Based on these data, does it appear that the world's population follows a Malthusian model?

SOLUTION One way to proceed is to check whether the growth rates over the 50-year periods are constant, or nearly so. Using the half-century as our unit of time makes the growth rate simply the ratio between the populations at the beginning and end of each half-century period. We find the following rates (Table 12.4):

Table 12.4

Time Period	Ratio of Populations	Growth Rate
1700 to 1750	$\frac{689}{579} \approx 1.19$	19%
1750 to 1800	$\frac{909}{689} \approx 1.32$	32%
1800 to 1850	$\frac{1086}{909} \approx 1.19$	19%
1850 to 1900	$\frac{1556}{1086} \approx 1.43$	43%
1900 to 1950	$\frac{2543}{1556} \approx 1.63$	63%
1950 to 2000	$\frac{6100}{2543} \approx 2.40$	140%

Since the growth rates vary widely, the Malthusian model, which assumes a constant growth rate, does not fit well and would be a very unreliable predictor.

PROBLEM SET 12.1

1. A population of 200 rodents is discovered living in the wall of your house. Despite your best efforts to eliminate the population, it grows at a rate of 95% each year. Fill in the following table to show the total number of rodents each year.

Time (in Years)	Total Number of Rodents
0	$(1 + 0.95)^0 \times 200 =$
1	$(1 + 0.95)^1 \times 200 =$
2	
3	
m	

2. The human population of a small town is initially 650. Suppose the population grows at a rate of 6% per year. Fill in the following table to show the total number of residents each year.

Time (in Years)	Total Number of Residents
0	$(1 + 0.06)^0 \times 650 =$
1	$(1 + 0.06)^1 \times 650 =$
2	
3	
m	

3. For a population of people living on an island, the number present after m years is given by $(1.45)^m \times 8500$. Find each of the following.
   a. The number of people initially on the island
   b. The annual growth rate
   c. The total island population after 2 years
   d. The total island population after 15 years

4. For a fast-growing population of mammals, the number present after m years is given by $(1.75)^m \times 50$. Find each of the following.
   a. The initial number of mammals
   b. The annual rate of growth
   c. The total mammal population after 3 years
   d. The total mammal population after 20 years

5. Suppose a population grows at the rate of 3% each year. If the initial population is 50,000, what is the population after 5 years? 20 years? 45 years?

6. Suppose a population grows at the rate of 4% each year. If the initial population is 75,000, what is the population after 12 years? 50 years? 100 years?

7. The population of the city of Limon, Costa Rica, grew from 168,000 in 1984 to 380,000 in 2004. Assuming a Malthusian population model, find the annual rate of growth.

8. The population of the city of Gifu, Japan, grew from 2,029,000 in 1985 to 2,112,100 in 2004. Assuming a Malthusian population model, find the annual rate of growth.

9. In 1995, two "sister" cities each had populations of approximately 35,000. Their annual growth rates differed, however. One city had an annual growth rate of 2%, while the other had an annual growth rate of 8%. Assuming that a Malthusian population model applies to these cities, predict the difference in their populations in 2005.

10. In the year 2000, city A had a population of 20,000 and city B had a population of 25,000. If their annual growth rates are 4.5% and 2.25%, respectively, in what year would you predict that the population of city A will exceed that of city B? Assume a Malthusian population model applies to these cities. Use trial and error.

11. In 1990, the population of the United States was approximately 249 million and was increasing at a rate of 0.7% per year. Assuming the growth rate remains the same, what is the anticipated size of the U.S. population in the years 2005 and 2010? Round to the nearest million.

12. In 2000, the population of the United States was approximately 281 million and was increasing at a rate of 1.1% per year. Assuming the growth rate remains the same, what is the anticipated size of the U.S. population in the years 2010 and 2015? Round to the nearest million.

13. In 2003, the population of Spain was approximately 40 million and was increasing at a rate of 0.16% per year.
   a. If the growth rate remains the same, predict the population of Spain in 2020. Round to the nearest thousand.
   b. If the growth rate from 2003 through the year 2010 is 0.16%, but the growth rate increases to 0.32% for the year 2011 and remains constant at 0.32% from 2011 through 2020, then predict the population in the year 2020. Round to the nearest thousand.

14. In 2000, the population of the United States was approximately 281 million and was increasing at a rate of 1.1% per year.
   a. If the growth rate remains the same, predict the population of the United States in 2020. Round to the nearest thousand.
   b. If the growth rate remains 1.1% through the year 2010, but decreases to 0.55% in 2011 and remains constant from 2011 to 2020, predict the population in the year 2020. Round to the nearest thousand.

15. In 1626, Peter Minuit of the Dutch East India Company purchased the island of Manhattan for the equivalent of $24 in trading goods. What would the value of these goods be in 2005 if their value had grown by a constant annual rate of 3% since that time? What would their value be if the annual rate of growth were 4%? Round to the nearest dollar.

16. Suppose you purchased a home in 1993 for $80,000. A review of real estate records for the past 10 years indicates that the average value of houses in your area has increased by an average of 4.5% per year. If this rate of growth continues, how much should the house be worth in 2007? Round to the nearest dollar.

17. The bacteria *E. coli* duplicates itself approximately every 20 minutes under ideal circumstances.

a. Fill in the following table by considering 20-minute intervals of time. Begin with one *E. coli* bacterium.

20-Minute Time Interval m	Total Number of *E. coli*
0	1
1	
2	
3	
4	

b. Use the formula for the growth rate to find the value of r in this case, where m is measured in 20-minute intervals of time.

c. Assuming that the rate of growth remains constant, write the Malthusian population growth formula of the form $P_m = (1 + r)^m \times P_0$. Use the rate you found in part (b) and let m represent the number of 20-minute time intervals.

d. Use your model from part (c) to find the total number of *E. coli* present after 4 hours and then after 1 day. Remember that time is measured in 20-minute intervals.

18. Under less-than-ideal conditions, such as lacking sufficient nutrients, *Nanobacterium sanguineum* can slow its growth rate so that it duplicates itself once every 6 days.

a. Fill in the following table by considering 6-day periods of time. Begin with one bacterium.

6-Day Time Interval m	Total Number of *N. sanguineum*
0	1
1	
2	
3	
4	

b. Use the formula for the growth rate to find the value of r in this case, where m is measured in 6-day intervals of time.

c. Assuming that the rate of growth remains constant, write the Malthusian population growth formula of the form $P_m = (1 + r)^m \times P_0$. Use the rate you found in part (b) and let m represent 6-day time intervals.

d. Use your model from part (c) to find the total number of *N. sanguineum* present after 33 days and then after 7 weeks. Remember that time is measured in 6-day intervals.

19. Your home appreciated from \$145,000 to \$215,000 in 5 years.

a. Find the annual rate of growth in the value of the house.

b. Assuming that a Malthusian population model applies to the value of your home, predict the value of your home after 5 more years.

20. A population of apes increased from 24 to 55 in 2 years.

a. Find the annual rate of growth in the ape population.

b. Assuming that a Malthusian population model applies to the ape population, predict the total number of apes after 2 more years.

21. The population of a city grew from 20,000 to 33,000 during the past 10 years. If growth continues at this rate, what will the population be in 20 more years?

22. The population of a given city grew from 35,000 to 42,000 during the past 15 years. If growth continues at this rate, what will the population be in 17 more years?

23. During the past 8 years, the population of Greenville grew from 28,000 to 35,500. If growth continues at this rate, what do you predict the population will be in 22 more years?

24. Between 1985 and 1993, the population of Braxton grew from 62,400 to 70,800. If this rate of growth continues, what do you predict the population will be in 2010?

25. According to Statistics Canada, the population estimates for Ontario in the years 1999 to 2002 were as follows: 11,527,900; 11,697,600; 11,894,900; and 12,068,300.

a. Calculate the annual rate of growth for each pair of consecutive years.

b. Can a Malthusian population model be assumed for this population? Explain why or why not.

26. According to Statistics Canada, the population estimates for British Columbia in the years 1999 to 2002 were as follows: 4,028,300; 4,060,100; 4,101,600; and 4,141,300.

a. Calculate the annual rate of growth for each pair of consecutive years.

b. Can a Malthusian population model be assumed for this population? Explain why or why not.

27. The world's population increased from about 4.4 billion in 1980 to about 6.3 billion in 2003.

a. Assuming the growth rate was constant, find the annual rate of growth for the world's population.

b. Use a Malthusian population model to predict the population of the world in the year 2010. Round to the nearest tenth of a billion.

c. There is concern that the world is overpopulated. Consider the total land area of Rhode Island, the smallest state, which is approximately 1045 square miles. If each of the world's 6.3 billion people stood on a 2-foot-by-2-foot square, how many square miles would be needed? (Recall that 5280 feet = 1 mile.) How many states the size of Rhode Island would this use?

28. In August 2004, the world's population was 6.38 billion. Assume the population grows at a constant annual rate of 1.13%.

a. Write the Malthusian population growth formula of the form $P_m = (1 + r)^m \times P_0$ that describes the world population growth.

b. The total land area of Alaska, the largest state, is approximately 586,412 square miles. If each person in the world stood on a 2-foot-by-2-foot square, in how many years would the world's population fill all the 2-foot-by-2-foot squares in Alaska? (Remember that 1 mile = 5280 feet.)

29. The current annual population growth rate in the United States is approximately 1.13%. If this rate remains constant, in how many years will the population of the United States double in size? Use trial and error.

30. Kenya, with an annual population growth rate of 4%, had the highest growth rate of any country in the world in 1994. Assuming that the growth rate remains constant, how long will it take Kenya's population to double? Use trial and error.

31. Suppose there are five investors in the first quarter of a Ponzi scheme. The investors are guaranteed a 40% rate of growth and time is measured in quarters of a year. What is the minimum number of people who must be investing in each of the next three quarters in order to pay off the investors from the previous quarter?

32. Suppose a Ponzi scheme has 80 investors during its first quarter. The investors are guaranteed a 40% rate of growth and time is measured in quarters of a year. What is the minimum number of people who must be investing in each quarter of the first 2 years in order to pay off the investors from the previous quarter?

33. If you started a Ponzi scheme like the one in the text (returning 40% in 90 days) with 10 investors, approximately how long would it take until every man, woman, and child in the United States would need to be an investor in order to keep the scheme going? Assume there are currently 290,000,000 people in the United States.

34. If you started a Ponzi scheme that guaranteed a return of 75% in 90 days, with 20 investors, approximately how long would it take until every man, woman, and child in the world would need to be an investor in order to keep the scheme going? Assume there are 6,280,000,000 people currently in the world.

35. Suppose a chain letter has 5 levels, asks you to send $2 to the person at the top of the list, and requires you to send out 10 new letters.

a. What is the payoff in rising from the bottom of the list to the top?

b. How many people must participate in order for an individual to rise from the bottom of the list to the top?

36. Suppose a chain letter has 6 levels, asks you to send $5 to the person at the top, and requires you to send out 4 new letters.

a. What is the payoff in rising from the bottom of the list to the top of the list?

b. How many people must participate in order for an individual to rise from the bottom of the list to the top?

37. Suppose a chain letter has 4 levels and requires you to send out 5 new letters. What are the payoffs in rising from the bottom of the list to the top of the list if you are required to pay the following amounts to the person at the top of the list?

a. \$1 **b.** \$5 **c.** \$10 **d.** \$50

38. Suppose a chain letter has 4 levels and requires you to pay \$5 to the person at the top of the list. What are the payoffs in rising from the bottom of the list to the top of the list if you are required to send out the following numbers of letters?

a. 1 **b.** 5 **c.** 10 **d.** 50

39. Suppose you want to create a chain letter that will give you a payoff of about \$90,000. You assume that you are more likely to encourage people to join if the cost is only \$3. Devise a scheme that will generate about \$90,000 for those who rise from the bottom of the list to the top.

40. Suppose you want to create a chain letter that will give you a payoff of about \$500,000. You assume that you are more likely to encourage people to join if they have to send out only 5 letters. Devise a scheme that will generate about \$500,000 for those who rise from the bottom of the list to the top.

Extended Problems

41. A sheet of paper is approximately 0.1 millimeter thick. If you fold the sheet of paper in half one time, you will have two layers, with a total thickness of approximately 0.2 millimeter. If you fold the paper in half again, you will have four layers, with a total thickness of approximately 0.4 millimeter. Create a table showing the thickness after each of the first 10 folds. If a piece of paper was big enough and you could fold it an unlimited number of times, how many folds would it take to create a thickness equal to 480 meters, which is the height of the Union Square Phase 7 building in Hong Kong?

42. The bacteria *E. coli* divides approximately every 20 minutes under ideal conditions. It measures about 2 micrometers in length and 0.8 micrometers in diameter, so it has a volume of about 1 cubic micrometer. A micrometer is one-millionth of a meter. Suppose one *E. coli* bacterium is present initially.

a. How many bacteria are present after 2 hours, and what is their total volume?

b. How many bacteria are present after 5 hours, and what is their total volume?

c. About how long would it take for the volume of cells to fill up a space equal to 1 cubic meter?

d. About how long would it take for the volume of cells to fill up a typical swimming pool, with a volume of 13,750 cubic meters?

43. *Nanobacterium sanguineum* is a unique bacteria species. It can slow or speed up its rate of growth depending on the conditions in which it finds itself. Research this interesting bacterium and write a short report. Be sure to include information about where this bacterium is found, what it does to its host, and what its strengths and weaknesses are. Include information about its different rates of growth. Create several tables to demonstrate growth patterns under different conditions and rates of growth, and explain what conditions lead to the different rates of growth. On the Internet, search keyword "Nanobacterium sanguineum."

44. The United States experienced an increase of approximately 33 million people between 1990 and 2000. However, population growth did not occur evenly throughout the country. Over the 10-year period from 1990 to 2000, the populations of some states grew much more rapidly than others. On the Internet, visit the U.S. Census Bureau website at www.census.gov, and find the populations for each state in 1990 and in 2000. Calculate the 10-year growth rate for each state. Which state(s) had the largest rates of growth? Which had the smallest rates of growth? Draw, or use the Internet to download, an outline map of the United States. Color groups of states with the greatest growth rates the same. Color groups of states with the smallest rates of growth the same. Summarize any patterns in growth rates you find in a short essay.

45. The prairie dog population has a low rate of growth compared to the rates for other mammals. A female prairie dog reproduces once a year beginning at the age of 2 years and generally has three to four pups. Typically, prairie dogs live about 4 years. Assume you begin with two prairie dogs: a 2-year-old male and a 2-year-old female. Create a table to show the total prairie dog population at the end of each year for the first 10 years. Assume that each litter contains two males and two females and that each prairie dog will die at the end of its fourth year. In the case of a female, assume that she has her litter of four pups before she dies in that fourth year. In your table, you will need to keep track of the age of each prairie dog to know when the females are able to have a litter, and when each prairie dog will die. Find the total population after 10 years and the annual rate of growth for this population of prairie dogs.

46. Research some get-rich-quick scams similar to the Ponzi scheme or the chain letters that were discussed in this section. There are many variations on these themes. On the Internet, search keywords "Ponzi schemes" or "chain letters" or "pyramid schemes." Write an essay that summarizes one of the scams, explain whether it is an example of Malthusian growth, and gives examples of situations in which it has been used.

12.2 Population Decrease, Radioactive Decay

INITIAL PROBLEM

The tranquilizer Librium (chlordiazepoxide HCl) has a half-life of between 24 and 48 hours. What is the hourly rate at which Librium leaves the bloodstream as the drug is metabolized by the body?

A solution of this Initial Problem is on page 776.

In the previous section, we developed a model to describe population growth and calculated rates of population growth. In this section, we will consider the possibility that a population is decreasing rather than increasing in size and modify the Malthusian population model to fit this situation. We will also see how to apply the same model to radioactive materials as they decay.

POPULATION DECLINE

Growing communities, such as Las Vegas, Nevada, have to plan ahead to make sure they have sufficient roadways, schools, utilities, and so on. On the other hand, some parts of the country suffer from a population decline due to factors such as a shortage of jobs. The next example shows how a community can make population projections to plan for a possible reduction in tax revenue, which would lead to a reduction in spending.

EXAMPLE 12.9 The population of Lake County, Oregon, was 7532 in the year 1980 and 7186 in the year 1990. Assume a Malthusian population model to do the following:

a. Determine the annual rate of decline in the Lake County population between 1980 and 1990.

b. Estimate the population of Lake County in the year 2000.