bet on the layout and adds to it 35 times as much as you bet. If you chose a number other than the winning number, your wager and the other losing bets are gathered in with a rake. If you bet \$100 on one number, what is your expected gain or loss?

SOLUTION This problem describes a probability experiment with numerical outcomes. The probability of winning is $\frac{1}{38}$, since there are 38 equally likely outcomes, namely, 00, 0, 1, 2, 3, . . ., 36. If you win, you win \$3500. The probability of losing is $\frac{37}{38}$. If you lose, you lose \$100 so we will assign this outcome a value of $-\$100$. We put this information into a table as shown in Table 10.10 and compute the expected value by multiplying the assigned values and their probabilities.

Table 10.10

Value	-100		3500		
Probability	$\frac{37}{38}$		$\frac{1}{38}$		
Products	$\frac{-3700}{38}$	$+$	$\frac{3500}{38}$	$=$	$\frac{-200}{38}$
					Expected value

Since the expected value is $\frac{-200}{38} \approx -\$5.26$, for every \$100 bet, you should expect to lose \$5.26 even though you lose \$100 on some bets and win \$3500 on others.

PROBLEM SET 10.3

1. An experiment consists of rolling two standard dice and noting the numbers showing on the two dice.

a. How many outcomes are in the sample space?

b. Let M be the event that the first die shows a multiple of 3. List the outcomes in M.

c. Let O be the event that the second die shows an odd number. List the outcomes in O.

d. List the outcomes in $M \cap O$.

e. Find $P(M)$, $P(O)$, and $P(M \cap O)$.

f. Find and interpret $P(M|O)$ and $P(O|M)$.

2. An experiment consists of drawing two cards, with replacement, from a pile of cards containing only the five cards 9, 10, J, Q, and K of diamonds.

a. How many outcomes are in the sample space?

b. Let F be the event that the first card is a face card. List the outcomes in F.

c. Let S be the event that the second card is a 10 or J. List the outcomes in S.

d. List the outcomes in $F \cap S$.

e. Find $P(F)$, $P(S)$, and $P(F \cap S)$.

f. Find and interpret $P(F|S)$ and $P(S|F)$.

3. A jar contains five white balls and three green balls. Two balls are drawn, in order, without replacement. Find the following:

a. the probability the second ball is green.

b. the probability the first ball is white.

c. the probability the first ball is white and the second ball is green.

d. the probability the second ball is green given the first ball is white.

e. the probability the first ball is white given the second ball is green.

f. the probability the first ball is green given the second ball is green.

4. Two standard dice are rolled. Find the following:

a. the probability that at least one of the dice is a five.

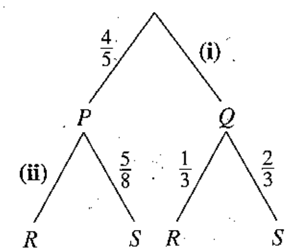
b. the probability the sum is eight.

c. the probability that at least one of the dice is a five and the sum is eight.

d. the probability that one of the dice is a five given the sum is eight.

e. the probability that the sum is eight given that at least one of the dice is a five.

5. Consider the following incomplete probability tree diagram.



a. Find (i) and (ii), the missing probabilities, in the tree diagram.

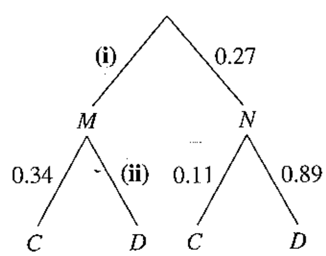
b. Find $P(R \cap P)$, $P(S \cap P)$, and $P(P)$.

c. Find $P(R \cap Q)$, $P(S \cap Q)$, and $P(R)$.

d. Find and interpret $P(S|P)$.

e. Find and interpret $P(Q|R)$.

6. Consider the following incomplete probability tree diagram.



a. Find (i) and (ii), the missing probabilities, in the tree diagram.

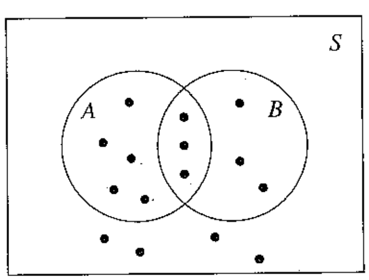
b. Find $P(C \cap M)$, $P(D \cap M)$, and $P(M)$.

c. Find $P(C \cap N)$, $P(D \cap N)$, and $P(D)$.

d. Find and interpret $P(C|M)$.

e. Find and interpret $P(N|D)$.

7. The diagram shows a sample space S of equally-likely outcomes and events A and B. Each outcome is represented by a dot in the figure.

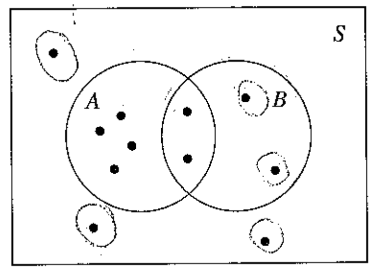


Find the following probabilities.

a. $P(A)$ **b.** $P(B)$ **c.** $P(A \cap B)$

d. $P(A|B)$ **e.** $P(B|A)$

8. The diagram shows a sample space S of equally likely outcomes and events A and B. Each outcome is represented by a dot in the figure.



Find the following probabilities.

a. $P(\overline{A})$

b. $P(B)$

c. $P(\overline{A} \cap B)$

d. $P(\overline{A}|B)$

e. $P(B|\overline{A})$

9. If L is the event that a person is left-handed, and M is the event that a person is male, then state *in words* what probabilities are expressed by each of the following:

a. $P(L|M)$

b. $P(M|L)$

c. $P(L \cap M)$

d. $P(\overline{L} \cap \overline{M})$

e. $P(\overline{L}|M)$

f. $P(\overline{L}|\overline{M})$

10. A student's name is chosen at random from a college registration list and the student is interviewed. If H is the event that the student completes his or her homework each night, and G is the event that the student gets good grades, then state *in words* what probabilities are expressed by each of the following:

a. $P(H \cap G)$

b. $P(H \cap \overline{G})$

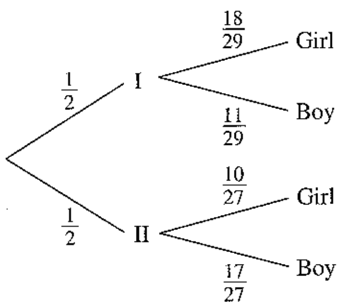
c. $P(G|H)$

d. $P(\overline{G}|H)$

e. $P(G|\overline{H})$

f. $P(\overline{G}|\overline{H})$

11. There are two fifth-grade classes in a school. Class I has 18 girls and 11 boys while class II has 10 girls and 17 boys. The principal will randomly select one fifth-grade class and one student from that class to represent the fifth grade on the student council. A probability tree diagram for this experiment is given next.



Find each of the following:

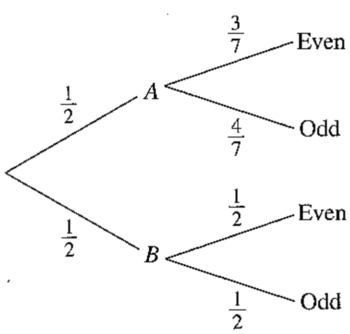
a. P(a girl is selected)

b. P(a boy is selected)

c. P(a girl is selected given that the student came from class II)

d. P(a boy is selected given that the student came from class I)

12. Box A contains 7 cards numbered 1 through 7, and box B contains 4 cards numbered 1 through 4. A box is chosen at random and a card is drawn. It is then noted whether the number on the card is even or odd. A probability tree diagram for this experiment is given next.



Find the following probabilities:

a. P(the number is even)

b. P(the number is odd)

c. P(the number is even given that it came from box A)

d. P(the number is odd given that it came from box B)

13. A triple test is a blood test, which can be offered to a woman who is in her 15th to 22nd week of pregnancy, to screen for such fetal abnormalities as Down syndrome. A positive triple test would indicate that the fetus might have a birth defect. The table below contains triple test results from a large sample of women. Assume that the results in the table are representative of the population as a whole.

Triple Test Results	Fetus with Down Syndrome	Fetus without Down Syndrome
Positive	5	208
Negative	1	2386

a. Given a positive test result, what is the probability that the fetus has Down syndrome?

b. Given a positive test result, what is the probability that the fetus does not have Down Syndrome?

c. Given that a fetus has Down syndrome, what is the probability that the test is negative?

14. In a fuse factory, machines A, B, and C manufacture 20%, 45%, and 35%, respectively, of the total fuses. Of the outputs for each machine, A produces 5% defectives, B produces 2% defectives, and C produces 3% defectives. A fuse is drawn at random.

a. Given that a fuse is defective, what is the probability it came from machine A?

b. Given that a fuse is defective, what is the probability it came from machine B?

c. Given that a fuse is defective, what is the probability it came from machine C?

15. A random sample of 400 adults are classified according to sex and highest education level completed as shown next.

Education	Female	Male
Elementary school	64	53
High school	99	87
College	28	39
Graduate school	14	16

If a person is picked at random from this group, find the probability that

a. the person is female given that the person has a graduate degree.

b. the person is male given that the person has a high school diploma as his or her highest level of education completed.

c. the person does not have a college degree given that the person is female.

16. A company would like to find out how the number of defective items produced varies between the day, evening, and night shifts. The following table shows the results of a sample of items taken from each shift.

	Day	Evening	Night
Defective	24	28	47
Nondefective	279	224	165

If an item is picked at random, find the probability that

a. the item is defective, given that it came from the night shift.

b. the item is not defective, given that it came from the day shift.

c. the item was produced by the night shift, given that it was not defective.

17. Suppose a screening test for a certain virus is 95% accurate for both infected and uninfected persons. If 10% of the population is infected and one person is selected at random, find the following:

a. the probability that the test result is positive.

b. the probability of a false positive test result.

c. the probability that a person is infected, given that the test result is positive.

d. the probability that the result is positive, given that the person is infected.

18. Suppose a screening test for a certain virus is 90% accurate for both infected and uninfected persons. If 2% of the population is infected, find the following:

a. the probability that the test result is negative.

b. the probability of a false negative test result.

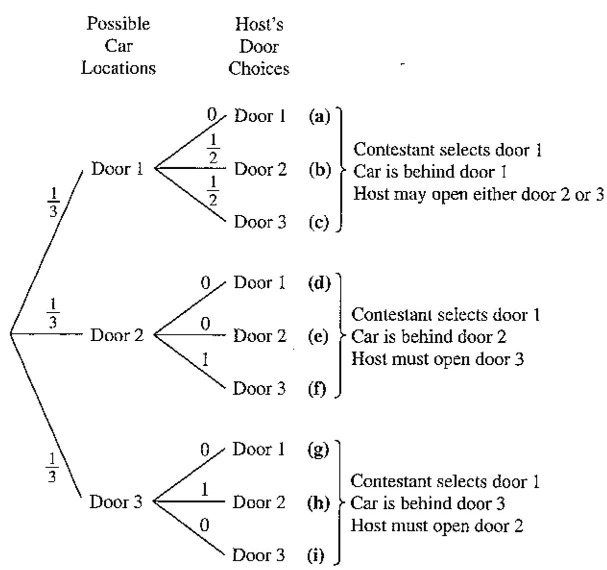
c. the probability that the person is uninfected, given that the test result is negative.

d. the probability that the test result is negative, given that the person is uninfected.

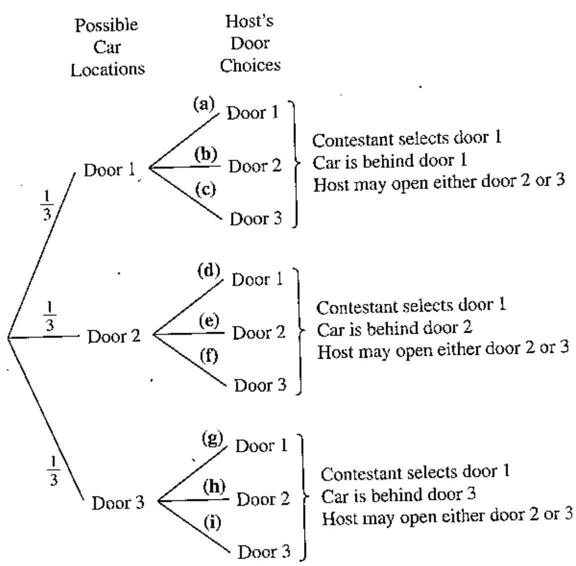
Problems 19 through 24

These problems relate to the three-door problem introduced at the start of this chapter.

19. Suppose a contestant on a game show is given the choice of three doors. Behind one door is a car, and behind each of the other two doors are goats. Assume the car and the goats have been randomly placed. The contestant picks door 1, and the host, who knows what is behind the doors, opens another door to reveal a goat. We assume that the host will always reveal a goat, and that when the host has a choice of revealing the goat behind either of the two doors the contestant did not choose, the host will make the choice randomly. The problem can be represented by the probability tree diagram below. Fill in the missing probabilities.



20. In problem 19, the assumption has been made that the host will always open a door to reveal a goat. Suppose we allow the case in which, after the contestant selects a door, the host randomly selects a different door. In this case, the host might open a door to reveal the car. Fill in the missing probabilities in the following tree diagram.



21. The contestant selects door 1 in problem 19. Suppose the host opens door 2.

a. Find $P(\text{Car is behind door 1} \mid \text{Host opens door 2})$.

b. Find $P(\text{Car is behind door 3} \mid \text{Host opens door 2})$.

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

22. Consider the tree diagram from problem 20. The contestant selects door 1. Suppose the host opens door 2.

a. Find $P(\text{Car is behind door 1} \mid \text{Host opens door 2})$.

b. Find $P(\text{Car is behind door 3} \mid \text{Host opens door 2})$.

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

23. The contestant selects door 1 in problem 19. Suppose the host opens door 3.

a. Find $P(\text{Car is behind door 1} \mid \text{Host opens door 3})$.

b. Find $P(\text{Car is behind door 2} \mid \text{Host opens door 3})$.

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

24. Consider the tree diagram from problem 20. The contestant selects door 1. Suppose the host opens door 3.

a. Find $P(\text{Car is behind door 1} \mid \text{Host opens door 3})$.

b. Find $P(\text{Car is behind door 2} \mid \text{Host opens door 3})$.

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

25. Two classes at a university are studying modern Latin-American fiction. Twenty of the 25 students in the first class speak Spanish, and 12 of the 18 students in the second class speak Spanish. If a student is selected at random from each of the 2 classes, what is the probability that both students speak Spanish? Show how you calculate your answer. What probability property did you use? Explain.

26. Two assembly lines are producing ink cartridges for a desktop printer. Five percent of the cartridges produced by the first assembly line are defective, while 10% of those produced from the second assembly line are defective. If a cartridge is selected randomly from each line, what is the probability that neither cartridge will be defective? Show how you calculate your answer. What probability property did you use? Explain.

27. Suppose you set your compact disc player to randomly play the 11 tracks on a CD. Tracks 1, 4, and 5 are your favorites. You listen to two songs.

a. Find the probability that the second song is one of your favorites.

b. Find the probability that the second song is one of your favorites, given that the first song was one of your favorites.

c. Are these events independent? Explain your reasoning.

28. Suppose the random-track-selection feature on your CD player is malfunctioning so that once a track is played, the same track is twice as likely to be selected next. After setting your compact disc player to randomly play the five tracks on a CD, you listen to two songs.

a. Find the probability track 3 plays first.

b. Find the probability track 3 plays second, given that track 3 played first.

c. Are these events independent? Explain your reasoning.

29. Suppose a standard die is rolled twice. Let A be the event a 3 occurs on the first roll, let B be the event that the sum of the two rolls is 7, and let C be the event that the same number is rolled both times.

a. Find $P(A)$, $P(B)$, $P(A \cap B)$, and determine if events A and B are independent.

b. Find $P(A)$, $P(C)$, $P(A \cap C)$, and determine if events A and C are independent.

c. Find $P(B)$, $P(C)$, $P(B \cap C)$, and determine if events B and C are independent.

30. In a box of computer chips, two are defective and five are not defective. Two chips are selected, without replacement. Let A be the event that a defective chip is chosen first, let B be the event that a nondefective chip is chosen second, and let C be the event that both chips are defective. Use the definition of independent events to

a. Find $P(A)$, $P(B)$, $P(A \cap B)$, and determine if events A and B are independent.

b. Find $P(A)$, $P(C)$, $P(A \cap C)$, and determine if events A and C are independent.

c. Find $P(B)$, $P(C)$, $P(B \cap C)$, and determine if events B and C are independent.

31. Complete the table below, in which the values in the first column shows the number of girls possible in a family with four children, and the second shows the probabilities for those outcomes. Assume that boys and girls are equally likely and the births are independent. Then find the expected number of girls in a family of four.

Number of Girls	Probability
0	
1	
2	
3	
4	

32. Suppose you play a game in which two fair standard dice are rolled. If the numbers showing on the dice are different, you lose \$2. If the numbers showing are the same, you win \$2 plus the dollar value of the sum of the dice. Complete the next table, in which the values in the first column are the outcomes for the rolls of the two dice. The second column has the probabilities for those outcomes, and the third column has the payoff values for each outcome. What is the expected value of the game?

Numbers Showing	Probability	Payoff
Different		
1, 1		
2, 2		
3, 3		
4, 4		
5, 5		
6, 6		

33. In a lottery, there are 50 prizes of \$10, 10 prizes of \$15, 5 prizes of \$30, and 1 prize of \$50. Suppose that 1000 tickets are sold.

a. What is a fair price to pay for one ticket?

b. What should the price of one ticket be if, on the average, people lose \$0.50?

c. If all 1000 tickets are sold, what can the lottery expect to gain if a ticket costs \$2?

34. A church conducts a drawing to raise money for the building fund. One thousand tickets are placed in a box. On each ticket is placed one of the following dollar amounts: \$0, \$5, \$10, \$50, or \$200. The table below shows the numbers of each type of ticket that will be placed in the box. Each participant will pay \$3 to draw one ticket from the box.

Number of Tickets	Dollar Amount
1	200
4	50
10	10
20	5
965	0

a. Find the expected value of one ticket.

b. If all the tickets are sold, how much money will the church make for the building fund?

c. If you buy a single ticket, what is the probability that you will not win any money?

35. For visiting a resort (and listening to a sales presentation), you will receive one gift. The probability of receiving each gift and the manufacturer's suggested retail value are as follows:

gift A, 1 in 52,000 (\$9272.00)
gift B, 25,736 in 52,000 (\$44.95)
gift C, 1 in 52,000 (\$2500.00)
gift D, 3 in 52,000 (\$729.95)
gift E, 25,736 in 52,000 (\$26.99)
gift F, 3 in 52,000 (\$1000.00)
gift G, 180 in 52,000 (\$44.99)
gift H, 180 in 52,000 (\$63.98)
gift I, 160 in 52,000 (\$25.00)

Find the expected value of your gift. Round to the nearest cent.

36. According to a publisher's records, 20% of the children's books published break even, 30% lose \$1000, 25% lose \$10,000, and 25% earn \$20,000. When a book is published, what is the expected income for the book?

37. Suppose you and a friend play a game. Two standard dice are rolled and the numbers showing on each die are multiplied. If the product is even, your friend gives you a quarter, but if the product is odd, you must give your friend one dollar.

a. What is the expected value of the game for you? Round to the nearest cent.

b. What is the expected value of the game for your friend? Round to the nearest cent.

c. How could you change the amount you pay your friend so that the expected value of the game for you is \$0.05?

38. At a carnival, you play a dice game in which you roll two standard dice. If you roll a total of 7, then you win \$1. If you roll double 6s, you lose \$5. If you roll any other combination, you win \$0.25.

a. What is the expected value of the game?

b. If the carnival wants to make sure that the player loses \$0.10 on average, how should the payoff for rolling a total of 7 be adjusted?

39. Suppose that the probability of an event is $\frac{1}{5}$.

a. What are the odds in favor of the event?

b. What are the odds against the event?

40. Suppose the probability of an event is $\frac{7}{19}$.

a. What are the odds in favor of the event?

b. What are the odds against the event?

41. If the odds against an event are 2 to 1, what is the probability of the event?

42. If the odds in favor of an event are 13:11, what is the probability of the complement of the event?

43. Suppose three coins are tossed.

a. What are the odds in favor of getting all heads?

b. What are the odds against getting only one head?

c. If event T is defined as getting exactly two tails, then what are the odds for $\overline{T}$?

44. Suppose two standard dice are rolled.

a. What are the odds in favor of getting a sum of 6?

b. What are the odds against getting a 3 on the second die?

c. If event L is defined as getting a total of at least 9, what are the odds in favor of the complement of L?

Extended Problems

45. In an effort to fight an apparent growth in the use of illegal drugs, many companies, professional sports teams, and schools have established drug-testing programs. Research the most common forms of drug testing: urinalysis, blood testing, saliva testing, and hair testing. For each test, what is the probability of getting a false positive result or a false negative result?

46. Two major trials in the latter half of the 1990s focused the attention of the nation on DNA testing: the 1995 murder trial of O. J. Simpson and the 1998–1999 impeachment trial of President Bill Clinton. Research DNA testing by searching keywords "DNA testing" on the Internet. How accurate are these tests in general, and what circumstances could lead to a false match? Describe what DNA is, and list some of the purposes for which DNA testing has been used, along with the probabilities involved. Write a report summarizing your findings.