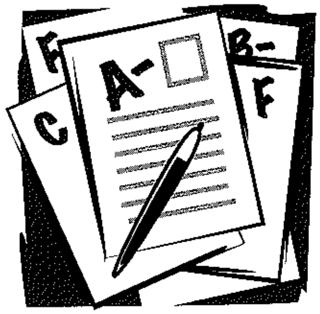
SOLUTION OF THE INITIAL PROBLEM



After returning exams to her large class of 90 students, Professor LaStat reports that the mean score on the test was 74 and the standard deviation was 8. At the students' request, she agrees to "curve" the test scores. She says that curving the scores will mean that all students whose scores are at least 1.5 standard deviations above the class mean will receive A's. Similarly, all students whose scores are at least 1.5 standard deviations below the class mean will receive F's. If Professor LaStat curves the grades in this manner, about how many students in the class will get A's? About how many will get F's?

SOLUTION Because the class is large, it is likely that the test scores have a normal distribution. (Large data sets are often normally distributed.) The percentage of scores in any given range of scores may be determined by comparing this normal distribution to the standard normal distribution. If the scores are "curved" as described, then the mean score of 74 will correspond to a score of 0 in the standard normal distribution. A score that is 1.5 deviations above the mean is $74 + 1.5 \times 8 = 86$ and corresponds to a score of 1.5 in the standard normal distribution.

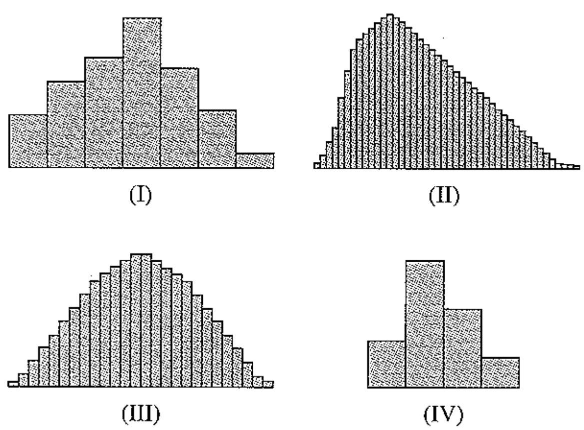
To determine the percentage of students who will receive A's, we can look at the standard normal distribution rather than the actual normal distribution of test scores. To find the percentage of A's, we must determine the area under the standard normal curve to the right of $z = 1.5$. From Table 11.3, we see that approximately 43.32% of the scores will fall between 0 and 1.5. By the symmetry of the bell-shaped curve, 50% of the scores in the standard normal distribution are greater than 0 and 50% are less than 0. Thus, approximately $50\% - 43.32\% = 6.68\%$ of the scores will be greater than 1.5, which means that approximately $0.0668 \times 90 \approx 6$ students will get A's.

By symmetry, the number of students who will receive F's will also be approximately $0.0668 \times 90 \approx 6$.

PROBLEM SET 11.1

Problems 1 and 2

Refer to the following histograms.

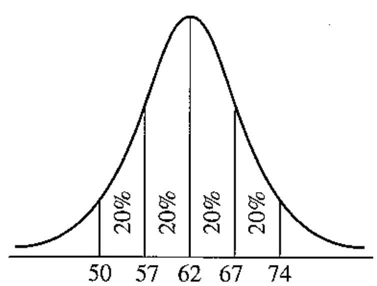


1. Which of histograms (I)–(IV) could best be approximated by the region under a smooth curve? Explain.

2. Which of histograms (I)–(IV) could best be approximated by the region under a smooth normal distribution curve? Explain.

Problems 3 through 6

Suppose the following figure represents the distribution of the weights (in pounds) of a certain large breed of dog.



3. What percentage of the dogs in this population weigh between 67 and 74 pounds?

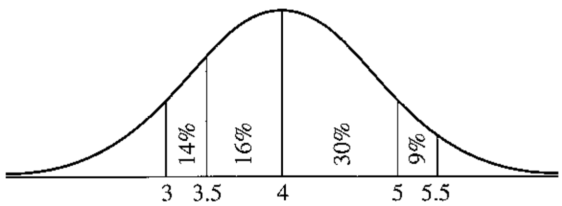
4. What percentage of the dogs in this population weigh between 57 and 62 pounds?

5. What percentage of the dogs in this population weigh between 50 and 67 pounds?

6. What percentage of the dogs in this population weigh between 50 and 74 pounds?

Problems 7 through 10

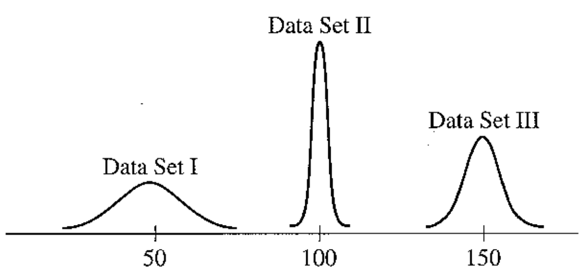
Suppose the following figure represents the distribution of body lengths (in inches) from a large population of a certain species of hamster.



7. What percentage of the hamsters in this population are between 3.5 and 4 inches long?
8. What percentage of the hamsters in this population are between 5 and 5.5 inches long?
9. What percentage of the hamsters in this population are between 3 and 5.5 inches long?
10. What percentage of the hamsters in this population are between 3.5 and 5 inches long?

Problems 11 and 12

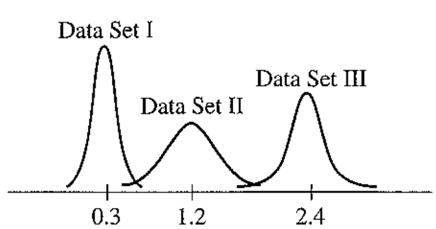
Refer to the following figure, where three normal distributions are sketched on the same x-axis.



11. Which data set has the largest mean? Which has the smallest mean? How can you tell?
12. Which data set has the largest standard deviation? Which has the smallest standard deviation? How can you tell?

Problems 13 and 14

Refer to the following figure, where three normal distributions are sketched on the same x-axis.

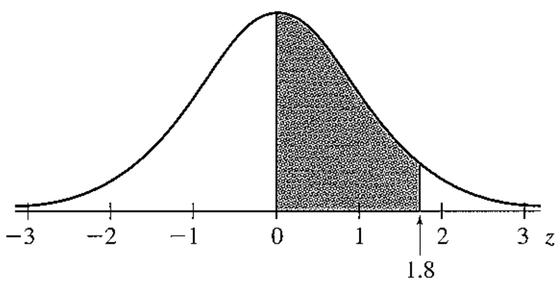


13. Which data set has the largest standard deviation? Which has the smallest standard deviation? How can you tell?
14. Which data set has the largest mean? Which has the smallest mean? How can you tell?
15. Sketch examples of two normal distributions that have the same mean, but have different standard deviations. Clearly label your graphs.
16. Sketch examples of two normal distributions that have different means, but have the same standard deviations. Clearly label your graphs.

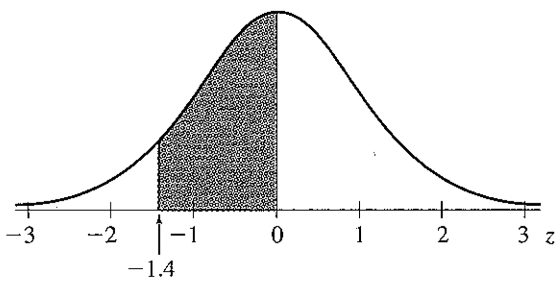
Problems 17 through 22

For the following standard normal curve, find the area of the shaded region using Table 11.3 and interpret the results.

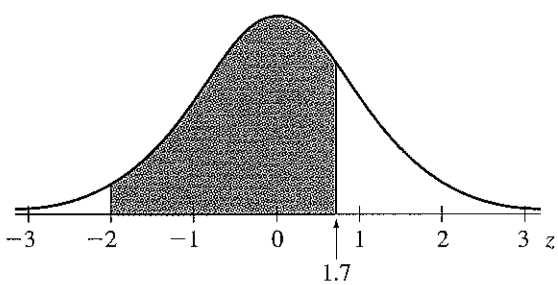
17.



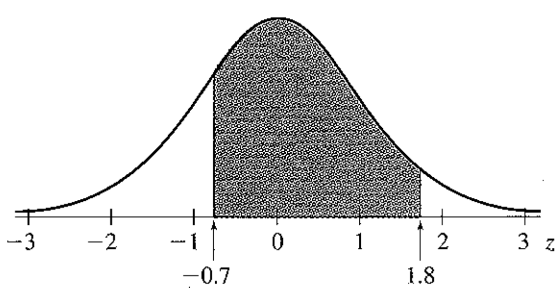
18.



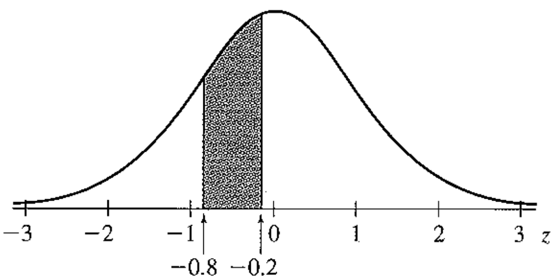
19.



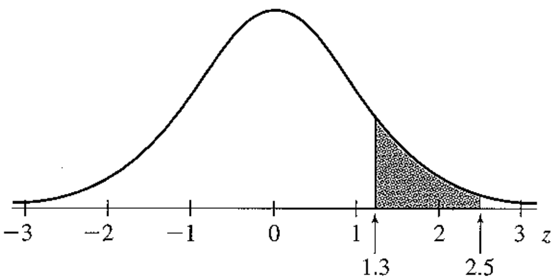
20.



21.



22.



23. Use Table 11.3 to find the area under the standard normal curve between 0 and 1.5. Then sketch the standard normal curve and shade the appropriate region.

24. Use Table 11.3 to find the area under the standard normal curve between −0.7 and 0. Then sketch the standard normal curve and shade the appropriate region.

25. Use Table 11.3 to find the area under the standard normal curve between −2.1 and 1.9. Then sketch the standard normal curve and shade the appropriate region.

26. Use Table 11.3 to find the area under the standard normal curve between 0.6 and 1.8. Then sketch the standard normal curve and shade the appropriate region.

27. Consider measurements taken from a population that has a standard normal distribution.

a. Find the percentage of the data that have a value between 1 and 3.

b. Find the percentage of the data that have a value larger than 2.

c. Find the percentage of the data that are not between −1 and 1.

28. Consider measurements taken from a population that has a standard normal distribution.

a. Find the percentage of the data that have a value between −2 and 3.

b. Find the percentage of the data that have a value less than 1.

c. Find the percentage of the data that are not between 0 and 1.

29. Consider measurements taken from a population that has a standard normal distribution.

a. Find the percentage of the data that have a value between 2 and 3.

b. Find the percentage of the data that have a value less than 2.

c. Find the percentage of the data that are not between −2 and 2.

30. Consider measurements taken from a population that has a standard normal distribution.

a. Find the percentage of the data that have a value between −3 and 1.

b. Find the percentage of the data that have a value less than −2

c. Find the percentage of the data that are not between 1 and 3.

Problems 31 through 38

Suppose measurements are taken from a population that has a standard normal distribution. Find the percentage of measurements that are in the specified interval.

31. **a.** between 0 and 1.2.
b. between −2.3 and 0.
c. between −0.7 and 1.8.

32. **a.** between 0 and 2.1.
b. between −1.3 and 0.
c. between −0.1 and 0.2.

33. **a.** greater than 1.8.
b. less than 1.8.

34. **a.** greater than −1.3.
b. less than 1.3.

35. **a.** between 0 and 1 standard deviation above the mean.
b. between 2 standard deviations below the mean and 0.
c. between 2 standard deviations below the mean and 1 standard deviation above the mean.

36. **a.** between 0 and 2 standard deviations above the mean.
b. between 1 standard deviation below the mean and 0.
c. between 1 standard deviation below the mean and 2 standard deviations above the mean.

37. **a.** within 1 standard deviation of the mean.
b. not within 3 standard deviations of the mean.

38. **a.** within 2 standard deviations of the mean.
b. not within 1 standard deviation of the mean.

39. For measurements taken from a population for which the distribution of measurements can be assumed to be normal, suppose the mean is 16 inches and the standard deviation is 2 inches.
a. What measurement is 1 standard above the mean?
b. What measurement is 2 standard deviations below the mean?
c. What percentage of measurements are within 2 standard deviations of the mean?
d. What percentage of measurements are greater than 16 inches?

40. For measurements taken from a population for which the distribution of measurements can be assumed to be normal, suppose the mean is 327 inches and the standard deviation is 54 inches.
a. What measurement is 2 standard deviations above the mean?
b. What measurement is 1 standard deviation below the mean?
c. What percentage of measurements are within 1 standard deviation of the mean?
d. What percentage of measurements are less than 327 inches?

41. Mensa, founded in 1946, was created as a society for intelligent people. The requirement for membership is a high IQ, which is defined as an IQ in the top 2% of the population. It is known that IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. Approximately what is the lowest IQ accepted for membership into Mensa?

42. The diastolic blood pressure of women between 18 and 70 years of age is normally distributed with a mean of 77 mm Hg and a standard deviation of 10 mm Hg. Suppose a drug company will test a new blood-pressure-lowering drug, and women with high blood pressures are needed for the study. If high blood pressure is defined as having a diastolic blood pressure in the top 7% of the population, approximately what is the lowest diastolic blood pressure that a woman could have and still be included in the study?

43. The serum total cholesterol for women ages 20 to 74 years is normally distributed with a mean of 197 mg/dL and a standard deviation of 43.1 mg/dL. A doctor will conduct a study to see if a high-fat and low-carbohydrate diet will increase levels of serum total cholesterol. For the study, women ages 20 to 74 with normal serum total cholesterol levels will follow the diet for 6 weeks. Suppose a normal serum total cholesterol level is defined as a level that is within 1.5 standard deviations of the mean.
a. What should the serum total cholesterol levels be for women who will be included in this study?
b. Out of 5000 women, approximately how many would be eligible to participate in the study?

44. The serum total cholesterol for men ages 20 to 74 years is normally distributed with a mean of 200 mg/dL and a standard deviation of 44.7 mg/dL. A doctor will conduct a study to see if a low-fat and high-carbohydrate diet will decrease levels of serum total cholesterol. For the study, men ages 20 to 74 with high serum total cholesterol levels will use the diet for 6 weeks. Suppose high serum total cholesterol is defined as a level that is greater than 2.5 standard deviations above the mean.
a. What should the serum total cholesterol levels be for men who will be included in this study?
b. Out of 3500 men, approximately how many would be eligible to participate in the study?

Extended Problems

45. The shape of the normal distribution is completely determined by specifying the mean and the standard deviation. The standard deviation is always a positive quantity and is a measure of the spread of the data. Programs are available on the Internet that allow a user to input a value for the mean and a value for the standard deviation and see the resulting normal distribution. One such program can be found at www.stattucino.com/berrie/dsl/, or use search keywords "normal distribution applet" on the Internet. To sketch a normal distribution, input a mean and a standard deviation and then click the "draw" button. Create several different normal curves by adjusting the values of the mean and standard deviation. Summarize your observations and include sketches.

46. In this section, you learned how to use a table (Table 11.3) to find the area in an interval and below a standard normal curve. There are calculators on the Internet that nicely calculate and illustrate areas under any normal curve between two values. One such site can be found at www.coe.tamu.edu/~strader/Mathematics/Statistics/NormalCurve/ or use search keywords "normal probability calculator." The user may input a mean and a standard deviation and then click and drag the vertical bars to see how the area changes. This program uses blue to show the area under the curve and between two values; it uses red to show the area under the curve and outside the interval. Visit this site to examine the relationship between areas and standard deviation. Consider the following explorations.

a. For the mean $\mu = 8$ and the standard deviation $\sigma = 2$, set the vertical bars to show the area under the curve from $\mu - \sigma = 6$ to $\mu + \sigma = 10$. Make a note of the calculated area.

b. For the mean $\mu = 8$ and the standard deviation $\sigma = 2$, set the vertical bars to show the area under the curve from $\mu - 2\sigma = 4$ to $\mu + 2\sigma = 12$. Make a note of the calculated area.

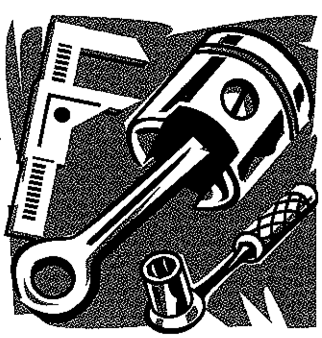
c. For the mean $\mu = 8$ and the standard deviation $\sigma = 2$, set the vertical bars to show the area under the curve from $\mu - 3\sigma = 2$ to $\mu + 3\sigma = 14$. Make a note of the calculated area.

d. Choose a different mean and standard deviation, and repeat parts (a), (b), and (c). How do the areas compare in each case to the areas you found for the normal curve with a mean of 8 and a standard deviation of 2? Is this a coincidence? Repeat parts (a) to (c) once more, using a different mean and standard deviation. What can you conclude?

47. At the beginning of this chapter, you read about Ronald A. Fisher and W. S. Gossett, who both made significant strides in the field of statistics. Research other famous statisticians, such as Carl Gauss, Karl Pearson, Florence Nightingale, Jerzy Neyman, and Thomas Bayes. What were their important contributions to statistics? Write a report to summarize your findings.

11.2 Applications of Normal Distributions

INITIAL PROBLEM



An automaker is considering two different suppliers of a certain critical engine part. The part must be within 0.012 mm of its required size or the engine will fail. The automaker will carefully measure all parts from the suppliers before installing them. Any parts outside the acceptable range will be discarded. Supplier A charges \$120 for 100 parts and guarantees that the actual part sizes will have a mean equal to the required size and a standard deviation of 0.004 mm. Supplier B charges \$90 for 100 parts and guarantees that the actual part sizes will have a mean equal to the required size and a standard deviation of 0.012 mm. Which supplier should the automaker choose?

A solution of this Initial Problem is on page 723.

THE RELATIONSHIP AMONG NORMAL DISTRIBUTIONS

We have seen that a critical property of all normal distributions is that if the endpoints of an interval are described by the number of standard deviations that they are above or below the mean, then the percentage of the data in that interval is the same for all normal distributions. In particular, this is true when comparing any normal distribution to the standard normal distribution, which has a mean of 0 and a standard deviation of 1. For example, in Section 11.1, we learned from the standard normal distribution tables that approximately 34% of the data in a standard normal distribution lies between $z = 0$ (the mean) and $z = 1$ (1 standard deviation more than the mean). Similarly, approximately 34% of the data in *any* normal distribution will lie between the mean and the value that is 1 standard deviation more than the mean. We next state this idea more precisely.