

# Radical Expressions

A **square root** of a number  $n$  is a number  $m$  such that  $m^2 = n$ . For example,  $9^2 = 81$  and  $(-9)^2 = 81$ , so the square roots of 81 are 9 and  $-9$ .

Every positive number has two square roots, one positive and one negative. Negative numbers have no real square roots. The square root of zero is zero.

The radical symbol,  $\sqrt{\quad}$ , represents a nonnegative square root:  $\sqrt{81} = 9$ . The opposite of a square root is negative:  $-\sqrt{81} = -9$ .

A **perfect square** is a number that is the square of an integer. So, 81 is a perfect square. A **radicand** is a number or expression inside a radical symbol.

Properties of Radicals	Simplest Form of a Radical Expression
For $a \geq 0$ and $b \geq 0$ : $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$	<ul style="list-style-type: none"> <li>• No perfect square factors other than 1 in the radicand</li> <li>• No fractions in the radicand</li> <li>• No radical signs in the denominator of a fraction</li> </ul>

## EXAMPLE Simplify the expression.

a.  $\sqrt{9 + 36} = \sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

b.  $\sqrt{50} - \sqrt{32} = \sqrt{25 \cdot 2} - \sqrt{16 \cdot 2} = 5\sqrt{2} - 4\sqrt{2} = (5 - 4)\sqrt{2} = 1\sqrt{2} = \sqrt{2}$

c.  $\sqrt{18} \cdot \sqrt{72} = \sqrt{18 \cdot 72} = \sqrt{1296} = 36$

d.  $(8\sqrt{3})^2 = 8^2 \cdot (\sqrt{3})^2 = 64 \cdot 3 = 192$

e.  $\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6 \cdot \sqrt{2}}{(\sqrt{2})^2} = \frac{6 \cdot \sqrt{2}}{2} = 3\sqrt{2}$

f.  $\frac{\sqrt{20}}{\sqrt{500}} = \frac{\sqrt{20}}{\sqrt{500}} = \sqrt{\frac{20}{500}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$

## PRACTICE

Find all square roots of the number or write *no square roots*.

1. 100

2. 64

3.  $\frac{1}{4}$

4.  $\frac{9}{25}$

5.  $-16$

6. 0

7. 0.81

8. 0.0016

Simplify the expression.

9.  $\sqrt{121}$

10.  $-\sqrt{169}$

11.  $-\sqrt{99}$

12.  $\sqrt{48}$

13.  $\sqrt{16 + 4}$

14.  $\sqrt{(-4)^2 + 6^2}$

15.  $\sqrt{175} - \sqrt{28}$

16.  $\sqrt{32} + \sqrt{162}$

17.  $\sqrt{8} \cdot \sqrt{10}$

18.  $4\sqrt{6} \cdot 2\sqrt{15}$

19.  $\sqrt{210 \cdot 420}$

20.  $(9\sqrt{3})^2$

21.  $\sqrt{137} \cdot \sqrt{137}$

22.  $\sqrt{12} \cdot \sqrt{48}$

23.  $5\sqrt{18} \cdot \sqrt{2}$

24.  $3\sqrt{7} \cdot 5\sqrt{11}$

25.  $\frac{\sqrt{192}}{\sqrt{3}}$

26.  $\sqrt{\frac{2}{49}}$

27.  $\frac{12}{\sqrt{6}}$

28.  $\frac{2}{\sqrt{5}}$