SOLUTION OF THE INITIAL PROBLEM
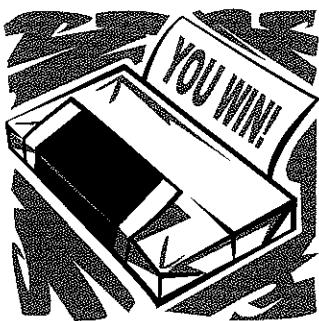
A candy bar company has a promotion in which letters printed inside some of the wrappers earn the buyer a prize. Suppose that you buy 400 of these candy bars and you discover that 25 wrappers are winners. You are thinking of buying another 1000 candy bars. How many wrappers would you expect to have letters printed on the inside?

SOLUTION Consider the first 400 bars that you unwrapped. Here, $n = 400$. The sample proportion is $\hat{p} = \frac{25}{400} = 0.0625$. Thus, we might estimate that of the next 1000 wrappers, 6.25% of them, or $0.0625(1000) = 62.5$, might be winners.

On the other hand, we may use the standard error to create a confidence interval for the number of winners. The standard error here is $\hat{s} = \sqrt{\frac{0.0625(1 - 0.0625)}{400}} \approx 0.0121$. We calculate our 95% confidence interval as

$$\hat{p} - 2\hat{s} = 0.0625 - 2(0.0121) = 0.0383,$$

and

$$\hat{p} + 2\hat{s} = 0.0625 + 2(0.0121) = 0.0867.$$

A 95% confidence interval for this situation is the interval of numbers between $0.0383 = 3.83\%$ and $0.0867 = 8.67\%$. Thus, out of 1000 candy bars, you should expect between $1000(0.0383) \approx 38$ and $1000(0.0867) \approx 87$ to have letters printed on the inside.

PROBLEM SET 11.3

1. Use your calculator to find each of the following. Round to the nearest thousandth.

   a. $\sqrt{\dfrac{(0.2)(0.8)}{50}}$

   b. $\sqrt{\dfrac{(0.14)(1 - 0.14)}{634}}$

   c. $0.49 - 3\sqrt{\dfrac{(0.49)(1 - 0.49)}{470}}$

2. Use a calculator to find each of the following. Round to the nearest thousandth.

   a. $\sqrt{\dfrac{(0.3)(0.7)}{30}}$

   b. $\sqrt{\dfrac{(0.81)(1 - 0.81)}{199}}$

   c. $0.26 - 3\sqrt{\dfrac{0.26(1 - 0.26)}{38}}$

3. All possible samples of size 250 are taken from a population that has a population proportion of 0.58.
   a. Find the mean of the set of sample proportions.
   b. Find the standard deviation of the set of sample proportions.

4. All possible samples of size 1358 are taken from a population that has a population proportion of 0.61.
   a. Find the mean of the set of sample proportions.
   b. Find the standard deviation of the set of sample proportions.

5. A factory produces 6000 cars during a certain week, and 300 of them have significant problems needing correction. Inspectors select 60 of the cars for a detailed inspection, and 5 have a problem needing correction.
   a. What is the population and what is the sample?
   b. What is the population proportion of cars having problems?
   c. What is the sample proportion of cars having problems?

6. A manufacturer creates an assortment of candies by mixing 500 caramels with 1000 chocolate-covered nuts. These are then put into half-pound packages. A package is opened and found to have 12 caramels and 18 chocolate-covered nuts.

a. What is the population and what is the sample?

b. What is the population proportion of caramels?

c. What is the sample proportion of caramels?

7. Suppose a certain city has 7140 registered voters, of which 3460 are Democrats, 3250 are Republicans, and 430 are Independents. A pre-election canvassing in a given neighborhood reveals the following numbers of registered voters: 185 Democrats, 210 Republicans, and 25 Independents.

a. What is the population and what is the sample?

b. Find the sample size.

c. What is the population proportion of registered Republicans?

d. What is the sample proportion of registered Republicans?

8. According to the 2000 U.S. Census, the number of Nevada residents age 25 or older was 1,310,176. Of those, the number who had earned at most a bachelor's degree was 158,078. Suppose a research group conducted a telephone survey of Nevada residents age 25 or older and it was determined that 299 had earned at most an associate's degree, 45 had earned at most a bachelor's degree, and 22 had earned a graduate degree.

a. What is the population and what is the sample?

b. Find the sample size.

c. What is the population proportion of people who earned at most a bachelor's degree?

d. What is the sample proportion of people who earned at most a bachelor's degree?

9. A factory produces 6000 boxes of cereal during a certain day and significantly underfills 300 of them. An inspector randomly selects a sample of 55 boxes for a detailed weighing.

a. Find the population proportion of boxes of cereal that are significantly underfilled.

b. Find the mean and standard deviation of the set of all sample proportions of underfilled boxes.

c. Is the sample size large enough so that we can conclude the distribution of sample proportions is approximately normal? Why or why not?

10. Out of the 3,450,000 registered voters in a state, 98,549 favor a moratorium on state executions until death penalty procedures are officially reviewed. A reporter will take a sample of 15 registered voters and ask each of them whether a moratorium is favored.

a. Find the population proportion of those who favor the moratorium.

b. Find the mean and standard deviation of the set of all the sample proportions of those who favor a moratorium.

c. Is the sample size large enough so that we can conclude the distribution of sample proportions is approximately normal? Why or why not?

11. Suppose that out of 27,560 people who live in a certain city, 22,048 support the continuation of the manned space shuttle program.

a. Find the population proportion of people who support the continuation of the manned space shuttle program.

b. Find the mean and standard deviation of the set of all the sample proportions when samples of size 150 are taken from the population.

c. Find the mean and standard deviation of the set of all the sample proportions when samples of size 3000 are taken from the population.

d. Compare the results from parts (b) and (c) and explain what you observe.

12. On July 1, 2002, the population of Oregon was estimated to be 3,504,700. Suppose that 2,278,055 Oregonians would not support a tax increase to fund schools.

a. Find the population proportion of Oregonians who would not support a tax increase to fund schools.

b. Find the mean and standard deviation of the set of all the sample proportions when samples of size 2400 are taken from the population.

c. Find the mean and standard deviation of the set of all the sample proportions when samples of size 10,000 are taken from the population.

d. Compare the results from parts (b) and (c) and explain what you observe.

13. Explain the difference between p and $\hat{p}$.

14. Explain the difference between the standard deviation of the set of all sample proportions and the standard error of the sample.

15. Suppose a student has five courses to complete as requirements in science and humanities and decides to take three of them next term. Since he registers early and there are multiple sections for each course, he feels free to choose any three of the five. The required courses are Science A (SA), Science B (SB), Humanities A (HA), Humanities B (HB), and Humanities C (HC). There are 10 ways in which he can select three of the five courses. Consider each selection of three courses as one sample.

a. Find the population proportion of required humanities courses he can take.

b. The following table contains his possible course selections. Find the sample proportion of required humanities courses for each selection (sample) of three classes.

Course Selections	Sample Proportion of Humanities Courses
SA, SB, HA	
SA, SB, HB	
SA, SB, HC	
SA, HA, HB	
SA, HA, HC	
SA, HB, HC	
SB, HA, HB	
SB, HA, HC	
SB, HB, HC	
HA, HB, HC	

c. Find the mean of the sample proportions and compare it to the population proportion. What do you observe?

16. Six students tie for top honors in a graduating class. The administration has decided that the top four students will give speeches during the graduation ceremonies, so school officials decide to pick four of the students at random. The six students are Ann (A), Betty (B), Carol (C), Diana (D), Eddie (E), and Fred (F). There are 15 ways in which the administration can select four students out of the six.

a. Find the population proportion of females among the top 6 students.

b. The following table contains the possible student selections. Find the sample proportion of females for each selection (sample) of four students.

Students	Sample Proportion of Females
A, B, C, D	
A, B, C, E	
A, B, C, F	
A, B, D, E	
A, B, D, F	
A, B, E, F	
A, C, D, E	
A, C, D, F	
A, C, E, F	
A, D, E, F	
B, C, D, E	
B, C, E, F	
B, C, D, F	
B, D, E, F	
C, D, E, F	

c. Find the mean of the sample proportions and compare it to the population proportion. What do you observe?

17. Find the standard error in a sample of size 40, given each of the following sample proportions.
a. 10%   **b.** 25%   **c.** 50%   **d.** 80%

18. Find the standard error in a sample of size 300, given each of the following sample proportions.
a. 10%   **b.** 25%   **c.** 50%   **d.** 80%

19. Find the standard error for a sample proportion of 45%, given each of the following sample sizes.
a. 50   **b.** 100   **c.** 500   **d.** 1000

20. Find the standard error for a sample proportion of 90%, given each of the following sample sizes.
a. 50   **b.** 100   **c.** 500   **d.** 1000

21. Five hundred students at a high school are randomly selected for a student services survey. Of those selected, 265 are females. Find the standard error for the proportion of females. Interpret the meaning of the standard error in this case.

22. Sixty cars are randomly selected from the weekly production of cars at a plant. Five of these cars have problems requiring corrections. Find the standard error for the proportion of cars requiring corrections. Interpret the meaning of the standard error in this case.

23. In a September 2003 Virginia Commonwealth University Life Sciences Survey, adults nationwide were asked, "In general, do you think that it is morally acceptable or morally wrong to use human cloning technology in developing new treatments for diseases?" Of the 1003 adults who were contacted, 361 thought it was morally acceptable. Find the sample proportion and the standard error for the proportion of adults who thought it was morally acceptable. Explain how you would interpret these two values. (*Source:* www.pollingreport.com.)

24. In a January 2003 CNN/*USA Today*/Gallup Poll, adults nationwide were asked the question, "Do you think that cloning that is designed specifically to result in the birth of a human being should be legal or illegal in the United States?" Of the 1000 adults surveyed, 110 said that it should be legal. Find the sample proportion and the standard error for the proportion of adults who thought it should be legal. Explain how you would interpret these two values. (*Source:* www.pollingreport.com.)

25. A general biology class is doing a project on physical characteristics. The students in the class were randomly assigned to the section after preregistration, so the instructor considers them to be a random sample of the student body. There are 42 students in the class, and 7 are left-handed.
    a. Find the sample proportion and standard error for the proportion of students who are left-handed.
    b. Find a 95% confidence interval for the population proportion of left-handed students.
    c. The instructor made an assumption that the students in the class were representative of the student body. Explain whether that assumption is reasonable.

26. An instructor for a government studies course at a community college surveys the 35 students in her class, asking whether they would approve of a tax increase to fund social programs. The students in the class range in age from 19 to 57 and seem to come from different social classes, so the instructor considers them to be a representative sample of adults in the city. Of the 35 students surveyed, 9 approved of a tax increase to fund social programs.
    a. Find the sample proportion and standard error for the proportion of adults who approve of a tax increase to fund social programs.
    b. Find a 95% confidence interval for the population proportion of adults who approve of a tax increase to fund social programs.

27. If a news organization reported that 39% of Americans believe the speed limit on all freeways should be raised to 85 miles per hour and gave the margin of error as 4%, what is the standard error of the sample? How big was the sample?

28. If a news organization reported that 47% of Americans believe the Alaskan Arctic Wildlife Refuge should be opened to oil and gas exploration and gave the margin of error as 3%, what is the standard error of the sample? How big was the sample?

29. The student services office of a university is concerned about student acceptance of new registration procedures. A random sample of students is selected and contacted. They are asked whether they find the new procedures satisfactory. Of the 280 students who respond, 172 are satisfied with the new procedures.
    a. Find the sample proportion and standard error.
    b. Find the 95% confidence interval for the percentage of students who are satisfied with the new registration procedures.
    c. Explain how to interpret the 95% confidence interval.

30. A company that produces flashlight batteries wishes to know what percentage of its batteries will last longer than 30 hours. It selects and tests a random sample of 1000 batteries. Of these batteries, 917 last 30 hours or more.
    a. Find the sample proportion and standard error.
    b. Find the 95% confidence interval for the percentage of batteries that last 30 hours or more.
    c. Explain how to interpret the 95% confidence interval.

31. In a February 2003 *Los Angeles Times* opinion poll, out of a sample of 1385 adults nationwide, 54% approved of President Bush's proposal to allow individuals to divert part of their Social Security payroll taxes into private accounts, which they could personally invest in the stock market for their retirement. (*Source:* www.pollingreport.com.)
    a. Determine a 95% confidence interval for the population proportion of adults who favor President Bush's proposal.
    b. Find the margin of error and explain its meaning.

c. The instructor made an assumption that the students in the class were representative of adults in the city. Explain whether that assumption is reasonable.

32. In a February 2003 *Los Angeles Times* opinion poll, out of a sample of 1385 adults nationwide, 43% supported a complete ban on all research into human cloning, without exception. (*Source:* www.pollingreport.com.)

    a. Determine a 95% confidence interval for the proportion of adults who support a complete ban on all research into human cloning.

    b. Find the margin of error and explain its meaning.

33. Administrators at a college are interested in the number of students who are working 10 or more hours per week while taking full-time class loads. A random sample of 240 full-time students reveals that 105 of the students are working 10 or more hours per week. Find a 95% confidence interval for the percentage of full-time students who are working 10 or more hours per week.

34. A random survey of 500 pregnant women conducted in a large northeastern city indicated that 145 of them preferred a female obstetrician to a male obstetrician. Find a 95% confidence interval for the percentage of pregnant women in the city who would prefer a female obstetrician.

35. Title IX is a federal law that prohibits discrimination on the basis of gender in any high school or college that receives federal funds. It has been used to ensure that women have equal opportunities in high school and college athletics and are not discriminated against. In a January 2003 *Wall Street Journal* poll, out of 500 adults surveyed nationwide, 68% approve of Title IX. Find the 95% confidence interval for the proportion of people who approve of Title IX. What is the margin of error?

36. In an effort to comply with the requirements of Title IX (problem 35), many schools have had to cut funding for men's athletics in order to increase funding for women's programs. In a January 2003 *Wall Street Journal* poll, out of 500 adults surveyed nationwide, 27% disapprove of cutbacks in men's athletics. Find the 95% confidence interval for the population proportion of people who disapprove of cutbacks in men's athletics for the purpose of satisfying the requirements of Title IX. What is the margin of error?

37. The United States has not had a military draft since 1970, and it is suspected that only about 15% of the population would favor its reinstatement. In order to estimate the proportion of people in the United States who would favor reinstating the draft, we must take a sample.

    a. How large a sample must be taken to have a 5% margin of error?

    b. How large a sample must be taken to have a 1% margin of error?

38. A woman has never held the position of President of the United States, and it is believed that 75% of Americans would be willing to vote for a female candidate for president if she were qualified for the job. In order to estimate the proportion of Americans who would be willing to vote for a female candidate for president if she were qualified for the job, we must take a sample.

    a. How large a sample must be taken to have a 5% margin of error?

    b. How large a sample must be taken to have a 1% margin of error?

Problems 39 through 42

Public opinion polls typically report a margin of error of 5% or less. The margin of error gives a value to the uncertainty about survey results. It is not surprising, then, that one important factor in determining the margin of error is the sample size.

39. Consider a sample proportion of 0.5. Notice in this exercise how the sample sizes and confidence intervals change as the margin of error decreases:

    a. Fill in the following table.

Margin of of Error	s	Sample Size $n = \dfrac{\hat{p}(1 - \hat{p})}{s^2}$	95% Confidence Interval
10%			
5%			
1%			
0.1%			

    b. Compare the sample size requirements and confidence intervals for the various margins of error in part (a). What do you conclude?

40. Consider a sample proportion of 0.4. Notice in this exercise how the sample sizes and confidence intervals change as the margin of error decreases:

a. Fill in the following table.

Margin of Error	s	Sample Size $n = \dfrac{\hat{p}(1-\hat{p})}{s^2}$	95% Confidence Interval
10%			
1%			
5%			
0.1%			

b. Compare the sample size requirements and confidence intervals for the various margins of error in part (a). What do you conclude?

41. Notice in this exercise how the margin of error changes as the sample proportion, $\hat{p}$, and the sample size, n, change.

a. For each combination of sample proportion and sample size in the following table, find the margin of error.

Margin of Error for Given Values of n and $\hat{p}$.

	$n = 30$	$n = 50$	$n = 100$	$n = 500$
$\hat{p} = 0.25$				
$\hat{p} = 0.3$				
$\hat{p} = 0.5$				

42. Notice in this exercise how the margin of error changes as the sample proportion, $\hat{p}$, and the sample size, n, change.

a. For each combination of sample proportion and sample size in the following table, find the margin of error.

Margin of Error for Given Values of n and $\hat{p}$.

	$n = 100$	$n = 500$	$n = 1000$	$n = 3000$
$\hat{p} = 0.2$				
$\hat{p} = 0.6$				
$\hat{p} = 0.9$				

b. Consider the margins of error from part (a). Discuss whether n or $\hat{p}$ causes the greatest change in the margin of error.

Extended Problems

43. Nielsen Media Research, a television ratings company, randomly selects 5000 "Nielsen Families" and carefully monitors their television viewing. Television shows are renewed or canceled based on Nielsen Family viewing habits. For sample sizes of 5000, find the margin of error for each of the following sample proportion values: 0.1, 0.3, 0.5, 0.7, and 0.9. Explain how only 5000 Nielsen Families can accurately reflect the television viewing habits of the entire population of the United States.
(Source: www.nielsenmedia.com.)

44. The Gallup Organization and other polling companies routinely use sample sizes of 1000 to 1500. Polling companies must balance accuracy with the cost of increasing the sample size. For a sample proportion of 0.4, find the margins of error that result from sample sizes of 1000, 2000, 4000, and 8000. Suppose that doubling the sample size also doubles the cost to conduct the poll. Comment on the increased cost versus the increased accuracy that results from taking a larger sample.

Problems 45 and 46

In previous problems, we have been concerned with finding a 95% confidence interval for a population proportion. Although a 95% confidence interval is most common (public opinion polls use it almost exclusively), other confidence intervals are easy to define and calculate.

We know that a 95% confidence interval contains the values that are within 2 standard errors of the sample proportion because the distribution of sample proportions is assumed to be normal, and 95% of the sample proportions are within 2 standard errors (standard deviations) of the population proportion. Similarly, we can define a 99.7% confidence interval, which would be based on 3 standard errors, since 99.7% of all sample proportions are within 3 standard errors of the population proportion. Other commonly used confidence intervals are a 99% confidence interval based on 2.58 standard errors and a 90% confidence interval based on 1.65 standard errors.

45. Refer to Problem 31.

   a. Find a 90% confidence interval for the proportion of adults who favor President Bush's proposal.

   b. Find a 99.7% confidence interval for the proportion of adults who favor President Bush's proposal.

   c. Interpret and compare the intervals from parts (a) and (b).

46. Refer to Problem 32.

   a. Find a 90% confidence interval for the proportion of adults who support a complete ban on all research into human cloning.

   b. Find a 99% confidence interval for the proportion of adults who support a complete ban on all research into human cloning.

   c. Interpret and compare the intervals from parts (a) and (b).

47. By the time George Gallup started his American Institute of Public Opinion in 1935, opinion polling had been around for over 100 years. Research the history of public opinion polls. When and for what purpose were opinion polls conducted originally? What is a "straw poll"? How have opinion polls changed over the years to become more scientific? For more information, on the Internet search keywords "history of opinion polls." Write a report to summarize your findings.

48. Opinion polls often survey a random sample of people nationwide and consider the opinions of the people in the sample as representative of the entire population. If the sample size is large enough, the distribution of the set of all the sample proportions is approximately normal and we can calculate percentages and confidence intervals using the normal distribution. Let's see how these concepts work in practice. The math class you are attending will be the population. Survey every person who is registered for the class and ask their opinion about a topic of interest to you. Some ideas for the question include, "Do you support a ban on cell phone use while driving?", "Do you drink coffee?", or "Do you have Internet access at home?" Create a paper ballot, such as the following, for each student.

> ## OPINION POLL
>
> Do you support a ban on cell phone use while driving?
>
> Yes ———————   No ———————

   a. Find the proportion of students in your class who favor a ban on cell phone use while driving. This value is the true population proportion.

   b. Using the results from part (a), find the mean and standard deviation of the set of all sample proportions of students who favor a ban on cell phone use while driving.

   c. Find the smallest sample size required so that the distribution of the set of sample proportions can be assumed to be approximately normal. Recall that the sample size will be considered sufficiently large if *both* of the following conditions are met

$$p - 3\sqrt{\frac{p(1 - p)}{n}} > 0$$

and

$$p + 3\sqrt{\frac{p(1 - p)}{n}} < 1.$$

   d. Thoroughly mix the ballots from the students in your class and randomly select a sample using the size you determined in part (c). Find the sample proportion of students who favor a ban on cell phone use while driving. Find a 95% confidence interval and calculate the margin of error. Interpret your results.

e. Repeat part (d) 10 times. What percent of confidence intervals did actually contain the true population proportion of students who favored a ban on cell phone use while driving?

f. Explain whether the results of your opinion poll can be used to represent the population of students in your school, your city, your state, or the nation.

49. A company decides to offer a "double your money back" guarantee on its high intensity beam flashlight. The flashlight costs $15, and the company promises to refund $30 to any customer who purchases a defective flashlight. To determine how much they might expect to pay out on this guarantee, the company tests 800 flashlights from a random sample. Of these, 28 are defective, and the rest are of high quality. Find a 99% confidence interval for the proportion of defective flashlights that are manufactured. Use this result to determine high and low estimates for the amount the company can reasonably expect to have to pay because of their stated guarantee. Suppose 10,000 flashlights will be sold with this guarantee.

50. Many websites for news organizations and polling organizations give information about survey results, sample sizes and margins of error. One such website is www.pollingreport.com. Using the survey results for five different surveys, construct the 95% confidence intervals and explain how each confidence interval should be interpreted with respect to each survey's question.