3| \leq 0$       69.  $|x - 9| > 0$


**CHALLENGE** Solve the inequality for  $x$  in terms of  $a$ ,  $b$ , and  $c$ . Assume  $a$ ,  $b$ , and  $c$  are real numbers and  $c > 0$ .

70.  $|ax + b| < c$  where  $a > 0$       71.  $|ax + b| \geq c$  where  $a > 0$   
 72.  $|ax + b| \leq c$  where  $a < 0$       73.  $|ax + b| > c$  where  $a < 0$


## PROBLEM SOLVING

**EXAMPLE 5**  
on p. 54  
for Exs. 74–78

74. **GYMNASTICS** The horizontal bar used in gymnastics events should be placed 110.25 inches above the ground, with a tolerance of 0.4 inch. Write an absolute value inequality for the acceptable bar heights.

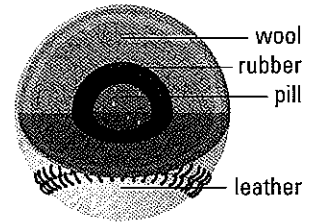
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75. **SOIL PH LEVELS** Cucumbers grow in soil having a pH level of 6.5, with a tolerance of 1 point on the pH scale. Write an absolute value inequality that describes the pH levels of soil in which cucumbers can grow.

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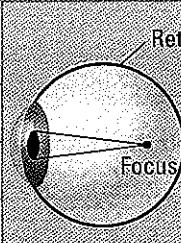
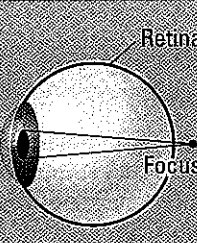
76. **MULTI-STEP PROBLEM** A baseball has a cushioned cork center called the *pill*. The pill must weigh 0.85 ounce, with a tolerance of 0.05 ounce.

- Write an absolute value inequality that describes the acceptable weights for the pill of a baseball.
- Solve the inequality to find the acceptable weights for the pill.
- Look back at Example 5 on page 54. Find the minimum and maximum percentages of a baseball's total weight that the pill can make up.



77. **MANUFACTURING** A regulation basketball should weigh 21 ounces, with a tolerance of 1 ounce. Write an absolute value inequality describing the weights of basketballs that should be *rejected*.

78. **MULTIPLE REPRESENTATIONS** The strength of eyeglass lenses is measured in units called *diopters*. The diopter number  $x$  is negative for nearsighted vision and positive for farsighted vision.

Nearsightedness (focus is in front of retina)		Retina		Farsightedness (focus is behind retina)	
Mild	$ x + 1.5  < 1.5$			Mild	$ x - 1  < 1$
Moderate	$ x + 4.5  < 1.5$			Moderate	$ x - 3  < 1$
Severe	$ x + 7.5  < 1.5$			Severe	$ x - 5  < 1$

- Writing Inequalities** Write an equivalent compound inequality for each vision category shown above. Solve the inequalities.
- Making a Graph** Illustrate the six vision categories by graphing their ranges of diopter numbers on the same number line. Label each range with the corresponding category name.

**EXAMPLE 6**  
on p. 54  
for Exs. 79–81

79. **SLEEPING BAGS** A manufacturer of sleeping bags suggests that one model is best suited for temperatures between 30°F and 60°F, inclusive. Write an absolute value inequality for this temperature range.

80. **TEMPERATURE** The recommended oven setting for cooking a pizza in a professional brick-lined oven is between 550°F and 650°F, inclusive. Write an absolute value inequality for this temperature range.

**81. AUDIBLE FREQUENCIES** An elephant can hear sounds with frequencies from 16 hertz to 12,000 hertz. A mouse can hear sounds with frequencies from 1000 hertz to 91,000 hertz. Write an absolute value inequality for the hearing range of each animal.

**82. CHALLENGE** The depth finder on a fishing boat gives readings that are within 5% of the actual water depth. When the depth finder reading is 250 feet, the actual water depth  $x$  lies within a range given by the following inequality:

$$|x - 250| \leq 0.05x$$

- Write the absolute value inequality as a compound inequality.
- Solve each part of the compound inequality for  $x$ . What are the possible actual water depths if the depth finder's reading is 250 feet?

## MIXED REVIEW

### PREVIEW

Prepare for  
Lesson 2.1  
in Exs. 83–94.

Plot the points in the same coordinate plane. (p. 987)

83. (4, 4)

84. (7, -8)

85. (-3, 0)

86. (0, -6)

87. (-2, -3)

88. (-5, 2)

Evaluate the expression for the given value of the variable. (p. 10)

89.  $6m - 10$ ;  $m = 4$

90.  $-4n + 18$ ;  $n = 3$

91.  $5p + 17$ ;  $p = 0$

92.  $7q + 3$ ;  $q = -4$

93.  $-2r - 3$ ;  $r = 7$

94.  $10t - 5$ ;  $t = -3$

Solve the equation for  $y$ . Then find the value of  $y$  for the given value of  $x$ . (p. 26)

95.  $5x + y = 14$ ;  $x = 8$

96.  $-3x + y = 12$ ;  $x = -9$

97.  $8x - 4y = 32$ ;  $x = -3$

98.  $-6x + 15y = 33$ ;  $x = 10$

## QUIZ for Lessons 1.6–1.7

Solve the inequality. Then graph the solution. (p. 41)

1.  $4k - 17 < 27$

2.  $14n - 8 \geq 90$

3.  $-9p + 15 \leq 96$

4.  $-8r - 11 > 45$

5.  $3(x - 7) < 6(10 - x)$

6.  $-25 - 4z > 66 - 17z$

Solve the equation or inequality. (p. 51)

7.  $|x - 6| = 9$

8.  $|3y + 3| = 12$

9.  $|2z + 5| = -9z$

10.  $|p + 7| > 2$

11.  $|2q - 3| \leq 3$

12.  $|5 - r| \geq 4$

**13. TEST SCORES** Your final grade in a course is 80% of your current grade, plus 20% of your final exam score. Your current grade is 83 and your goal is to get a final grade of 85 or better. Write and solve an inequality to find the final exam scores that will meet your goal. (p. 41)

**14. GROCERY WEIGHTS** A container of potato salad from your grocer's deli is supposed to weigh 1.5 pounds, with a tolerance of 0.025 pound. Write and solve an absolute value inequality that describes the acceptable weights for the container of potato salad. (p. 51)

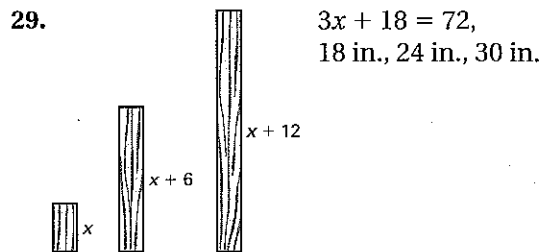
49.  $1\frac{2}{3}$  51. 6; 15, 8, 15, 8 53. 2; 6, 6, 3 55. 4 57. 2  
 59. 4 61. 2.9 63. no solution 65. all real numbers  
 67.  $x = \frac{d-b}{a-c}$ ;  $a = c$  and  $b \neq d$ ;  $a = c$  and  $b = d$   
 1.3 Problem Solving (pp. 23-24) 69. 3 h 71. 9 h  
 73. a.  $3c + 2g = 8$  b.  $2\frac{1}{4}$ ;  $\frac{1}{2}$ ;  $2\frac{5}{12}$ ;  $\frac{1}{4}$  75. 18 min

- 1.4 Skill Practice (pp. 30-31) 1. formula 3.  $l = \frac{A}{w}$   
 5 mm 5.  $h = \frac{2A}{b_1 + b_2}$ ; 6 cm 7.  $y = 26 - 3x$ ; 5  
 9.  $y = -\frac{6}{5}x + \frac{31}{5}$ ; 11 11.  $y = \frac{3}{2}x - \frac{21}{2}$ ; -3  
 13.  $y = \frac{7}{4}x - \frac{11}{4}$ ; 6 17. The variable  $y$  should only appear on one side of the equation, not both;  
 $4y - xy = 9$ ,  $y(4 - x) = 9$ ,  $y = \frac{9}{4 - x}$ . 19.  $h = \frac{S}{\pi r} - k$ ;  
 about 4.96 cm 21.  $y = \frac{40 + 3x}{x}$ ; 11 23.  $y = \frac{16x + 28}{3x}$ ;  
 $7\frac{2}{3}$  25.  $y = \frac{15}{1 - 2x}$ ; 5 27. Method 1:  $y = \frac{5}{3}x - 3$ ,  
 $y = \frac{5}{3} \cdot 2 - 3$ ,  $y = \frac{1}{3}$ ; Method 2:  $15 \cdot 2 - 9y = 27$ ,  
 $30 - 9y = 27$ ,  $-9y = -3$ ,  $y = \frac{1}{3}$ ; *Sample answer:*  
 Method 1 is more efficient because it is already solved for  $y$ . 29.  $z = \frac{x + y}{xy - 1}$  31.  $z = \frac{xy}{xy - y - x}$

- 1.4 Problem Solving (pp. 31-32) 33.  $d = \frac{C}{\pi}$ ; about 36 in.  
 35.  $C = \frac{5}{9}(F - 32)$ ;  $10^\circ\text{C}$  37.  $R = 80c + 150d$ ;  
 $d = \frac{R - 80c}{150}$ ; 80 designer tuxedos; 160 designer  
 tuxedos; 240 designer tuxedos 39.  $V = \frac{\ell^2 w}{4\pi}$ ;  $V = \frac{w^2 \ell}{4\pi}$

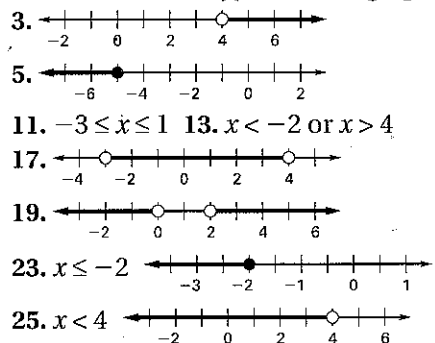
- 1.5 Skill Practice (pp. 37-38) 1. verbal model  
 3. 0.5 h 5. 90 mi 7. 54 ft 9. 20 m 11.  $y = 4x + 11$   
 13.  $y = 46 - 10x$  17.  $4x + 9 = 12$ , 0.75 ft 19. The  
 pattern shows the output is decreased by 10 each  
 time; an equation that represents the table is  
 $y = 75 - 10x$ . 23.  $y = 7x - 16$

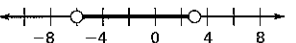
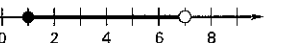
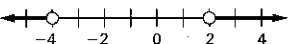
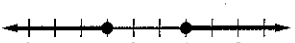
- 1.5 Problem Solving (pp. 38-39) 25. 3.75 km/min  
 27.  $y = 1.5x + 15$ ; no; the bamboo shoot will  
 eventually slow its growth rate and stop growing.



31.  $40x + 7(20 - x) = 404$ ; 8 boxes of books,  
 12 boxes of clothes 33. about 4.07 in.

1.6 Skill Practice (pp. 44-45) 1. graph



35. The inequality symbol should not be reversed  
 when subtracting;  $10 > 2x$ ,  $5 > x$ .  
 37.  $-6 < x < 3$    
 39.  $1 \leq x < 7$    
 43.  $x < -4$  or  $x > 2$    
 45.  $x \leq -\frac{1}{2}$  or  $x \geq 1$    
 49. no solution 51. no solution

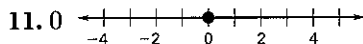
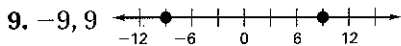
- 1.6 Problem Solving (pp. 46-47) 53.  $45x + 35 \leq 250$ ,  
 $x \leq 4\frac{7}{9}$  days; 4 or fewer days 55. a.  $0 \leq e < 500$

- b.  $1400 \leq e < 2429$  c.  $0 \leq e < 500$  or  $1400 \leq e < 2429$   
 57.  $50 \leq F \leq 95$ ;  $10 \leq C \leq 35$   
 59. a. Amy:  $0.65(84) + 0.15(80) + 0.2w \geq 85$ ,  
 Brian:  $0.65(80) + 0.15(100) + 0.2x \geq 85$ , Clara:  
 $0.65(75) + 0.15(95) + 0.2y \geq 85$ , Dan:  $0.65(80) +$   
 $0.15(90) + 0.2z \geq 85$  b.  $w \geq 92$ ;  $x \geq 90$ ;  $y \geq 110$ ;  $z \geq 97.5$   
 c. Amy, Brian, and Dan. *Sample answer:* It is  
 impossible to score over 100 points on a test, so Clara  
 will not be able to achieve a grade of 85 or better.

1.6 Problem Solving Workshop (p. 49)

1.  $y = -35x + 200$ ;  $x > 20$  3.  $x \geq \$7000$

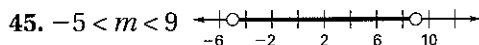
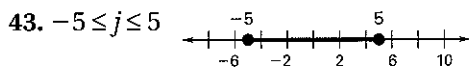
- 1.7 Skill Practice (pp. 55-56) 1. An apparent solution  
 that must be rejected because it does not satisfy  
 the original equation. 3. solution 5. not a solution  
 7. solution



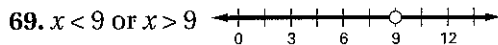
21.  $-4, 9$  23.  $\frac{6}{7}, 2$  25.  $-7, 4$  27.  $-7, 2$  29.  $1\frac{4}{9}, 3$

31.  $-20, 4$  33. No; the equation has no solutions because an absolute value will never be negative.

35.  $-3$  37.  $-1\frac{1}{2}, -\frac{1}{2}$  39.  $-\frac{1}{6}, -3\frac{3}{4}$  41. When writing the second equation, the right side of the equation should be  $-x - 3$ ;  $5x - 9 = -x - 3$ ,  $6x - 9 = -3$ ,  $6x = 6$ ,  $x = 1$ , the solutions are 3 and 1.



65.  $c > 0, c = 0, c < 0$  67. no solution



71.  $x \leq \frac{-c-b}{a}$  or  $x \geq \frac{c-b}{a}$  73.  $x < \frac{c-b}{a}$  or  $x > \frac{-c-b}{a}$

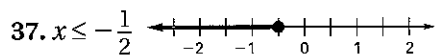
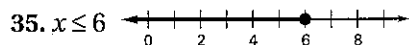
75.  $|p - 6.5| \leq 1$  77.  $|b - 21| > 1$  79.  $|x - 45| \leq 15$

81.  $|e - 6008| \leq 5992$ ,  $|m - 46,000| \leq 45,000$

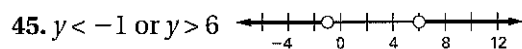
**Chapter Review** (pp. 61–64) 1. exponent, base  
3. extraneous solution 5. *Sample answer:*  $3(x - 4)$   
and  $3x - 12$  7. Inverse property of multiplication  
9. Distributive property 11.  $3x - 6y$  13.  $18b - 33$   
15.  $-2t^4 + 5t^2$  17.  $-\frac{1}{6}$  19. 9 21.  $-1$  23. \$74.99

25.  $y = -10x + 7$ ;  $-23$  27.  $y = \frac{-15}{x - 6}$ ; 15 29.  $y = \frac{5}{2}x - 5$ ;

$-20$  31.  $h = \frac{S - 2\pi r^2}{2\pi r}$ ; about 7.73 cm 33. 602 mi



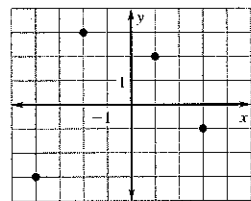
41.  $-3, 1\frac{2}{3}$  43. no solution



47.  $|v - 26| \leq 0.5$ ,  $25.5 \text{ in.} \leq v \leq 26.5 \text{ in.}$

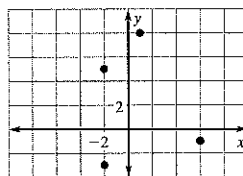
## Chapter 2

**2.1 Skill Practice** (pp. 76–78) 1. independent, dependent  
3. domain:  $-4, -2, 1, 3$ , range:  $-3, -1, 2, 3$



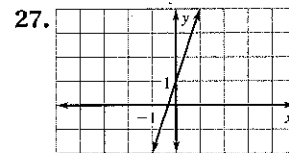
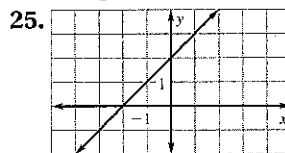
Input	Output
-4	-3
-2	-1
1	2
3	3

5. domain:  $-2, 1, 6$ , range:  $-3, -1, 5, 8$



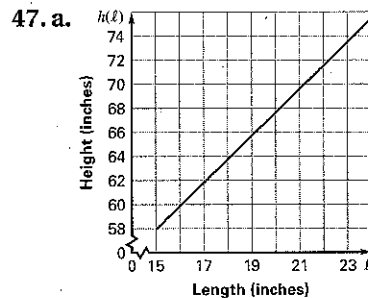
Input	Output
-2	-3
1	-1
6	5
6	8

11. Yes; each input has exactly one output. 13. Yes; each input has exactly one output. 15.  $x$  is the input and  $y$  is the output, so there should be one value of  $y$  for each value of  $x$ ; the relation given by the table is not a function because the inputs 1 and 0 each have more than one output. 17. No; the input  $-2$  has more than one output. 19. No; the input  $-1$  has more than one output. 21. function 23. not a function



35. not linear; 10 37. linear; 6 39. linear;  $-3$

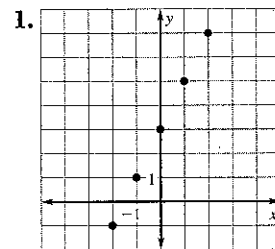
**2.1 Problem Solving** (pp. 78–79) 43. Yes; each input has exactly one output. 45. About 905;  $V(6)$  represents the volume of a sphere with radius 6.



domain:  
 $15 \leq l \leq 24$ ,  
range:  
 $57.95 \leq h(l) \leq 75.5$

b. 59 in. or 4 ft 11 in. c. 21.7 in. 49. a. domain: 11,350,000, 12,280,000, 12,420,000, 15,980,000, 18,980,000, 20,850,000, 33,870,000, range: 20, 21, 27, 31, 34, 55 b. Yes; each input  $p$  has exactly one output. c. No; the input 21 has more than one output.

**Extension** (p. 81)



discrete;  $-1, 1, 3, 5, 7$