An "L" at the end of a branch indicates that outcome results in your losing the bet, and a "W" at the end of a branch indicates that you win with that outcome. When we add the probabilities at the ends of each branch labeled L, we see that the probability the game ends as a loss is $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$, while the probability is only $\frac{5}{16}$ that you will win. Because the chances of losing are more than twice the chance of winning, a payoff of two dollars against our one-dollar bet does not seem like a good deal. If you only want to play when the game is "fair," you should not take the bet.

PROBLEM SET 10.2

Problems 1 and 2

Draw one-stage tree diagrams to represent the possible outcomes of the given experiments.

1. **a.** Toss one dime and observe whether the coin lands heads or tails.
   b. Pull a dollar bill from your wallet, and note the last digit in the serial number.
2. **a.** Draw a marble from a bag containing red, green, black, and white marbles, and observe the color.
   b. Open a yearlong wall calendar and note the month.

Problems 3 and 4

Draw two-stage tree diagrams to represent the possible outcomes of the given experiments.

3. **a.** Toss a coin twice, and observe on each toss whether the coin lands heads or tails.
   b. Select paint colors for an historic home from navy, stone, peach or blue and then select trim colors from light gray, rosedust, or ivory.
4. **a.** Draw a marble from a box containing yellow and green marbles and observe the color. Then draw a marble from a box containing yellow, red, and blue marbles and observe the color.
   b. Build a computer and printer package, choosing a computer from Dell, Apple, or Hewlett Packard and a printer from Epson, Brother, or Hewlett Packard.
5. Different branches on a tree diagram need not have the same number of stages. For example, suppose that from a box containing one red, one white, and one blue ball, we will draw balls (without replacement) until the red ball is chosen. Draw the tree diagram to represent the possible outcomes of this experiment.
6. Tree diagrams need not be finite. For example, consider the experiment of tossing a coin until it lands heads. This usually takes only a few tosses (in fact, two on average), but it could take any number of tosses. Draw at least three stages of the tree diagram representing the possible outcomes of this experiment to show the pattern.
7. For your vacation, you will travel from your home to New York City, then to London. You may travel to New York City by car, train, bus, or plane, and from New York to London by ship or plane.
   a. Draw a tree diagram to represent all possible travel arrangements.
   b. How many different travel arrangements are possible?
   c. Apply the Fundamental Counting Principle to find the number of possible travel arrangements. Does your answer agree with your result in part (b)?
8. Suppose that a frozen yogurt dessert can be ordered in three sizes (small, medium, large), two flavors (vanilla, chocolate), and with any one of four topping options (plain, sprinkles, hot fudge, chocolate chips).
   a. Draw a tree diagram to represent all possible yogurt desserts.
   b. How many different desserts are possible?
   c. Apply the Fundamental Counting Principle to find the number of possible desserts. Does your answer agree with your result in part (b)?
9. An experiment consists of tossing a coin and then rolling two dice. How many outcomes are possible for the following? Use the Fundamental Counting Principle.
   a. tossing the coin
   b. rolling the first die
   c. rolling the second die
   d. conducting the experiment

10. An experiment consists of selecting one card from a standard deck, flipping a coin, and spinning a three-color spinner. How many outcomes are possible for the following? Use the Fundamental Counting Principle.

a. selecting the card

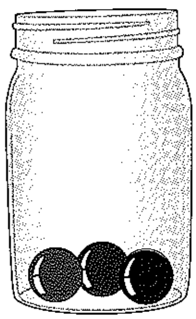
b. tossing the coin

c. spinning the spinner

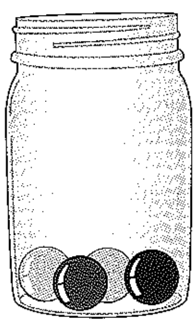
d. conducting the experiment

11. One marble is selected from each of the following containers.

a. Draw a one-stage probability tree diagram and find the probability of drawing the blue marble.

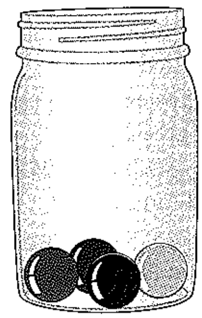


b. Draw a one-stage probability tree diagram and find the probability of drawing the blue marble. Find the probability of drawing a yellow marble. Find the probability of drawing the blue marble or a yellow marble.

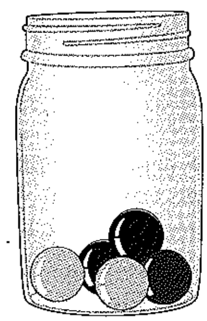


12. One marble is selected from each of the following containers.

a. Draw a one-stage probability tree diagram and find the probability of drawing the blue marble.



b. Draw a one-stage probability tree diagram and find the probability of drawing a red marble. Find the probability of drawing a yellow marble. Find the probability of drawing a red marble or a yellow marble.



13. Refer to the container from problem 11(b). A marble is drawn and replaced, and then a second marble is drawn.

a. Draw a two-stage probability tree diagram to represent this experiment.

b. What is the probability that both marbles selected are yellow?

c. What is the probability that the second marble selected is blue?

d. What is the probability that both selected marbles are the same color?

e. Repeat the problem if a marble is drawn and *not* replaced, and then a second marble is drawn.

14. Refer to the container from problem 12(b). A marble is drawn and replaced, and then a second marble is drawn.

a. Draw a two-stage probability tree diagram to represent this experiment.

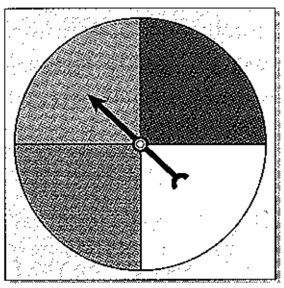
b. What is the probability that both marbles selected are red?

c. What is the probability that the second marble selected is green?

d. What is the probability that both selected marbles are the same color?

e. Repeat the problem if a marble is drawn and *not* replaced, and then a second marble is drawn.

15. The following spinner is spun twice.



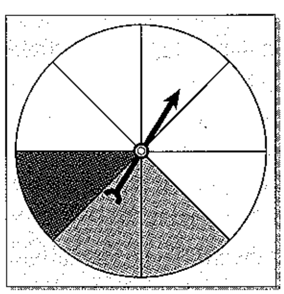
a. Draw a two-stage probability tree diagram to represent this experiment.

b. Find the probability the spinner lands on yellow both times.

c. Find the probability the spinner lands on red on the second spin.

d. Find the probability the spinner lands on blue and then green or lands on green and then blue.

16. The following spinner is spun twice.



a. Draw a two-stage probability tree diagram to represent this experiment.

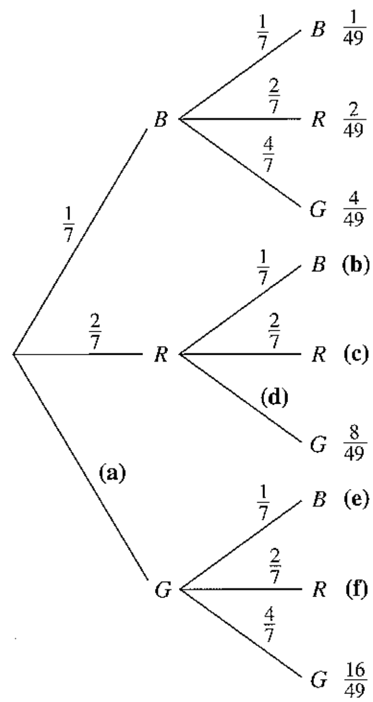
b. Find the probability the spinner lands on yellow both times.

c. Find the probability the spinner lands on red on the second spin.

d. Find the probability the spinner lands on blue and then yellow or lands on yellow and then red.

$P((B,R) \cup (Y,R))$ $\frac{1}{4}\ \frac{3}{8}$ $\frac{3}{8}\ \frac{1}{8}$ $\frac{3}{32} + \frac{3}{64} = \frac{9}{64}$

17. Consider the following two-stage probability tree diagram for the experiment of drawing two marbles from a box. The diagram is unfinished. Fill in the missing probabilities for parts (a) through (f) to complete the diagram.



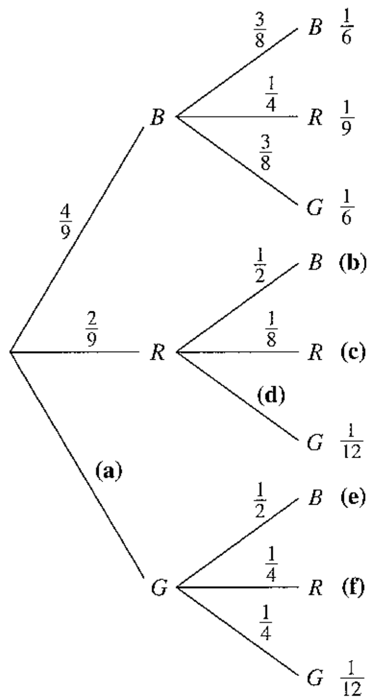
g. How many outcomes are in the sample space?

h. Were the marbles drawn with replacement or without replacement? Explain.

i. Can you tell how many marbles of each type were in the box? Explain.

j. Find the probability of getting two marbles that are the same color.

18. Consider the following two-stage probability tree diagram for the experiment of drawing two marbles from a box. The diagram is unfinished. Fill in the missing probabilities for parts (a) through (f) to complete the diagram.



g. How many outcomes are in the sample space?

h. Were the marbles drawn with replacement or without replacement? Explain.

i. Can you tell how many marbles of each type were in the box? Explain.

j. Find the probability of getting two marbles that are the same color.

19. A fair coin is flipped four times.

a. Use the Fundamental Counting Principle to find the number of possible outcomes.

b. Find the probability of getting four heads.

c. Find the probability of getting exactly two heads.

d. Find the probability of getting exactly three tails.

20. A bowl contains three marbles (red, blue, green). A box contains four numbered tickets (1, 2, 3, 4). One marble is selected at random, and then a ticket is selected at random.

a. Use the Fundamental Counting Principle to find the number of possible outcomes.

b. Find the probability that the green marble is selected.

c. Find the probability that the ticket numbered 2 is selected.

d. Find the probability that the red marble and the ticket numbered 3 are selected.

21. A box of 20 chocolates contains three different varieties: nut-filled, nougat, and caramel, but all the chocolates appear identical on the outside. Near the nutrition information, the package reads "This box contains 10% nut-filled, 30% caramel, and 60% nougats." Suppose you select two chocolates.

a. How many stages does this experiment have?

b. How many chocolates of each type are in the box?

c. Create the probability tree diagram for this experiment.

d. Find the probability of selecting two nut-filled chocolates.

e. Find the probability of selecting a caramel and a nut-filled chocolate.

f. Find the probability of selecting a nougat or a caramel.

22. While planning the landscaping for your front yard, you select tulip bulbs at a nursery. Your plan is to plant 10 red and 8 white tulips in one flower bed and to plant 2 purple tulips in a small pot near your door. On the way home from the nursery, the bulbs roll out of their bags in the trunk and are mixed up. You cannot predict the color of the tulips just by looking at the bulbs. Suppose you select 2 bulbs to plant in the pot.

a. How many stages does this experiment have?

b. Create the probability tree diagram for this experiment.

c. How many outcomes are there in the sample space?

d. Find the probability that both bulbs are purple.

e. Find the probability of selecting a red and a white bulb.

f. Find the probability of selecting a purple or a red bulb.

23. A pinochle deck contains 48 cards. The cards are arranged in the usual four suits. Each suit contains two of each of the following cards: 9, 10, jack, queen, king, and ace. Suppose two cards are drawn without replacement from a standard pinochle deck.

a. How many outcomes are possible for this experiment?

b. How many outcomes correspond to the event that both cards are face cards?

c. What is the probability that both cards are face cards?

d. What is the probability of getting a pair?

e. What is the probability of getting an identical pair of cards?

24. Consider the experiment of drawing two cards without replacement from a standard deck of 52 cards.

a. How many outcomes are possible for this experiment?

b. How many outcomes correspond to the event that the cards are both face cards?

c. What is the probability that both cards are face cards?

d. What is the probability of getting a pair?

e. Find the probability of drawing two cards that have the same suit.

25. A light bulb is selected from box 1 and another from box 2. In box 1, 30% of the bulbs are defective. In box 2, 45% of the bulbs are defective. Each of the selected bulbs is recorded as defective or not defective.

a. Draw a probability tree diagram for this experiment.

b. Find the probability that both bulbs are defective.

c. Find the probability that the first bulb is defective and the second is not defective.

26. While still half asleep, you randomly select a black sock from your drawer. After you remove that sock, the drawer contains two white socks and four more black socks. Without replacement you continue to randomly select one sock at a time from your drawer until another black sock is selected. Draw a probability tree diagram to represent this experiment. Find the probability of each of the following events.

a. Exactly one draw is needed to get another black sock.

b. Exactly two draws are needed to get another black sock.

c. Exactly three draws are needed to get another black sock.

27. A game at a carnival consists of throwing darts at balloons. Eight balloons are arranged in such a way that the player will always pop one of them. The popped balloon is replaced after each dart is thrown. Two stars are hidden behind the balloons. If the player pops a balloon that reveals a star, he wins a prize. A player pays 50¢ for three darts. Assuming that skill is not involved, find the probability that the player

a. wins a prize (gets a star) after just one shot.

b. wins in exactly two shots.

c. wins in exactly three shots.

d. does not win.

28. As a Back-to-School promotion, a cereal manufacturer distributes 350,000 boxes of cereal that contain prizes. Fifty boxes contain a certificate for a free desktop computer. Fifty thousand boxes contain a certificate for a dictionary. The rest of the boxes contain a CD spelling program. Suppose you select two boxes.

a. Find the probability that you win two computers.

b. Find the probability that you win a computer and a CD spelling program.

c. Find the probability that you win at least one CD spelling program.

d. Find the probability that you win two dictionaries.

29. Each individual letter of the word MISSISSIPPI is placed on a piece of paper, and all 11 pieces of paper are placed in a bowl. Two letters are selected at random from the bowl without replacement. Find the probability of

a. selecting the two Ps.

b. selecting the same letter in both selections.

c. selecting two consonants.

30. Four families get together for a barbecue. Every member of each family puts his or her name in a hat for a prize drawing. The Martell family has three members, the Werner family four, the Borschowa family four, and the Griffith family six. Two names are drawn from the hat. Find the probability of

a. selecting two members of the Borschowa family.

b. selecting two members from the same family.

c. selecting two members from different families.

31. A pair of dice is constructed so that each die is marked with a 1 on one side, a 2 on two sides, and a 3 on three sides. The dice are rolled. Find the probability that

a. two 3s are rolled.

b. the same number appears on each die.

c. two odd numbers are rolled.

32. A pair of dice is constructed as in problem 31. The two dice are rolled. What is the probability that

a. neither die shows a 2?

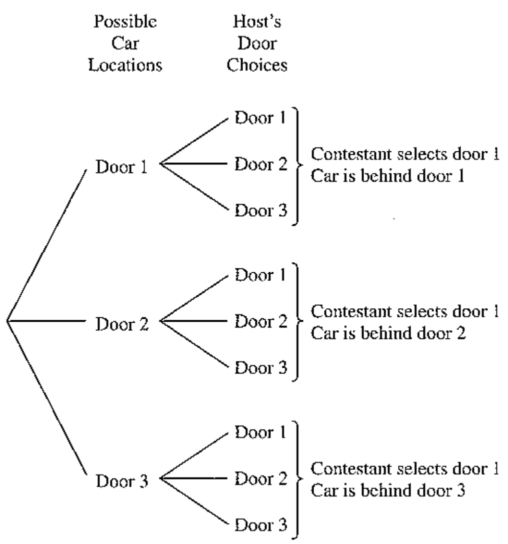
b. the numbers showing on the two dice are different?

c. one die shows an odd number and the other shows an even number?

Extended Problems

33. Consider the three-door problem, which was introduced at the beginning of this chapter. Suppose a contestant on a game show is given the choice of three doors. Behind one door is a car, and behind each of the other two doors are goats. Assume the car and the goats have been randomly placed. The contestant picks door 1, and the host, who knows what is behind the doors, opens another door to reveal a goat. We assume that the host will always reveal a goat, and that when the host has a choice of revealing the goat behind either of the two doors the contestant did not choose, the host will make the choice randomly.

a. In the following, the first stage represents the possible car locations. The car could be behind door 1, door 2, or door 3. The second stage represents the host's possible selections. Throughout, we assume that the contestant selects door 1. Assign a probability to each branch of the following tree diagram.



b. Find the probability that the car is behind door 1.

c. Find the probability that the car is behind door 2 and the host opens door 3.

d. Find the probability that the car is behind door 3 and the host opens door 3.

e. Find the probability that the host opens door 2.

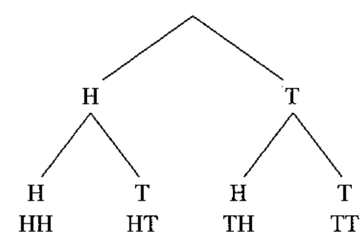
f. Find the probability that the host opens door 2 or 3.

34. The study of probability theory is generally considered to have begun around 1654 with the correspondence between the mathematicians Blaise Pascal and Pierre de Fermat involving several problems concerning dice games. Write a brief report on the history of probability and the major mathematicians who contributed to its development.

35. One of the most important and interesting counting devices in algebra and probability is known as **Pascal's triangle**. Several rows in the triangle are shown next.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

The top row, containing only the number 1, is generally referred to as the "0" row, so that the entries in the "first" row are 1 and 1. Entries in the "second" row are 1, 2, and 1. One interpretation of Pascal's Triangle relates to the results of **binomial experiments**. A binomial experiment is one that has two possible outcomes, such as tossing a coin and observing either a head or a tail. Now suppose you toss a coin two times and count the number of heads. The tree diagram for this experiment is shown below.



Notice that you can get two heads in one way, exactly one head in two ways, and no heads in one way. The numbers of ways of obtaining two heads, one head, or no heads are given in the "second" row of Pascal's triangle.

1	2	1
two heads	one head	no heads
(one way)	(two ways)	(one way)

a. Draw the tree diagram for the experiment of tossing three coins. Show that the numbers of ways of obtaining three, two, one, or no heads are given by the entries of the third row of Pascal's triangle.

b. Draw the tree diagram for the experiment of tossing four coins. Find the numbers of ways of obtaining four, three, two, one, or no heads. In which row of Pascal's triangle are these values found?

c. Study the entries in Pascal's triangle. There is a way to generate each row of numbers in the triangle by using the entries in the row above it. Determine the relationship and give the entries in the fifth, sixth, and seventh rows of Pascal's Triangle without constructing a tree diagram.