SOLUTION OF THE INITIAL PROBLEM

An automaker is considering two different suppliers of a certain critical engine part. The part must be within 0.012 mm of its required size or the engine will fail. The automaker will carefully measure all parts from the suppliers before installing them. Any parts outside the acceptable range will be discarded. Supplier A charges \$120 for 100 parts and guarantees that the actual part sizes will have a mean equal to the required size and a standard deviation of 0.004 mm. Supplier B charges \$90 for 100 parts and guarantees that the actual part sizes will have a mean equal to the required size and a standard deviation of 0.012 mm. Which supplier should the automaker choose?

SOLUTION At first glance, supplier B looks like the best choice because the parts from this supplier are less expensive. Notice that parts from supplier B cost $\frac{\$90}{100}$, or \$0.90 each, while parts from supplier A cost $\frac{\$120}{100}$, or \$1.20 each.

However, let's take a closer look. We will consider how many parts from each supplier are acceptable and what each acceptable part costs. We assume that in each case the part sizes are normally distributed. For supplier A, the tolerance of 0.012 mm is 3 times the standard deviation of 0.004. Thus, all parts from supplier A that are within 3 standard deviations of the mean will be usable. By the 68–95–99.7 rule, we know that 99.7 of every 100 parts (on average) will be within 3 standard deviations of the mean and will be acceptable. So, the average cost of each acceptable part from supplier A is $\frac{\$120}{99.7}$, or about \$1.20.

On the other hand, for supplier B, the tolerance of 0.012 is exactly equal to the standard deviation. Thus, only parts from supplier B that are within 1 standard deviation of the mean will be usable by the automaker. Again, by the 68–95–99.7 rule, 68 of every 100 parts (on average) will be within 1 standard deviation of the mean and will be acceptable. So, the average cost of each acceptable part from supplier B is $\frac{\$90}{68}$, or about \$1.32. The average cost of the acceptable parts from supplier A is less than the average cost of acceptable parts from supplier B. In addition, fewer parts from supplier A will be discarded. Therefore, the automaker should choose supplier A.

PROBLEM SET 11.2

1. A data set is represented by a normal distribution with a mean of 25 and a standard deviation of 3. For each of the following data values, how many standard deviations above or below the mean is it?
   a. 28 **b.** 31 **c.** 22
   d. 26.5 **e.** 20.5 **f.** 16

2. A data set is represented by a normal distribution with a mean of 206 and a standard deviation of 22. For each of the following data values, how many standard deviations above or below the mean is it?
   a. 184 **b.** 272 **c.** 239
   d. 217 **e.** 140 **f.** 206

3. Suppose a data set is represented by a normal distribution with a mean of 52 and a standard deviation of 10.
   a. What data value is 3 standard deviations above the mean?
   b. What data value is 2 standard deviations below the mean?
   c. What data value is 1.5 standard deviations below the mean?
   d. What data value is 2.5 standard deviations above the mean?
   e. What data value is $\frac{1}{4}$ of a standard deviation above the mean?

4. Suppose a data set is represented by a normal distribution with a mean of 125 and a standard deviation of 7.
   a. What data value is 2 standard deviations above the mean?
   b. What data value is 3 standard deviations below the mean?
   c. What data value is 1.5 standard deviations below the mean?
   d. What data value is 2.5 standard deviations above the mean?
   e. What data value is $\frac{1}{5}$ of a standard deviation below the mean?

5. Approximately 20% of the data in a standard normal distribution are between $-\frac{1}{4}$ and $\frac{1}{4}$, or within $\frac{1}{4}$ of a standard deviation of the mean. Suppose the measurements on a population are normally distributed with mean 84 and standard deviation 8.
   a. What data value is $\frac{1}{4}$ of a standard deviation above the mean?
   b. What data value is $\frac{1}{4}$ of a standard deviation below the mean?
   c. What percentage of the measurements in the population lie between 82 and 86?

6. Approximately 50% of the data in a standard normal distribution are between $-\frac{2}{3}$ and $\frac{2}{3}$, or within $\frac{2}{3}$ of a standard deviation of the mean. Suppose the measurements on a population are normally distributed with mean 145 and standard deviation 12.
   a. What data value is $\frac{2}{3}$ of a standard deviation above the mean?
   b. What data value is $\frac{2}{3}$ of a standard deviation below the mean?
   c. What percentage of the measurements of the population lie between 137 and 153?

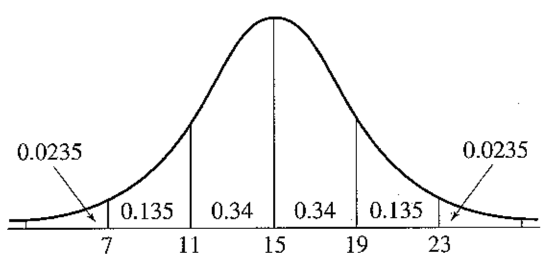
7. Suppose that turkeys from a certain ranch have weights that are normally distributed with a mean of 12 pounds and a standard deviation of 2.5 pounds. Use the 68–95–99.7 rule.
   a. What percentage of turkeys have weights between 9.5 pounds and 14.5 pounds?
   b. What percentage of turkeys have weights between 4.5 pounds and 19.5 pounds?
   c. What percentage of turkeys have weights between 7 pounds and 12 pounds?

8. Suppose that heights of a certain type of tree are normally distributed with a mean of 154 feet and a standard deviation of 8.2 feet. Use the 68–95–99.7 rule.
   a. What percentage of trees have heights between 129.4 feet and 178.6 feet?
   b. What percentage of trees have heights between 145.8 feet and 162.2 feet?
   c. What percentage of trees have heights between 154 feet and 170.4 feet?

9. A certain population has measurements that are normally distributed with a mean of μ and a standard deviation of σ. Use the 68–95–99.7 rule.
   a. Find the percentage of measurements that are between $\mu - \sigma$ and $\mu + 2\sigma$.
   b. Find the percentage of measurements that are between $\mu - 3\sigma$ and $\mu + \sigma$.
   c. Find the percentage of measurements that are not between $\mu - \sigma$ and $\mu + \sigma$.

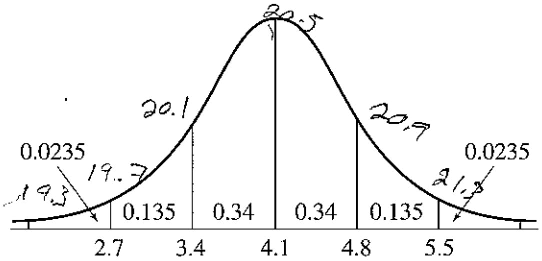
10. A certain population has measurements that are normally distributed with a mean of μ and a standard deviation of σ. Use the 68–95–99.7 rule.
   a. Find the percentage of measurements that are between $\mu - 2\sigma$ and $\mu + 2\sigma$.
   b. Find the percentage of measurements that are between $\mu - 3\sigma$ and $\mu + 2\sigma$.
   c. Find the percentage of measurements that are not between $\mu - 3\sigma$ and $\mu + \sigma$.

11. Consider the following normal distribution.



   a. What is the mean of the population? How do you know?
   b. What is the standard deviation of the population? How do you know?
   c. What percentage of the measurements are between 11 and 23?
   d. What percentage of the measurements are between 7 and 19?
   e. What percentage of the measurements are not between 7 and 23?

12. Consider the following normal distribution.



a. What is the mean of the population? How do you know?

b. What is the standard deviation of the population? How do you know?

c. What percentage of the measurements are between 4.1 and 5.5?

d. What percentage of the measurements are between 2.7 and 4.8?

e. What percentage of the measurements are not between 3.4 and 5.5?

13. Suppose a normal distribution has mean 10 and standard deviation 2. Find the z-scores of the measurements 9, 10, 11, 14, and 17.

14. Suppose a normal distribution has mean 20.5 and standard deviation 0.4. Find the z-scores of the measurements 19.3, 20.2, 20.5, 21.3, and 23.

15. Recall that IQ scores are normally distributed with mean 100 and standard deviation 15. Find the z-scores of the IQ scores 64, 80, 96, 111, 136, and 145.

16. Suppose that the weights of rabbits have a normal distribution with a mean of 7.1 pounds and a standard deviation of 0.6 pounds. Find the z-scores of the weights 5.9, 6.2, 7.1, 7.2, 8.4, and 9 pounds.

17. Suppose that the weights of checked luggage for individuals checking in at a particular airport have a normal distribution of 55.6 pounds and a standard deviation of 11.3 pounds. Find the z-scores of weights 45.16, 49.82, 55.20, and 58.63 pounds.

18. Suppose that insect lifetimes for a particular species have a normal distribution with a mean of 812 hours and a standard deviation of 19 hours. Find the z-scores for lifetimes of 749, 766, 791, 801, 833, and 842 hours.

19. In a normally distributed data set, find the value of the mean if the following additional information is given.

a. The standard deviation is 4.25 and the z-score for a data value of 52.1 is 1.9.

b. The standard deviation is 0.6 and the z-score for a data value of 2 is -2.3.

20. In a normally distributed data set, find the value of the mean if the following additional information is given.

a. The standard deviation is 3.5 and the z-score for a data value of 15.3 is 1.2.

b. The standard deviation is 0.8 and the z-score for a data value of 4.9 is -0.6.

21. In a normally distributed data set, find the value of the standard deviation if the following additional information is given.

a. The mean is 9.8 and the z-score for a data value of 10.3 is 2.

b. The mean is 577 and the z-score for a data value of 533 is -0.5.

22. In a normally distributed data set, find the value of the standard deviation if the following additional information is given.

a. The mean is 226.2 and the z-score for a data value of 230 is 0.2.

b. The mean is 14.6 and the z-score for a data value of 5 is -0.3.

23. Suppose that there are 100 franchises of Betty's Boutique in similar shopping malls across America. The gross Saturday sales of these boutiques are approximately normally distributed with a mean of \$4610 and a standard deviation of \$370.

a. Find the z-scores of each of the following gross Saturday sales amounts: \$3870, \$4425, and \$5535.

b. What percentage of the Betty's Boutique franchises had gross Saturday sales between \$4425 and \$5535? Use the z-scores you found in part (a) and Table 11.3.

c. What percentage had gross Saturday sales between \$3870 and \$5535? Use the z-scores you found in part (a) and Table 11.3.

d. What percentage of stores had gross Saturday sales less than \$5535?

24. The lifetime of a certain brand of passenger tire is approximately normally distributed with a mean of 41,500 miles and a standard deviation of 1950 miles.

a. Find the z-scores of each of the following tire lifetimes: 38,575; 41,500; and 46,765.

b. What percentage of this brand of tires will have lifetimes between 38,575 and 41,500 miles? Use the z-scores you found in part (a) and Table 11.3.

c. What percentage of tires will have lifetimes between 38,575 and 46,765 miles? Use the z-scores you found in part (a) and Table 11.3.

d. What percentage of tires will have lifetimes of more than 46,765 miles?

25. Suppose that a certain breed of dog has a mean weight of 11 pounds with a standard deviation of 3.5 pounds. Also, suppose that the weights of this breed of dog are approximately normally distributed.

a. What percentage of dogs will weigh between 5.75 and 16.25 pounds?

b. What percentage of dogs will weigh between 11 and 18 pounds?

c. What percentage of dogs will weigh more than 15.55?

d. What percentage of dogs will weigh less than 1.2 pounds?

26. Suppose that a certain insect has a mean lifespan of 5.6 days with a standard deviation of 1.2 days. Assume the lifespan of this insect is approximately normally distributed.

a. Calculate the percentage of insects with a lifespan between 3.2 and 8 days.

b. Calculate the percentage of insects with a lifespan between 2.6 and 8.6 days.

c. What percentage of insects will live longer than 3.32 days?

d. What percentage of insects will live less than 6.56 days?

Problems 27 through 32

The SAT is a standardized, 3-hour test designed to measure verbal and mathematical abilities of college-bound seniors. Many colleges and universities use the scores from the SAT as part of their admissions processes because the result of the test is one predictor of how well incoming students may do in college. Until 2005, students could earn at most 1600 points on the SAT: 800 for the verbal section and 800 for the math section. The scores for the SAT are normally distributed. In the year 2003, a total of 1,406,324 high school students took the SAT. (*Source:* www.collegeboard.com.)

27. For the 2003 SAT scores in the math section only, the mean was 507 and the standard deviation was 111.

a. What percentage of students had a math score between 300 and 500 points? Round your z-scores to the nearest tenth.

b. What percentage of students had a math score less than 200 points? Round your z-score to the nearest tenth.

c. Approximately how many students had a math score of at least 400? Round your z-score to the nearest tenth.

28. For the 2003 SAT scores in the verbal section only, the mean was 519 and the standard deviation was 115.

a. What percentage of students had a verbal score between 300 and 500 points? Round your z-scores to the nearest tenth.

b. What percentage of students had a verbal score less than 200 points? Round your z-score to the nearest tenth.

c. Approximately how many students had a verbal score of at least 400? Round your z-score to the nearest tenth.

29. Boston College considers a variety of factors when deciding which students to admit. In 2003, a student was considered "competitive" in the application process if he or she had both a math SAT score and a verbal SAT score in the mid to high 600s. Refer to problems 27 and 28 for mean and standard deviation information. Round your z-score to the nearest tenth.

a. What percentage of students had a math score of at least 650?

b. What percentage of students had a verbal score of at least 650?

30. Harvard University considers a variety of factors, including SAT scores, when deciding which students to admit to the freshman class. Harvard does not have any cutoff SAT scores, but they consider a student "competitive" in the application process if he or she had math scores as well as verbal score in the 700s. Refer to problems 27 and 28 for mean and standard deviation information. Round your z-scores to the nearest tenth.

a. What percentage of students had a math SAT score of at least 700?

b. What percentage of students had a verbal SAT score of at least 700?

31. Of the students whose parents did not earn a high-school diploma, the mean verbal score on the 2003 SAT was 413 with a standard deviation of 100, and the mean math score on the 2003 SAT was 443 with a standard deviation of 114.

a. What percentage of these students had a verbal score between 500 and 600? Round your z-scores to the nearest tenth.

b. What percentage of these students had a math score between 500 and 600? Round your z-scores to the nearest tenth.

c. What percentage of these students had a verbal score less than 300 points? Round your z-score to the nearest tenth.

d. What percentage of these students had a math score less than 300 points? Round your z-score to the nearest tenth.

32. Of the students whose parents have a graduate degree, the mean verbal score on the 2003 SAT was 559 with a standard deviation of 107, and the mean math score on the 2003 SAT was 569 with a standard deviation of 111.

a. What percentage of these students had a verbal score between 500 and 600? Round your z-scores to the nearest tenth.

b. What percentage of these students had a math score between 500 and 600? Round your z-scores to the nearest tenth.

c. What percentage of these students had a verbal score less than 300 points? Round your z-score to the nearest tenth.

d. What percentage of these students had a math score less than 300 points? Round your z-score to the nearest tenth.

33. Assume the distribution of the annual total rainfall in Guatemala is approximately normal with a mean of 955 mm and a standard deviation of 257 mm. In 2001, a 4-month drought during the rainy season damaged crops and caused 65,000 people to suffer life-threatening malnutrition. In what percentage of years would Guatemala suffer from drought conditions, that is, receive less than 600 mm of rain? Round your z-score to the nearest tenth.

34. A study recorded the serum total cholesterol levels for 7429 females ages 20 to 74. The study found that the serum total cholesterol was approximately normally distributed, with a mean of 204 mg/dL and a standard deviation of 44.2 mg/dL. According to the American Heart Association, high total serum cholesterol is 240 mg/dL or above. Estimate the percentage of females in this study who had a high total serum cholesterol level.

35. A study partially funded by Novartis Animal Health monitored the number of fleas when flea-infested dogs and cats were treated monthly with imidacloprid. Scientists counted fleas in the animal's surroundings and on the animal. The following table summarizes the counts of fleas on day 0, 14, and 28, where μ = the mean number of fleas observed. Assume flea counts are approximately normally distributed.

Flea Counts	Day 0	Day 14	Day 28
On animal	$\mu = 41.3$ $\sigma = 58.8$	$\mu = 1.4$ $\sigma = 2.5$	$\mu = 1.7$ $\sigma = 4.7$
In surroundings	$\mu = 13.8$ $\sigma = 11$	$\mu = 12.7$ $\sigma = 14$	$\mu = 1$ $\sigma = 1.7$

(Source: www.vet.ksu.edu/depts/dmp/personnel/faculty/docs/imidfip.doc.)

a. Compare the percentages of animals that had at most one flea observed on them on day 14 and on day 28. Round your z-scores to the nearest tenth. What can you conclude?

b. Compare the percentages of animals that had at most one flea observed in their surroundings on day 14 and on day 28. Round your z-scores to the nearest tenth. What can you conclude?

c. If you treat 50 dogs with imidacloprid, and you count fleas on the animals 28 days later, approximately how many dogs might you find with fewer than 5 fleas? At least 11 fleas?

36. Housefly wing-length measurements are approximately normally distributed with mean length 4.55 mm and standard deviation 0.392 mm. (*Source:* www.seattlecentral.edu.)

a. What percentage of housefly wings are longer than 5 mm? Round your z-scores to the nearest tenth.

b. What percentage of housefly wings are between 3 mm and 4 mm? Round your z-scores to the nearest tenth.

c. If you swat 30 houseflies and measure their wing lengths, approximately how many houseflies might you find with wing lengths between 3.77 mm and 4.16 mm? Between 4.16 mm and 4.55 mm?

Extended Problems

Problems 37 and 38

We can use the normal distribution to estimate the number of measurements in a sample less than a given number. For example, if 16% of measurements from a normal distribution are less than 200, we can expect about 16 measurements out of 100 to be less than 200. Similarly, we can expect about 160 measurements out of 1000 to be less than 200.

37. In problem 24, we looked at mileage ratings for automobile tires. Suppose that the tire company guarantees tires to last at least 38,000 miles and will replace any tire that does not last this long (on a properly aligned automobile). What percentage of tires will have to be replaced? If the cost of such a replacement is $86, how much will the company expect to pay on each lot of 1000 tires?

38. Suppose a snack food manufacturer claims their boxes of crackers are filled to a mean weight of 1.3 pounds with a standard deviation of 0.2 pound. In an ad campaign, they promise to reimburse customers if the actual weight of the box of crackers is less than 1 pound. If a box of crackers costs \$2.50 and there are 1 million boxes sold in a year, what is the expected cost of such a program to the manufacturer?

39. Graphics calculators can graph normal distributions. The equation for the normal distribution with mean μ, standard deviation σ, and variable x is as follows.

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

a. If $\mu = 0$ and $\sigma = 1$, the equation represents the standard normal curve. Substitute the values $\mu = 0$ and $\sigma = 1$ into the equation and simplify. Graph the equation on your calculator. Think about your window settings. What values of x will make sense? Sketch and label this graph.

b. Keep the graph of the standard normal curve on the screen and graph two more normal curves. For these, continue to use $\mu = 0$, but select different values for σ in each equation. You may have to adjust your window settings to see all three graphs. Sketch and label the three graphs. What can you conclude?

c. Keep the graph of the standard normal curve from part (a) on the screen and delete the other two graphs. Graph two more normal curves: use $\sigma = 1$ and select different values for μ in each equation. You may have to adjust your window settings. Sketch and label the three graphs. What can you conclude?

d. Keep the graph of the standard normal curve from part (a) on the screen and delete the other two graphs. Change both the mean and standard deviation, predict how the new normal curve will compare to the standard normal curve, and graph the new normal curve. Was your prediction correct?

40. Fourteen teams were in the 2003–2004 Women's National Basketball Association (WNBA). Create a frequency histogram of heights for all of the players in the WNBA for the current season and determine if the distribution is approximately normal. Find the mean and the standard deviation using the statistical features of your calculator. Summarize the heights using the 68–95–99.7 rule. On the Internet go to http://sports.espn.go.com/wnba/teams for a list of women on each team's roster and their heights.

41. Research the history of the normal distribution. Who is credited with the "discovery" of the normal distribution? What were some of its earliest applications? Summarize your findings in a report.

42. Cereal and potato chip manufacturers want consumers to be happy with their products. They specify the weight of the contents on the label. However, packaging processes fill cereal boxes and potato chip bags imperfectly. If a consumer opened a bag of potato chips to find only 9 ounces instead of the stated 13.25 ounces, he or she would not be very happy. How much variation in weight is there from bag to bag? Contact, by phone or e-mail, manufacturers of two of the packaged products you usually buy. Ask them for statistical data related to packaging, specifically the mean weight of the contents and the standard deviation. Then calculate the percent of packages that would contain less than 90%, less than 80%, less than 70%, and less than 60% of the stated weight. How many packages of the product do you buy in a year? Suppose, for example, that you buy approximately 40 of the 13.25-ounce packages in a year. Determine how many bags will likely contain less than 12 ounces and how much you are overpaying for the product. Summarize your findings in a table and write a report.

43. We have seen that we can use tables to determine areas under the standard normal curve. Many Internet sites will also calculate percentages related to a normal distribution. Search the Internet to find a site that will allow you to input the mean, standard deviation, and endpoints of an interval and that will return the percentage of values in that interval under the normal curve. Many sites will also graph the information for you. Search the Internet using keywords "normal distribution probability applets." Use the applet to obtain more exact solutions to problems 33 through 36.