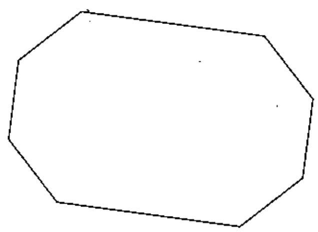
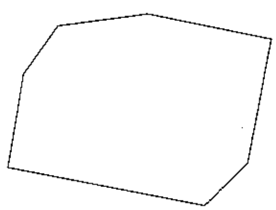
PROBLEM SET 2.1

1. **a.** Find the number of sides and the number of vertices, and give the name of the polygon.
   b. Copy the polygon, pick one vertex, and draw all the diagonals from that vertex. How many triangles are formed?
   c. What is the sum of the vertex angles in the polygon?



2. **a.** Find the number of sides and the number of vertices, and give the name of the polygon.
   b. Copy the polygon, pick one vertex, and draw all the diagonals from that vertex. How many triangles are formed?
   c. What is the sum of the vertex angles in the polygon?



3. Find the sum of the measures of the vertex angles in:
   a. A dodecagon (12-gon)
   b. A decagon (10-gon)
   c. 16-gon
   d. 24-gon

4. Find the sum of the measures of the vertex angles in:
   a. An icosagon (20-gon)
   b. A nonagon (9-gon)
   c. 18-gon
   d. 30-gon

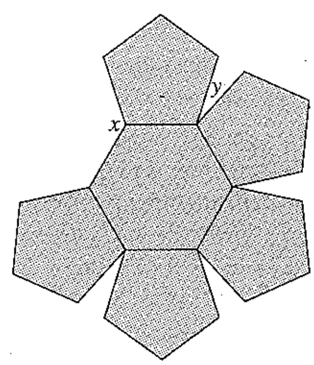
5. **a.** Find the measure of each vertex angle in a regular nonagon (9-gon).
   b. The sum of all but one of the n vertex angles of a regular n-gon is 3078°. Find the value of n and the measure of the remaining vertex angle.

6. **a.** Find the measure of each vertex angle of a regular dodecagon (12-gon).
   b. The sum of all but one of the n vertex angles of a regular n-gon is 5950°. Find the value of n and the measure of the remaining vertex angle.

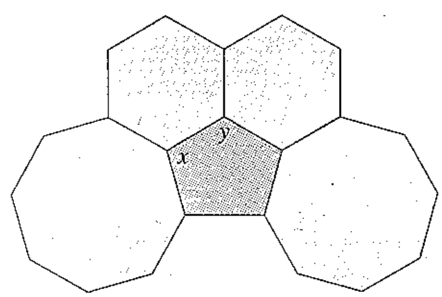
7. Is it possible to sketch a triangle with three congruent sides that is not a regular polygon? Explain.

8. Is it possible to sketch a triangle with three congruent angles that is not a regular polygon? Explain.

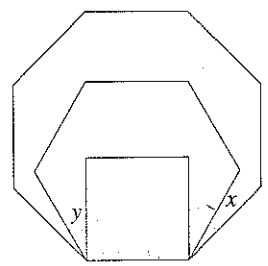
9. A soccer ball is made up of regular hexagons and regular pentagons. If the seams of a soccer ball are cut and the material is laid flat, the regular pentagons no longer meet at an edge. Find the degree measures x and y in the following figure. Explain why there are gaps in the figure.



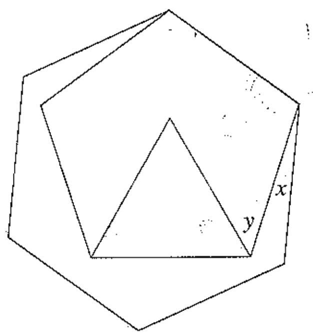
10. Find the degree measures x and y in the following figure, which is composed of regular octagons, regular hexagons, and a pentagon. Is the shaded pentagon a regular polygon? Explain.



11. The following figure is made up of regular polygons. Find the degree measures x and y.

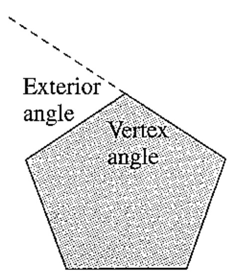


12. The following figure is made up of regular polygons. Find the degree measures x and y.



Problems 13 and 14

Another way to approach the question of the sum of the vertex angles of an n-gon is to think of beginning at one vertex and traveling around the perimeter of the polygon. When you come to the next vertex, you must make a turn of a certain number of degrees. If you extend the edge you just traveled, you will note that there are two angles formed with respect to the vertex, the side you extended, and the next side. One of these is the vertex angle, and the other we will call the **exterior angle** as shown next.



The measures of the vertex angle and the exterior angle add to 180°. Since there are n vertices, the sum of the measures of the vertex angles and exterior angles combined is $n(180°)$. As you travel around the perimeter of the polygon, you must turn 360°. Therefore, the sum of the measures of the vertex angles in an n-gon is found to be $n(180°) - 360° = n(180°) - 2(180°) = (n - 2)180°$. Since the sum of the measures of the n exterior angles of an n-gon is 360°, we have the following result for any regular n-gon: $n \times$ (measure of an exterior angle) = 360° or $n = \dfrac{360°}{\text{measure of an exterior angle}}$.

13. **a.** What is the measure of an exterior angle for a regular hexagon?

b. If the measure of each vertex angle in a regular polygon is 144°, how many sides does the polygon have?

c. How many sides does a regular polygon have if an exterior angle has a measure of 40°?

14. **a.** What is the measure of an exterior angle for a regular pentagon?

b. If the measure of each vertex angle in a regular polygon is 160°, how many sides does the polygon have?

c. How many sides does a regular polygon have if an exterior angle has a measure of 24°?

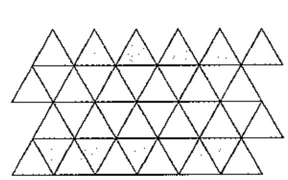
15. Refer to Figure 2.11(b) in which a regular tiling by triangles that is not edge-to-edge is constructed by sliding alternating rows of an edge-to-edge tiling to the right.

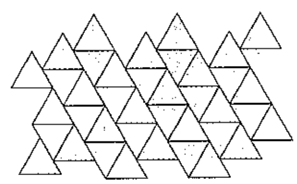
a. If arbitrary rows were shifted, would the result still be a regular tiling?

b. If the top vertex of each triangle were not in the middle of a side, would the result still be a regular tiling?

c. Explain your reasoning for parts (a) and (b).

16. Consider the edge-to-edge tiling from Figure 2.11(a). Slide alternating diagonal rows of triangles so that the vertex of each triangle is in the middle of a side of another triangle as shown.


Edge-to-Edge Tiling


Slide Diagonal Rows

a. Is the result of sliding the diagonal rows an edge-to-edge tiling?

b. Is this tiling different from the one in Figure 2.11(b)? Explain your answer. (*Hint:* Slowly rotate and observe the tiling for a full 360°. What do you notice?)

17. Explain why a regular edge-to-edge tiling cannot be composed of regular heptagons (7-gons).

18. Explain why a regular edge-to-edge tiling cannot be composed of regular octagons.

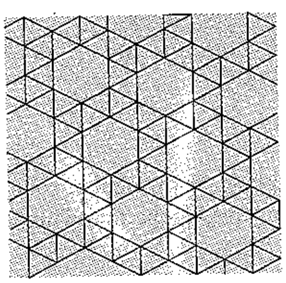
19. **a.** Show that an edge-to-edge semiregular tiling cannot be composed of regular pentagons and equilateral triangles that meet at a vertex.

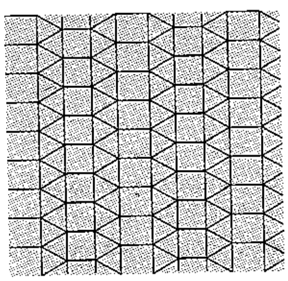
    b. Determine whether an edge-to-edge semiregular tiling can be made up of squares and regular octagons. If so, provide a sketch. If not, provide an explanation.

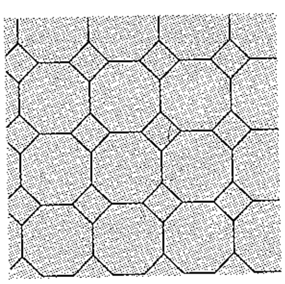
20. **a.** Show that an edge-to-edge semiregular tiling cannot be composed of regular hexagons and squares that meet at a vertex.

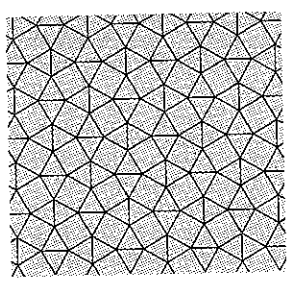
    b. Determine whether an edge-to-edge semiregular tiling can be made up of squares and regular pentagons. If so, provide a sketch. If not, provide an explanation.

21. Explain why each of the following tilings from Figure 2.14 is a semiregular tiling. For each tiling, sketch the vertex figure.

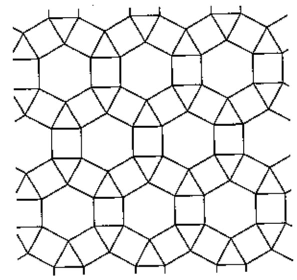

(a)

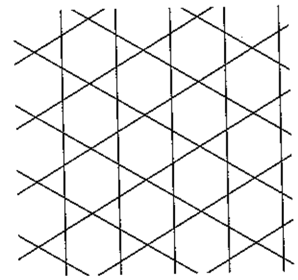

(b)

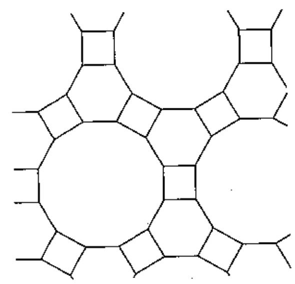

(c)

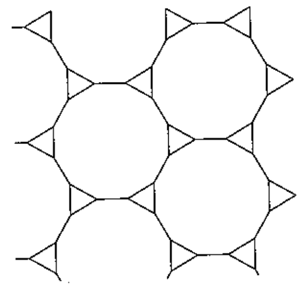

(d)

22. Explain why each of the following tilings from Figure 2.14 is a semiregular tiling. For each tiling, sketch the vertex figure.

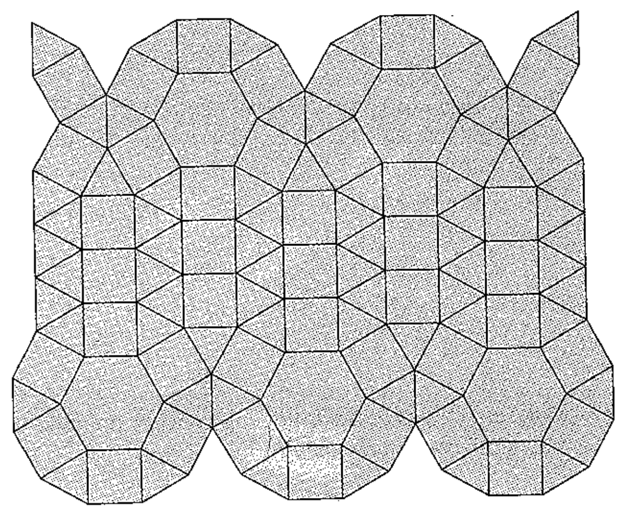

(a)


(b)

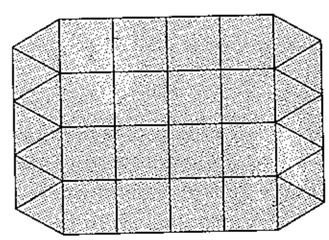

(c)


(d)

23. Determine whether the tiling in the following figure is a semiregular tiling. Give a reason for your answer.



24. Determine whether the tiling in the following figure is a semiregular tiling. Give a reason for your answer.

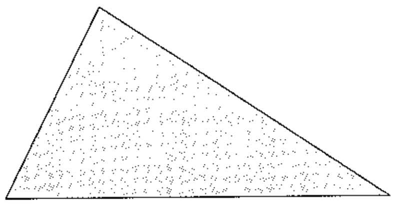


Problems 25 and 26

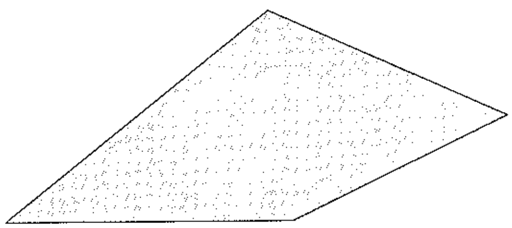
a. Any triangle can form a tiling. Make six copies of the given triangle, cut them out, and paste them on your paper to form a tiling.

b. Make four copies of the given convex quadrilateral, cut them out, and paste them on your paper around a point to demonstrate how they will form a tiling

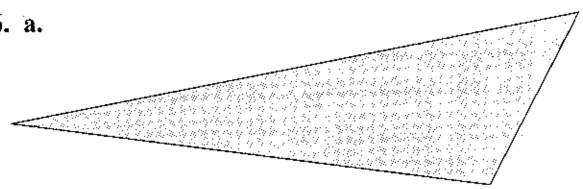
25. a.



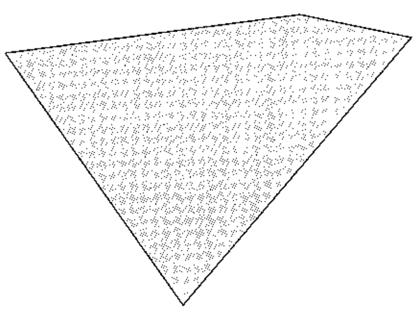
b.



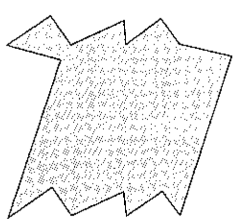
26. a.



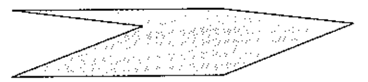
b.



27. Make six copies of the following concave polygon and demonstrate how they form a tiling.

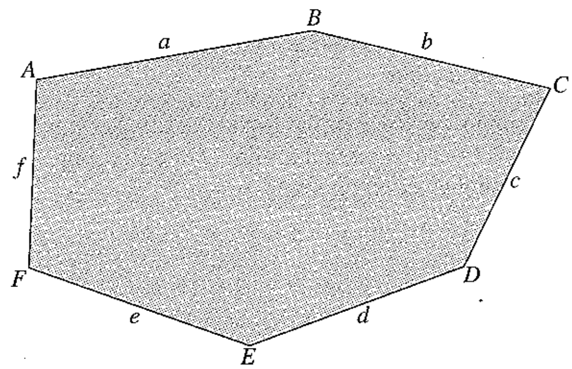


28. Make six copies of the following concave polygon and demonstrate how they form a tiling.



Problems 29 and 30

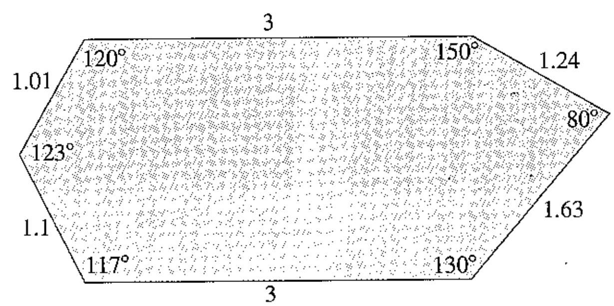
There are exactly three kinds of irregular, convex hexagons that tile the plane. Consider the following hexagon with vertex angles labeled with capital letters and side lengths labeled with lower case letters.



All regular hexagons will tile a plane. For irregular, convex hexagons to tile the plane, any one of the following sets of conditions must hold:

(I) $A + B + C = 360°$, and $f = c$
(II) $A + C + D = 360°$, $a = d$, and $b = f$
(III) $A = C = E = 120°$, $a = f$, $b = c$, and $e = d$

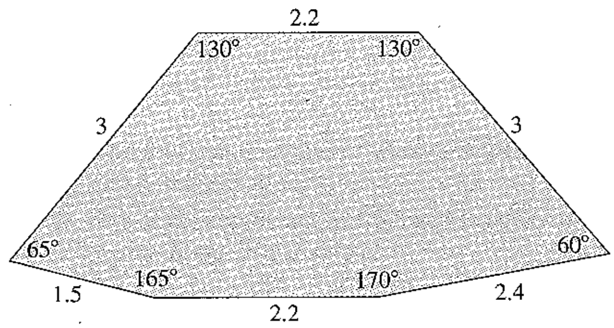
29. Consider the following convex, irregular hexagon.



a. Which set of conditions I, II, or III does the hexagon satisfy?

b. Copy, cut out, and paste at least six copies of the hexagon on your paper to form a tiling.

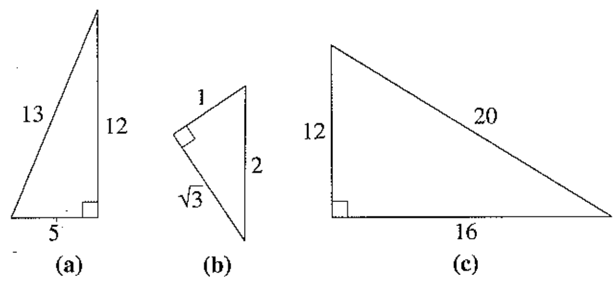
30. Consider the following convex, irregular hexagon.



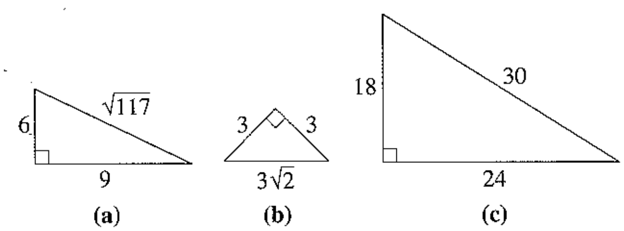
a. Which set of conditions I, II, or III does the hexagon satisfy?

b. Copy, cut out, and paste at least six copies of the hexagon on your paper to form a tiling.

31. Verify that the Pythagorean theorem is satisfied for each of the following right triangles.



32. Verify that the Pythagorean theorem is satisfied for each of the following right triangles.



33. Use the Pythagorean theorem to find the length of the hypotenuse of a right triangle whose sides have the following lengths.

a. 15, 20 **b.** 2, 7 **c.** 5, 5

34. Use the Pythagorean theorem to find the length of the hypotenuse of a right triangle whose sides have the following lengths.

a. 3, 8 **b.** 36, 48 **c.** 8, 8

35. Use the Pythagorean theorem to find the length of the third side of a right triangle if the hypotenuse and second side have the following lengths.

a. Hypotenuse = 35, Side = 28

b. Hypotenuse = 4, Side = 2

c. Hypotenuse = $\sqrt{113}$, Side = 7

36. Use the Pythagorean theorem to find the length of the third side of a right triangle if the hypotenuse and second side have the following lengths.

a. Hypotenuse = 45, Side = 27

b. Hypotenuse = 20, Side = 10

c. Hypotenuse = $\sqrt{84}$, Side = 9

37. Use the converse of the Pythagorean theorem to determine whether a triangle with the given side lengths is a right triangle.

a. 10, 24, 26

b. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

c. 6, 8, 12

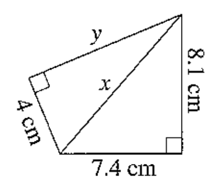
38. Use the converse of the Pythagorean theorem to determine whether a triangle with the given side lengths is a right triangle.

a. 10, 20, 30

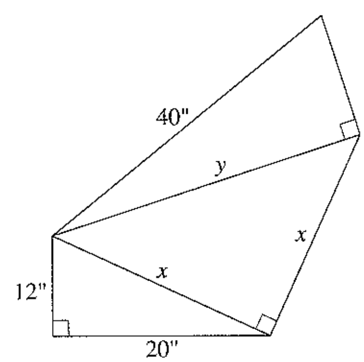
b. $\sqrt{7}$, $\sqrt{8}$, $\sqrt{56}$

c. 1, 5, $\sqrt{26}$

39. Find the missing lengths x and y in the figure. Round your answers to the nearest hundredth.



40. Find the missing lengths x and y in the figure. Round your answers to the nearest hundredth.



41. A baseball diamond is a square that measures 90 feet on a side. How far must the catcher throw the ball from home plate to second base to pick off a runner? Round your answer to the nearest hundredth of a foot.

42. A 90-foot-tall antenna on the flat roof of a building is to be secured with four cables. Each cable runs from the top of the antenna to a spot on the roof 30 feet from the base of the antenna. How much cable is needed? Round your answer to the nearest tenth of a foot.

43. A 16-foot ladder will be used to paint a house. If the foot of the ladder must be placed at least 4 feet away from the house to avoid flowers and shrubs, what is the highest point on the house that the top of the ladder will reach? Round your answer to the nearest tenth of a foot.

44. If the diagonals of a square are 40 feet long, what is the length of each side? Round your answer to the nearest tenth of a foot.

Extended Problems

45. We have seen that regular pentagons will not form a tiling. However, some irregular, convex pentagons will form a tiling. Currently, 14 general kinds of irregular, convex pentagons are known to tile the plane. Students, a physicist, and a homemaker discovered these tilings. Research tilings that involve irregular pentagons by using search keywords "pentagon tiling" on the Internet. List the angle and side length requirements for at least five of the pentagons that will form tilings and provide sketches of the tilings.

Problems 46 and 47

We saw that a vertex angle of a regular n-gon has measure $\frac{(n-2)180}{n}$ degrees. If three regular polygons surround a vertex, the three vertex angle measures must add to 360°. Therefore, if three regular polygons with a sides, b sides, and c sides surround a vertex we have the following.

$$\frac{(a-2)180}{a} + \frac{(b-2)180}{b} + \frac{(c-2)180}{c} = 360$$

Simplifying this equation gives $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$.

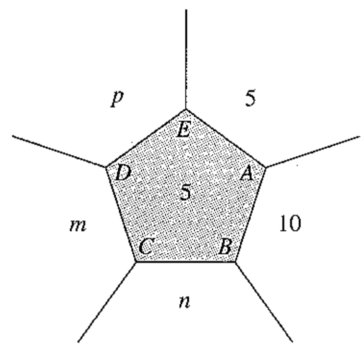
46. **a.** Suppose one of the polygons is an equilateral triangle, so $a = 3$. Find all values for b and c that satisfy the equation.

b. Suppose one of the polygons is a square, so $a = 4$. Find all values for b and c that satisfy the equation.

c. Suppose one of the polygons is a regular pentagon, so $a = 5$. Find all values for b and c that satisfy the equation.

d. Suppose one of the polygons is a regular hexagon, so $a = 6$. Find all values for b and c that satisfy the equation.

47. Consider the possibility of creating a semiregular tiling of three regular polygons such that $a = 5$, $b = 5$, and $c = 10$. Notice that $\frac{1}{5} + \frac{1}{5} + \frac{1}{10} = \frac{1}{2}$. Consider the following figure, where m, n, and p represent the number of sides in the polygon.



a. Point A is surrounded by two regular pentagons and a regular decagon. If point B is surrounded similarly, what is n?

b. If point C is surrounded similarly, what is m?

c. If point D is surrounded similarly, what is p?

d. What is the arrangement around E? What can you conclude?

e. For every triple of numbers a, b, and c that satisfies the equation $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$ in problem 46, determine whether a regular a-gon, a regular b-gon, and a regular c-gon will form a semiregular tiling.