PROBLEM SET 10.1

Problems 1 through 4

Calculating probabilities often requires that you perform operations with fractions. The following problems are designed to help you brush up on fractions. Perform the given operations by hand. If the result is a fraction, express it in lowest terms.

1. **a.** $\frac{1}{8} + \frac{3}{4}$ **b.** $\frac{1}{3} + 1 - \frac{11}{15}$

c. $\frac{3}{10} \cdot \frac{2}{9}$ **d.** $\dfrac{\frac{4}{7}}{\frac{3}{2}}$

2. **a.** $\frac{2}{3} + \frac{1}{5}$ **b.** $\frac{5}{8} + 1 - \frac{1}{12}$

c. $\frac{5}{8} \cdot \frac{4}{15}$ **d.** $\dfrac{\frac{3}{5}}{\frac{2}{3}}$

3. **a.** $\dfrac{\frac{8}{15}}{\frac{11}{45}}$

b. $\frac{1}{4} \cdot \frac{2}{5} + \frac{3}{4} \cdot \frac{3}{5}$

c. $\frac{1}{3}(240) + \frac{2}{5}(-50)$

4. **a.** $\dfrac{\frac{2}{7}}{1 - \frac{2}{7}}$

b. $\frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3}$

c. $\frac{3}{8}(500) + \frac{7}{16}(80) + \frac{3}{16}(-1200)$

5. According to the weather report, there is a 20% chance of snow in the county tomorrow. Which of the following statements would be an appropriate interpretation of this statement?

a. Out of the next 5 days, it will snow 1 of those days.

b. Out of the next 24 hours, snow will fall for 4.8 hours.

c. Of past days when conditions were similar, 1 of 5 had some snow in the county.

d. It will snow on 20% of the area of the county tomorrow.

6. The doctor says, "There is a 40% chance that your problem will get better without surgery." Which of the following statements would be an appropriate interpretation of this statement?

a. You can expect to feel 40% better.

b. In the future, you will feel better on 2 of every 5 days.

c. Among you and the next four other patients with the same problem, two will get better without surgery.

d. Among patients with symptoms similar to yours who have participated in research studies of non-surgical treatments, about 40% got better.

7. List the elements of the sample space and one possible event for each of the following experiments.

a. A quarter is tossed, and the result is recorded.

b. A single die with faces labeled *A*, *B*, *C*, *D*, *E*, *F* is rolled, and the letter on the top face is recorded.

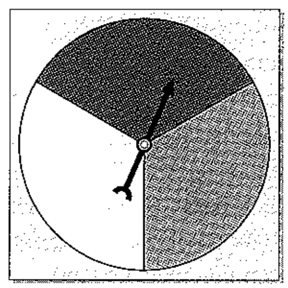
c. A telephone number is selected at random from a telephone book, and the fourth digit is recorded.

8. List the elements of the sample space and one possible event for each of the following experiments.

a. A $20 bill is obtained from an automatic teller machine, and the right-most digit of the serial number is recorded.

b. Some white and black marbles are placed in a jar, mixed, and a marble is chosen without looking. The color of the marble is recorded.

c. The following "Red–Blue–Yellow" spinner is spun once, and the color is recorded. (All central angles in the spinner are 120°.)



9. An experiment consists of drawing a slip of paper from a bowl in which there are 10 slips of paper labeled *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, and *J* and recording the letter on the paper. List each of the following:

a. The sample space

b. The event that a vowel is drawn

c. The event that a consonant is drawn

d. The event that a letter between *B* and *G* (excluding *B* and *G*) is drawn

e. The event that a letter in the word ZOOLOGY is drawn

10. An experiment consists of drawing a ping-pong ball out of a box in which 12 balls were placed, each marked with a number from 1 to 12, inclusive, and recording the number on the ball. List the following:

a. The sample space

b. The event that an even number is drawn

c. The event that a number less than 8 is drawn

d. The event that a number divisible by 2 and 3 is drawn

e. The event that a number greater than 12 is drawn

11. An experiment consists of tossing four coins and noting whether each coin lands with a head or a tail showing. List each of the following:
   - **a.** The sample space
   - **b.** The event that the first coin shows a head
   - **c.** The event that three of the coins show heads
   - **d.** The event that the fourth coin shows a tail
   - **e.** The event that the second coin shows a head and the third coin shows a tail

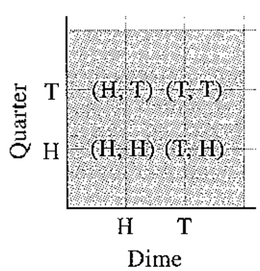
12. An experiment consists of flipping a coin and rolling an eight-sided die and noting whether the coin lands with a head or a tail showing and which number faces up on the die. The eight faces on each die are labeled 1, 2, 3, 4, 5, 6, 7, and 8 as shown.

Eight-Sided Die

List each of the following:
   - **a.** The sample space
   - **b.** The event that a 2 faces up on the die
   - **c.** The event that the coin lands with a head showing
   - **d.** The event that the coin lands with a tail showing and an odd number faces up on the die
   - **e.** The event that the coin lands with a head showing or a 7 faces up on the die

Problems 13 and 14

One way to find the sample space of an experiment involving two parts is to plot the possible outcomes of one part of the experiment horizontally and the outcomes of the other part vertically, then fill in the pairs of outcomes in a rectangular array. For example, suppose that an experiment consists of tossing a dime and a quarter. The sample space could be plotted as:



The sample space of the experiment is {(H, H), (H, T), (T, H), (T, T)}.

13. Use the method just described to construct the sample space for the experiment of tossing a coin and rolling a four-sided die with faces labeled 1, 2, 3, and 4.

14. Use the method just described to construct the sample space for the experiment of tossing a coin and drawing a marble from a jar containing purple, green, and yellow marbles.

15. A standard six-sided die is rolled 60 times with the following results.

Outcome	Frequency
1	10
2	9
3	10
4	12
5	8
6	11

   - **a.** Find the experimental probability of the following events.
     1. Getting a 4
     2. Getting an odd number
     3. Getting a number greater than 3
   - **b.** Based on the experimental probability in (a), if the die is rolled 250 times, how many times would you expect to get an even number?

16. A dropped thumbtack will land with the point up or the point down. The results for tossing a thumbtack 60 times are as follows.

Outcome	Frequency
Point up	42
Point down	18

   - **a.** What is the experimental probability that the thumbtack lands
     1. point up?
     2. point down?
   - **b.** Based on the experimental probability in (a), if the thumbtack is tossed 100 times, about how many times would you expect it to land
     1. point up?
     2. point down?

17. An experiment consists of tossing three fair coins.

- **a.** List the outcomes in the sample space and the theoretical probability for each outcome in a table.
- **b.** Find the theoretical probability for the event of getting at least one head.
- **c.** Find the theoretical probability for the event of getting exactly two heads.

18. A jar contains three marbles: one red, one green, and one yellow. An experiment consists of drawing a marble from the jar, noting its color, placing it back in the jar, mixing, and drawing a second marble.

- **a.** List the outcomes in the sample space and the theoretical probabilities for each outcome in a table.
- **b.** Find the theoretical probability for the event of getting at least one red marble.
- **c.** Find the theoretical probability for the event of getting no red marbles.

Problems 19 and 20

Refer to Example 10.2(d), which gives the sample space for the experiment of rolling two standard dice. Assume the dice are fair, and give the theoretical probabilities of the listed events.

19. **a.** Getting a 4 on the second die

- **b.** Getting an even number on each die
- **c.** Getting a total of at least 7 dots
- **d.** Getting a total of 15 dots

20. **a.** Getting a 5 on the first die

- **b.** Getting an even number on one die and an odd number on the other die
- **c.** Getting a total of no more than 7 dots
- **d.** Getting a total greater than 1

21. An experiment consists of rolling two standard dice and noting the numbers that show on the top faces. Assume the dice are fair.

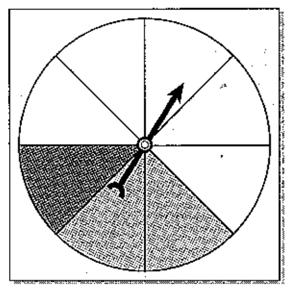
- **a.** List the elements in the sample space.
- **b.** Find the theoretical probability of the event that the product of the two numbers is even.
- **c.** Find the theoretical probability of the event that the product of the two numbers is odd.
- **d.** Find the theoretical probability of the event that the product of the two numbers is a multiple of 5.

22. An experiment consists of rolling an eight-sided die and a standard six-sided die and noting the numbers that show on the top faces. Assume the dice are fair.

- **a.** List the elements in the sample space.
- **b.** Find the theoretical probability of the event that the sum of the two numbers is greater than 6.
- **c.** Find the theoretical probability of the event that the sum of the two numbers is less than 7.
- **d.** Find the theoretical probability of the event that the product of the two numbers is a multiple of 5.

23. Refer to the following spinner.

- **a.** What is the probability of the spinner landing on yellow?
- **b.** Explain why the probability of getting white is the same as the probability of getting blue.



24. Refer to the preceding spinner.

- **a.** What is the probability of the spinner landing on white?
- **b.** Explain why the probability of getting red is less than the probability of getting any other color.

Problems 25 and 26

Two twelve-sided dice having the numbers 1–12 on their faces are rolled and the numbers facing up are added. Assume the dice are fair and find the probabilities for the listed events.

Twelve-Sided Die

25. **a.** The total is 5.

- **b.** The total is a perfect square.
- **c.** The total is a prime number.

26. **a.** The total is 11.

- **b.** The total is a multiple of 7.
- **c.** The total is even or 19.

27. A six-sided die is constructed that has two faces marked with 2s, three faces marked with 3s, and one face marked with a 5. If this die is rolled once, find the following probabilities:

a. Getting a 2
b. Not getting a 2
c. Getting an odd number
d. Not getting an odd number

28. A 12-sided die is constructed that has three faces marked with 1s, two faces marked with 2s, three faces marked with 3s, and four faces marked with 4s. If this die is rolled once, find the following probabilities:

a. Getting a 4
b. Not getting a 4
c. Getting an odd number
d. Not getting an odd number

29. A couple planning their wedding decides to randomly select a month in which to marry. If T is the event the month is 30 days long, and Y is the event the month ends in the letter y, find and interpret $P(T)$, $P(Y)$, and $P(T \cup Y)$.

30. A family planning a vacation randomly selects one of the states in the United States as their destination. If O is the that event the state borders the Pacific Ocean and N is the event that the state's name contains the word "New," find and interpret $P(O)$, $P(N)$, and $P(O \cup N)$.

31. In a class consisting of girls and boys, 6 of the 14 girls and 7 of the 11 boys have done their homework. A student is selected at random. Consider the following events.

G: The student is a girl.

D: The student has done his or her homework.

a. Find $P(G)$ and $P(D)$.
b. Find and interpret $P(G \cup D)$ and $P(G \cap D)$.
c. Are events G and D mutually exclusive? Explain.

32. For an experiment in which a fair coin is tossed and a fair standard die is rolled, consider the following events.

H: The coin lands heads up.

F: The die shows a number greater than 4.

a. Find $P(H)$ and $P(F)$.
b. Find and interpret $P(H \cup F)$ and $P(H \cap F)$.
c. Are events H and F mutually exclusive?

33. Consider the sample space for the experiment in Example 10.2(c) and the following events.

A: getting a green on the first spin

B: getting a yellow on the second spin

a. List the sample space and find $P(A)$, $P(B)$, $P(A \cap B)$, and $P(A \cup B)$.
b. Verify that the equation $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ holds for the probabilities in part (a).

34. Suppose a jar contains 20 marbles, numbered 1 through 20, with each odd-numbered marble colored red, and each even-numbered marble colored black. A marble is drawn from the jar and its color and number are noted.

a. List the sample space.
b. Consider the following events.

A: getting a black marble

B: getting a number divisible by 3

Find $P(A)$, $P(B)$, $P(A \cap B)$, and $P(A \cup B)$.

c. Verify that the equation $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ holds for the probabilities in part (b).

35. a. Suppose 45% of people have blood type O. Let E be the event that a person has type O blood. Describe $\overline{E}$ and find $P(\overline{E})$.

b. Approximately 8% of babies are born left-handed. Let F be the event that a baby is left-handed. Describe $\overline{F}$ and find $P(\overline{F})$.

36. Based on research conducted after the 1989 Loma Prieta earthquake, U.S. Geological Survey (USGS) results indicate that there is a 62% probability of at least one quake of magnitude 6.7 or greater striking the San Francisco Bay region before 2032. Describe the complement of this event and give its probability.

37. In Example 10.6, a jar contains four marbles: one red, one green, one yellow, and one white. Two marbles are drawn from the jar, one after another, without replacing the first one drawn. Let A be the event the first marble is green, let B be the event the first marble is green and the second marble is white, and let C be the event the second marble is red.

a. Are events B and C mutually exclusive? Explain.
b. Are events A and C mutually exclusive? Explain.
c. Describe in words the complement of event A.
d. Find the probability of the event A and the event $\overline{A}$.
e. Verify that the equation $P(\overline{A}) = 1 - P(A)$ holds for the probabilities you found in (d).

38. A card is drawn from a standard deck. Consider the sample space in Figure 10.4. Let A be the event the card is a diamond, let B be the event the card is a club, and let C be the event the card is a jack, queen, or king.

a. Are events A and B mutually exclusive? Explain.

b. Are events A and C mutually exclusive? Explain.

c. Describe in words the complement of event A.

d. Find the probability of the event A and the event $\overline{A}$.

e. Verify that the equation $P(\overline{A}) = 1 - P(A)$ holds for the probabilities you found in (d).

39. Consider the experiment of randomly placing one car (C) and two goats (G) behind three curtains so that one object is behind each curtain. The results are recorded in order.

a. List all possible outcomes in the sample space.

b. Let E be the event the car is hidden behind curtain number 1. List the outcome(s) of the sample space that correspond to event E.

c. Describe $\overline{E}$ and list the outcome(s) of the sample space that correspond to $\overline{E}$.

d. Find $P(E)$ and $P(\overline{E})$.

40. Consider the experiment of randomly placing one silver dollar (D) and three rocks (R) inside four drawers so that one object is in each drawer. The results are recorded in order.

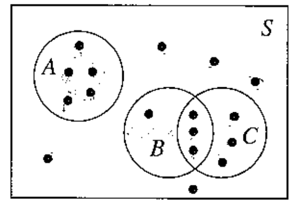
a. List all possible outcomes in the sample space.

b. Let E be the event the silver dollar is hidden in the first drawer. List the outcome(s) of the sample space that correspond to event E.

c. Describe $\overline{E}$ and list the outcome(s) of the sample space that correspond to $\overline{E}$.

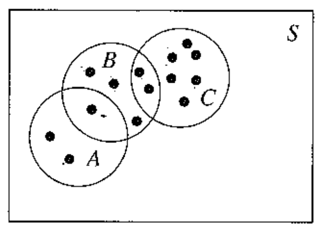
d. Find $P(E)$ and $P(\overline{E})$.

41. Consider the sample space S, as shown, for an experiment with equally likely outcomes. Events A, B, and C are indicated. Outcomes are represented by points. Find the probability of each of the following events.



a. A	**b.** B	**c.** C	**d.** S
e. $A \cup B$	**f.** $B \cap C$	**g.** $A \cap C$	**h.** $\overline{A}$
i. $\overline{B}$	**j.** $\overline{C}$		

42. Consider the sample space S, as shown, for an experiment with equally likely outcomes. Events A, B, and C are indicated. Outcomes are represented by points. Find the probability of each of the following events.



a. A	**b.** B	**c.** C	**d.** S
e. A ∪ B	**f.** B ∩ C	**g.** A ∩ C	**h.** $\overline{A}$
i. $\overline{B}$	**j.** $\overline{C}$		

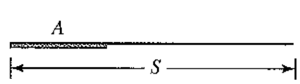
Extended Problems

Problems 43 and 44

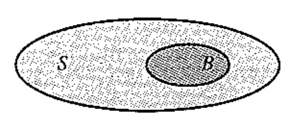
When the probability of an event is proportional to a measurement such as length or area, the probability is determined as follows. Let A be an event that can be measured as a length or as an area. Let $m(A)$ and $m(S)$ represent the measure of the event A and of the sample space S, respectively. Then

$$P(A) = \frac{m(A)}{m(S)}.$$

For example, in the following figure marked (a), if the length of S is 12 cm and the length of A is 4 cm, then $P(A) = \frac{4}{12} = \frac{1}{3}$. Similarly, in the figure marked (b), if the area of region B is 10 square centimeters and the area of the region S is 60 square centimeters, then $P(B) = \frac{10}{60} = \frac{1}{6}$.



(a)



(b)

43. A bus travels between Albany and Binghamton, a distance of 100 miles. Suppose the bus breaks down. Answer the questions below to find the probability $P(A)$ that the bus breaks down within 10 miles of either city.

   a. The road from Albany to Binghamton is the sample space. What is $m(S)$?

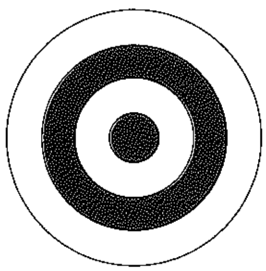
   b. Event A is that part of the road within 10 miles of either city. What is $m(A)$?

   c. Find $P(B)$.

44. The following dartboard is made up of circles with radii of 1, 2, 3, and 4 inches. Suppose a dart hits the board randomly.

   a. Find the probability the dart hits the bull's eye. (*Hint:* The area of a circle with radius r is πr^2.)

   b. Find the probability the dart hits the outer ring.



45. Search parties have the difficult job of deciding exactly where to search for a hiker lost in the woods. Many factors must be taken into account, such as the terrain and the hiker's age, fitness, and personality. The selection of specific areas to search is based on assigning a probability to each area. A newer method of assigning probabilities is called **Trail-Based Probability of Areas**. To use this method, the search party has to assume that the hiker was following an established trail from a known point of origin. Research the Trail-Based Probability of Areas assignment procedure by searching keywords "search and rescue trail-based probability of areas" on the Internet or go to www.sarinfo.bc.ca/Trailpoa.htm for more information. How are probabilities assigned to areas? How is the procedure different for a single-trail search or a multiple-trail search? Summarize your findings in a report.

46. The terms *10-year flood*, *50-year flood*, *100-year flood*, and *500-year flood* describe the estimated probability of a flood happening in any given year. A 10-year flood is defined as a flood that has a 1 in 10 chance, or a 10% probability, of occurring in any given year. A 50-year flood is defined as a flood that has a 1 in 50 chance, or a 2% probability, of occurring in any given year. How are these probabilities determined? What is the probability that a 100-year and a 500-year flood will both occur in any given year? Floods are classified in this way primarily to determine flood insurance rates in areas where floods can occur. For a home insured against floods, what is the probability a 100-year flood will occur during a 30-year mortgage payoff period? The Missouri River has had six 100-year floods since 1945. Which other rivers in the United States have experienced frequent 100-year or 500-year floods? Research floods by using search keywords "100-year flood" on the Internet or go to http://water.usgs.gov/pubs/FS/FS-229-96/ for more information. Write a report of your findings.

47. A classic problem in geometric probability is the **Buffon Needle Problem**. If a needle of a certain length, 2 inches for example, is dropped at random on a floor made of planks wider than the needle, what is the probability the needle will fall across a crack between two planks? To simulate the experiment without a planked floor, use a large piece of paper and draw parallel lines 3 inches apart across the whole paper.

   a. Drop a needle onto the "floor" from a consistent height (about 5 feet). Repeat the experiment 60 times, recording whether the needle falls across a crack. Compute the experimental probability for the event.

   b. Repeat part (a) with a longer needle (but a needle that is also shorter than the distance between the "planks").

   c. Divide the length of the longer needle by the length of the shorter one (measure the lengths carefully). Next, divide the probability you found in part (b) by the probability from part (a). How do the two compare? Is a relationship or generalization suggested?

48. In our examples and problems, we have generally assumed that coins and dice were "fair"; that is, we assumed each face was equally likely to appear on top. Conduct the following experiment with a die that is possibly not fair.

   a. Find a wooden cube from a set of children's blocks or a hardware store, and number its faces 1 through 6.

   b. Roll this "die" 100 times, record the results, and calculate the experimental probability of each number on your die.

   c. Hollow out the center of the face marked with a "6"; this can be done with a drill. Make the hollowed region about $\frac{1}{2}$ inch deep and over about half the face. Be sure you do not cut an edge.

d. Roll the hollowed die 100 times, record the results, and compute the experimental probability of each number on your die.

e. Compare your results from part (d) with those from part (b). To what could you attribute any differences?

49. If you drop a thumbtack, it will land either with the point up or with the point down. Are these events equally likely? Conduct an experiment to find out. Drop a handful of thumbtacks 20 times and record the number of tacks that land with the point up and the number that land with the point down. Calculate the experimental probability that a tack will land with the point up and the experimental probability that the tack will land with the point down. Do the events appear to be equally likely? Conduct the experiment again. Do your results of the second experiment differ significantly from the results of the first experiment? Based on your experimental results, how many tacks would land with the point down if you dropped 1000 tacks? Write a short essay that explains your experiment, and summarize your results in a table.

50. At the beginning of this chapter, the three-door problem was defined as follows. Suppose that a contestant on a game show is given a choice of three doors. Behind one door is a car, and behind each of the other two doors are goats. The contestant randomly picks door number 1, but does not open it, and the host, who knows what is behind the doors, opens door number 3 to reveal a goat. The host then says to the contestant, "Do you want to keep the door you selected or switch to door number 2?"

Conduct a simulation to determine if it is to the contestant's advantage to switch from the original door choice to the other door. For this simulation, three people are needed: a contestant, a host, and a data recorder. Gather the following materials: three paper cups (labeled 1, 2, and 3), one penny, and one standard die. The contestant must not look while the host rolls the die until a 1, 2, or 3 is rolled. The penny will be placed under the corresponding cup. The contestant then rolls the die until a 1, 2, or 3 is rolled and selects the corresponding cup but does not look under it. The host deliberately lifts a cup with no penny underneath from the two cups not selected by the contestant.

a. After the host lifts a cup with no penny underneath, suppose the contestant does not want to switch to the other cup and wants to stay with the original selection. Simulate this "stay" strategy 50 times. Instead of asking the contestant whether he or she wants to switch, the contestant lifts the cup originally selected. The contestant wins if the penny is under the cup and loses if there is no penny. The data recorder should use a table to keep track of the number of wins and losses.

b. After the host lifts a cup with no penny underneath, suppose the contestant does want to switch to the other cup. Simulate this "switch" strategy 50 times. Instead of asking the contestant whether he or she wants to switch, the contestant lifts the other cup that was not selected. The contestant wins if the penny is under the cup and loses if there is no penny. The data recorder should use a table to keep track of the number of wins and losses.

c. Calculate $P(\text{win})$ for the situation when the contestant "stays." Calculate $P(\text{win})$ for the situation in which the contestant "switches."

d. Consider the experimental probabilities calculated in part (c), and explain whether it is to the contestant's advantage to switch his or her choice.

51. The four human blood types are O, A, B, and AB. For each blood type, there are two Rh factors: positive and negative.

a. What percent of the human population falls into each of the eight categories? Search the Internet using keywords "blood types" or go to the American Red Cross's website at www.redcross.org/home/ for more information. Make a table listing each blood type and Rh factor combination and the percentage of the population that falls into each category.

b. What is the probability that one person, selected at random, will have A-positive blood? Answer the same question for each of the other seven classifications of blood.

c. In a movie theater containing 450 people, approximately how many people will have each blood type?

d. Survey at least 50 people and keep track of how many people fall into each category. Calculate the percentage of people in each blood category. Do your survey results match the Red Cross's percentages?

e. Research the history of blood type categorization. When were the differences in blood types first recognized? People with blood type A can receive what types of blood? Answer the same question for blood types B, AB, and O. For a person with blood type A, what it the probability that he or she can receive blood from a randomly selected person? Answer the same question for persons with blood type B, AB, and O.