EXAMPLE 12.15 Archaeologists find bones from animals killed by ancient hunters at the site of an ancient village. Testing of the bones reveals that 35% of the original ^{14}C remains undecayed. Estimate the age of the bones and, hence, the age of the village.

SOLUTION From Table 12.6, we see that when 35% of the original ^{14}C remains, the sample is 1.51 half-lives old. Since the half-life of ^{14}C is 5730 years, we see that the age of the sample is $1.51 \times 5730 \approx 8652$ years. ■

SOLUTION OF THE INITIAL PROBLEM

The tranquilizer Librium (chlordiazepoxide HCl) has a half-life of between 24 and 48 hours. What is the hourly rate at which Librium leaves the bloodstream as the drug is metabolized by the body?

SOLUTION We apply the decay rate formula using the given half-life, but with the time units of years replaced by hours. First, we set $h = 24$ and find that the corresponding decay rate is

$$d = 1 - \left(\frac{1}{2}\right)^{\frac{1}{24}} \approx 0.028.$$

Then we set $h = 48$ and find that the corresponding decay rate is

$$d = 1 - \left(\frac{1}{2}\right)^{\frac{1}{48}} \approx 0.014.$$

We conclude that Librium leaves the bloodstream at a rate of 1.4% to 2.8% an hour.

PROBLEM SET 12.2

1. The population of Russia fell from about 148.7 million in 1992 to about 144 million in 2002. Assume a Malthusian population model.
   - **a.** Find the rate of decline for the population.
   - **b.** Predict the total population in the year 2003.
   - **c.** In the year 2003, the population of Russia was estimated to be about 142.9 million. How close was your predicted value to the actual value? What factors might cause the values to differ?

2. Potato blight was the name given to a fungus that destroyed potato crops in Ireland in the 1840s and caused widespread famine. In 1841, the population of Ireland was 8,175,124. By 1851, because of famine, death, disease, and emigration, the population had fallen to 6,552,385. Assume a Malthusian population model.
   - **a.** Find the rate of decline for the population.
   - **b.** Predict the total population in the year 1926.
   - **c.** The population of Ireland continued to decline for many years. The actual population in 1926 was about 4,228,553. How close was your predicted value to the actual value? What factors might cause the values to differ?

3. In 1978 there were approximately 30,000 black rhinoceroses in existence. By 1984, the population had dropped to 9000. Assume a Malthusian population model.
   - **a.** Find the rate of decline for the population.
   - **b.** Using the rate found in part (a), write the equation of the form $P_m = (1 + r)^m \times P_0$ that represents the black rhinoceros population after m years, where $m = 0$ in 1978. Predict the population in the years 2010 and 2020.
   - **c.** If the population continues to decline at the same rate, in approximately what year will there be only one black rhinoceros left? Use the guess-and-test method and your calculator.

4. In 1961, there were approximately 50,000 prairie chickens in New Mexico. By 1979, the population had dropped to about 10,000. Assume a Malthusian population model.

a. Find the rate of decline for the population.

b. Using the rate found in part (a), write the equation of the form $P_m = (1 + r)^m \times P_0$ that represents the prairie chicken population after m years, where $m = 0$ in 1961. Predict the population in the years 2010 and 2031.

c. If the population continues to decline at the same rate, in approximately what year will there be only one prairie chicken left? Use the guess-and-test method and your calculator.

5. Suppose a radioactive substance has an annual decay rate of 3%. Initially, there are 27 kilograms of the substance. Fill in the following table to show the amount of the substance left at the end of each year. Use the radioactive decay formula $A_m = (1' - d)^m \times A_0$.

Time in Years	Amount of Substance Present, in Kilograms
0	$(1 - 0.03)^0 \times 27 =$
1	$(1 - 0.03)^1 \times 27 =$
2	
3	
m	

6. Suppose a radioactive substance has an annual decay rate of 1.35%. Initially, there are 16 grams of the substance. Fill in the following table to show the amount of the substance left at the end of each year. Use the radioactive decay formula $A_m = (1 - d)^m \times A_0$.

Time in Years	Amount of Substance Present, in Grams
0	$(1 - 0.0135)^0 \times 16 =$
1	$(1 - 0.0135)^1 \times 16 =$
2	
3	
m	

7. If the amount, in grams, of a radioactive substance present after m years is given by $A_m = (0.955)^m \times 1400$, find each of the following.

a. How much of the radioactive substance was present initially?

b. What is the decay rate?

c. How much of the substance had decayed after 1 year?

d. How much of the substance remained after 20 years?

e. How much of the substance remained after 51 years?

8. If the amount, in grams, of a radioactive substance present after m years is given by $A_m = (0.85)^m \times 500$, find each of the following.

a. How much of the radioactive substance was present initially?

b. What is the decay rate?

c. How much of the substance had decayed after 1 year?

d. How much of the substance remained after 3 year?

e. How much of the substance remained after 4.265 years?

9. Suppose that a radioactive substance has an annual decay rate of 2.5%. If 300 grams of the substance are present initially, how much of the substance will remain after 5 years? 10 years? 15 years?

10. Suppose that a radioactive substance has an annual decay rate of 0.17%. If 5000 grams of the substance are present initially, how much of the substance will remain after 20 years? 100 years? 3500 years?

11. The half-life of strontium-90 is 28 years. What is the annual decay rate for strontium-90?

12. The half-life of uranium-233 is 160,000 years. What is the annual decay rate for uranium-233?

13. The half-life of plutonium-241 is 13 years. A sample contains 50 grams of the substance.

a. What is the annual decay rate of plutonium-241?

b. How much of the sample will remain after 6.5 years?

c. How much of the sample will remain after 13 years? 26 years? 39 years?

14. The half-life of radium-226 is 1620 years. A sample contains 2500 grams of the substance.

a. What is the annual decay rate of radium-226?

b. How much of the sample will remain after 25,920 years?

c. How much of the sample will remain after 810 years? 1620 years? 3240 years?

15. The half-life of sodium-22 is 2.6 years.
- **a.** What is the annual decay rate for sodium-22?
- **b.** If there are 200 grams of sodium-22 in the year 2000, how much will remain in the year 2035?

16. Suppose the half-life of a radioactive substance is 73 days.
- **a.** What is the annual decay rate for this substance?
- **b.** If 200 grams of the substance were available on January 1, 2001, how much would remain on December 3, 2001?

17. Suppose the half-life of a radioactive element is 18 minutes. If there are 10 grams of the substance initially, how much will be left 4 hours later?

18. The half-life of argon-41 is 1.8 hours. If 50 grams of the substance were available at noon, how much would remain at midnight?

19. Plutonium-241 has a half-life of 13 years. If a sample of 100 grams was produced in 1950, how much of the sample remained in 2003?

20. Suppose the half-life for a radioactive substance is 400 years. How much will remain after 2000 years if 100 grams were present initially?

Problems 21 through 24

Use the half-life radioactive decay formula

$$A_m = \left(\frac{1}{2}\right)^{\frac{m}{h}} \times A_0.$$

21. Suppose that initially 500 grams of a radioactive substance are contained in a sample, so $A_0 = 500$. For each of the following situations, calculate how much of the substance will remain after $m = 100$ years.
- **a.** Suppose the half-life is 100 years.
- **b.** Suppose the half-life is 50 years.
- **c.** Suppose the half-life is 10 years.
- **d.** Suppose the half-life is 1 year.

22. Suppose that initially 30 grams of a radioactive substance are contained in a sample, so $A_0 = 30$. For each of the following situations, calculate how much of the substance will remain after $m = 50$ days.
- **a.** Suppose the half-life is 100 days.
- **b.** Suppose the half-life is 50 days.
- **c.** Suppose the half-life is 10 days.
- **d.** Suppose the half-life is 1 day.

23. Suppose the half-life of a certain radioactive substance is 2 years, so $h = 2$. Initially there are $A_0 = 100$ grams of the substance. Calculate how much of the substance will remain after each of the following periods.
- **a.** 1 year
- **b.** 18 months
- **c.** 2 years
- **d.** 4 years
- **e.** 8 years

24. Suppose the half-life of a certain radioactive substance is 5000 years, so $h = 5000$. Initially there are $A_0 = 100$ grams of the substance. Calculate how much of the substance will remain after each of the following periods.
- **a.** 1 year
- **b.** 100 years
- **c.** 1000 years
- **d.** 10,000 years

Problems 25 and 26

Use the half-life approximation formula:

$$h \approx \frac{0.693}{d + \frac{d^2}{2}}.$$

25. A radioactive substance has an annual decay rate of 2.445%.
- **a.** What is the approximate half-life for this substance?
- **b.** Compare the half-life with those given in Table 12.5. Which element could this be?

26. A radioactive substance has an annual decay rate of 0.04278%.
- **a.** What is the approximate half-life for this substance?
- **b.** Compare the half-life with those given in Table 12.5. Which element could this be?

27. A fossil is tested for the level of ^{14}C. It is found that the fossil contains about 90% of its original amount. Estimate the age of the fossil.

28. If 500 milligrams of ^{14}C are present in a sample from a skull at the time of death, how many milligrams of ^{14}C would be present in the skull after each of the following periods?
- **a.** 5000 years
- **b.** 25,000 years
- **c.** 50,000 years

29. An archaeologist discovers a burial site that she believes to be 8000 years old. Examination of bones from the site shows that 40% of its ^{14}C is still present. Is the archeologist's belief reasonable? Justify your answer.

30. Charcoal from a suspected ancient campfire is tested and found to contain only 0.5% of the original amount of ^{14}C. Determine the age of the charcoal.

Extended Problems

31. Predictions about the future of the world's population have led to warnings about overpopulation and a lack of resources. Although the world's population is increasing, many individual countries are experiencing decreasing populations.

a. Use the Internet to research world population growth. List five countries with large rates of growth, and list several countries currently experiencing declining populations. Describe any geographic or economic similarities between the countries with fast-growing populations. What similarities can be found between countries with declining populations? Why might the countries with declining populations be concerned about their lack of growth? For information on the Internet, search keywords "world population growth."

b. Write an essay describing the factors often considered when researchers make predictions about the world's population. List several current predictions about the future of the world's population and explain why they differ. For information on the Internet use search keywords "world population predictions."

32. The Shroud of Turin is a piece of linen cloth that bears the negative image of the front and back of a man. The image is that of a man with a beard, long hair and a mustache, and wounds on his body, most of which are consistent with having been flogged and crucified. Many Christians believe that the cloth was the fabric in which Jesus was wrapped after his crucifixion. The cloth has been tested in an attempt to establish its age. Research the Shroud of Turin, and write a report about attempts to estimate its age. Include information about dating techniques that have been used and conclusions that have been drawn. On the Internet use search keywords "Shroud of Turin."

33. Carbon dating is only one of about 40 different radiometric methods used to date a sample. Research three of the other methods. Which methods are commonly used today? What assumptions must be made when relying on these methods? Which are thought to be the most accurate? According to these methods, how old is the earth? For information on the Internet use search keywords "radiometric dating techniques."

34. Techniques for dating fossils and bones are controversial. The use of carbon-14 to date organic objects requires several assumptions. If these assumptions cannot be made, then any dates obtained as a result of carbon-14 dating may not be accurate. Research carbon dating. Write a report that summarizes the following:

What items cannot be accurately dated using this technique? What are the ideal conditions under which carbon-14 dating can be used to estimate the age of a sample? What assumptions must be made in order to rely on the dates obtained using this method? On the Internet use search keywords "carbon dating controversy."

35. Roll a set of 30 dice and remove all the dice showing a 1. Record the number of dice you have left in the row for roll number 1 in the table. Roll the remaining dice, remove all 1s, and record the number of dice remaining in the row for roll number 2. Continue until you have only a few dice left.

Roll Number	Number of Dice Remaining
0	30
1	
2	
3	

a. Plot the data from the table. Put the roll number on the x-axis and the number of dice remaining on the y-axis. Sketch a smooth curve that seems to fit the trend in the scatter plot.

b. You can find the decay rate by using the radioactive decay formula. You will need to use the fact that you started with 30 dice and then select one other roll number and its corresponding number of dice remaining in the formula.

c. Locate on your graph the point at which about half of the original number of dice remain. After how many rolls did this occur? What do we call this?

36. The northern spotted owl was listed as a threatened species in 1990. Some experts speculate that the primary cause of the decline in the owl population is the loss of old-growth forest habitat. Research the northern spotted owl, and write a report to summarize your findings. On the Internet, search keywords "northern spotted owl." Be sure to include answers to the following questions.

a. What measures are being taken to protect the owl? Do they appear to be working?

b. What is the current rate of growth or decline for this population? What is the current estimated population of northern spotted owls? What was the approximate owl population in 1990, when the northern spotted owl was listed as a threatened species?

c. Use the current population estimate and the current growth rate to predict the northern spotted owl population for the next 10 years. Plot these values on a graph. What might you conclude about the future of the owl population? Use your graph to support your conclusion.

d. Describe the difference between a *threatened* species and an *endangered* species. Based on your research, do you think this population will be classified as endangered in the future? Explain.

12.3 Logistic Population Models

INITIAL PROBLEM

Consider the following list of world population figures (Table 12.7).

Table 12.7

Year	Population (in millions)
1700	579
1750	689
1800	909
1850	1086
1900	1556
1950	2543
2000	6100

Based on these data, does it appear that the world's population follows a logistic model?

A solution of this Initial Problem is on page 790.