6.2 Apply Properties of Rational Exponents

Before You simplified expressions involving integer exponents.
Now You will simplify expressions involving rational exponents.
Why? So you can find velocities, as in Ex. 84.

Key Vocabulary
- **simplest form of a radical**
- **like radicals**

The properties of integer exponents you learned in Lesson 5.1 can also be applied to rational exponents.

KEY CONCEPT *For Your Notebook*

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those listed on page 330, but now apply to rational exponents as illustrated.

Property	Example
1. $a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2 + 3/2)} = 5^2 = 25$
2. $(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
3. $(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2 - 1/2)} = 4^2 = 16$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

EXAMPLE 1 Use properties of exponents

Use the properties of rational exponents to simplify the expression.

a. $7^{1/4} \cdot 7^{1/2} = 7^{(1/4 + 1/2)} = 7^{3/4}$

b. $(6^{1/2} \cdot 4^{1/3})^2 = (6^{1/2})^2 \cdot (4^{1/3})^2 = 6^{(1/2 \cdot 2)} \cdot 4^{(1/3 \cdot 2)} = 6^1 \cdot 4^{2/3} = 6 \cdot 4^{2/3}$

c. $(4^5 \cdot 3^5)^{-1/5} = [(4 \cdot 3)^5]^{-1/5} = (12^5)^{-1/5} = 12^{[5 \cdot (-1/5)]} = 12^{-1} = \frac{1}{12}$

d. $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} = 5^{(1 - 1/3)} = 5^{2/3}$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\frac{42}{6}\right)^{1/3}\right]^2 = (7^{1/3})^2 = 7^{(1/3 \cdot 2)} = 7^{2/3}$

EXAMPLE 2 Apply properties of exponents

BIOLOGY A mammal's surface area S (in square centimeters) can be approximated by the model $S = km^{2/3}$ where m is the mass (in grams) of the mammal and k is a constant. The values of k for some mammals are shown below. Approximate the surface area of a rabbit that has a mass of 3.4 kilograms (3.4×10^3 grams).

Mammal	Sheep	Rabbit	Horse	Human	Monkey	Bat
k	8.4	9.75	10.0	11.0	11.8	57.5

Solution

$S = km^{2/3}$ — **Write model.**

$= 9.75(3.4 \times 10^3)^{2/3}$ — **Substitute 9.75 for k and 3.4×10^3 for m.**

$= 9.75(3.4)^{2/3}(10^3)^{2/3}$ — **Power of a product property**

$\approx 9.75(2.26)(10^2)$ — **Power of a power property**

≈ 2200 — **Simplify.**

▶ The rabbit's surface area is about 2200 square centimeters.

✓ GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

1. $(5^{1/3} \cdot 7^{1/4})^3$
2. $2^{3/4} \cdot 2^{1/2}$
3. $\dfrac{3}{3^{1/4}}$
4. $\left(\dfrac{20^{1/2}}{5^{1/2}}\right)^3$
5. **BIOLOGY** Use the information in Example 2 to approximate the surface area of a sheep that has a mass of 95 kilograms (9.5×10^4 grams).

PROPERTIES OF RADICALS The third and sixth properties on page 420 can be expressed using radical notation when $m = \frac{1}{n}$ for some integer n greater than 1.

KEY CONCEPT *For Your Notebook*

Properties of Radicals

Product property of radicals

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Quotient property of radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}},\ b \neq 0$$

EXAMPLE 3 Use properties of radicals

Use the properties of radicals to simplify the expression.

a. $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$ — **Product property**

b. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\dfrac{80}{5}} = \sqrt[4]{16} = 2$ — **Quotient property**

SIMPLEST FORM A radical with index n is in **simplest form** if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

EXAMPLE 4 Write radicals in simplest form

Write the expression in simplest form.

a. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5}$ **Factor out perfect cube.**

$= \sqrt[3]{27} \cdot \sqrt[3]{5}$ **Product property**

$= 3\sqrt[3]{5}$ **Simplify.**

REVIEW RADICALS
For help with rationalizing denominators of radical expressions, see p. 266.

b. $\dfrac{\sqrt[5]{7}}{\sqrt[5]{8}} = \dfrac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \dfrac{\sqrt[5]{4}}{\sqrt[5]{4}}$ **Make denominator a perfect fifth power.**

$= \dfrac{\sqrt[5]{28}}{\sqrt[5]{32}}$ **Product property**

$= \dfrac{\sqrt[5]{28}}{2}$ **Simplify.**

LIKE RADICALS Radical expressions with the same index and radicand are **like radicals.** To add or subtract like radicals, use the distributive property.

EXAMPLE 5 Add and subtract like radicals and roots

Simplify the expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10} = (1 + 7)\sqrt[4]{10} = 8\sqrt[4]{10}$

b. $2(8^{1/5}) + 10(8^{1/5}) = (2 + 10)(8^{1/5}) = 12(8^{1/5})$

c. $\sqrt[3]{\mathbf{54}} - \sqrt[3]{2} = \sqrt[3]{\mathbf{27}} \cdot \sqrt[3]{\mathbf{2}} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = (3 - 1)\sqrt[3]{2} = 2\sqrt[3]{2}$

✓ **GUIDED PRACTICE** for Examples 3, 4, and 5

Simplify the expression.

6. $\sqrt[4]{27} \cdot \sqrt[4]{3}$ **7.** $\dfrac{\sqrt[3]{250}}{\sqrt[3]{2}}$ **8.** $\sqrt[5]{\dfrac{3}{4}}$ **9.** $\sqrt[3]{5} + \sqrt[3]{40}$

VARIABLE EXPRESSIONS The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	Example
When n is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When n is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Absolute value is not needed when all variables are assumed to be positive.

EXAMPLE 6 Simplify expressions involving variables

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} = 4y^2$

b. $(27p^3q^{12})^{1/3} = 27^{1/3}(p^3)^{1/3}(q^{12})^{1/3} = 3p^{(3 \cdot 1/3)}q^{(12 \cdot 1/3)} = 3pq^4$

c. $\sqrt[4]{\frac{m^4}{n^8}} = \frac{\sqrt[4]{m^4}}{\sqrt[4]{n^8}} = \frac{\sqrt[4]{m^4}}{\sqrt[4]{(n^2)^4}} = \frac{m}{n^2}$

d. $\frac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$

EXAMPLE 7 Write variable expressions in simplest form

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^{10}b^4c^5}$ **Factor out perfect fifth powers.**

$= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4}$ **Product property**

$= ab^2c\sqrt[5]{4a^3b^4}$ **Simplify.**

AVOID ERRORS
You must multiply both the numerator *and* denominator of the fraction by *y* so that the value of the fraction does not change.

b. $\sqrt[3]{\frac{x}{y^8}} = \sqrt[3]{\frac{x \cdot y}{y^8 \cdot y}}$ **Make denominator a perfect cube.**

$= \sqrt[3]{\frac{xy}{y^9}}$ **Simplify.**

$= \frac{\sqrt[3]{xy}}{\sqrt[3]{y^9}}$ **Quotient property**

$= \frac{\sqrt[3]{xy}}{y^3}$ **Simplify.**

EXAMPLE 8 Add and subtract expressions involving variables

Perform the indicated operation. Assume all variables are positive.

a. $\frac{1}{5}\sqrt{w} + \frac{3}{5}\sqrt{w} = \left(\frac{1}{5} + \frac{3}{5}\right)\sqrt{w} = \frac{4}{5}\sqrt{w}$

b. $3xy^{1/4} - 8xy^{1/4} = (3 - 8)xy^{1/4} = -5xy^{1/4}$

c. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2} = 12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2} = (12z - 3z)\sqrt[3]{2z^2} = 9z\sqrt[3]{2z^2}$

✓ GUIDED PRACTICE for Examples 6, 7, and 8

Simplify the expression. Assume all variables are positive.

10. $\sqrt[3]{27q^9}$ **11.** $\sqrt[5]{\frac{x^{10}}{y^5}}$ **12.** $\frac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$ **13.** $\sqrt{9w^5} - w\sqrt{w^3}$

6.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS12 for Exs. 5, 27, and 85
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 51, 69, 86, and 89

SKILL PRACTICE

1. **VOCABULARY** Are $2\sqrt{5}$ and $2\sqrt[3]{5}$ like radicals? *Explain* why or why not.

2. ★ **WRITING** Under what conditions is a radical expression in simplest form?

EXAMPLE 1 on p. 420 for Exs. 3–14

PROPERTIES OF RATIONAL EXPONENTS Simplify the expression.

3. $5^{3/2} \cdot 5^{1/2}$
4. $(6^{2/3})^{1/2}$
5. $3^{1/4} \cdot 27^{1/4}$
6. $\frac{9}{9^{-4/5}}$
7. $\frac{80^{1/4}}{5^{-1/4}}$
8. $\left(\frac{7^3}{4^3}\right)^{-1/3}$
9. $\frac{11^{2/5}}{11^{4/5}}$
10. $(12^{3/5} \cdot 8^{3/5})^5$
11. $\frac{120^{-2/5} \cdot 120^{2/5}}{7^{-3/4}}$
12. $\frac{64^{5/9} \cdot 64^{2/9}}{4^{3/4}}$
13. $(16^{5/9} \cdot 5^{7/9})^{-3}$
14. $\frac{13^{3/7}}{13^{5/7}}$

EXAMPLE 3 on p. 421 for Exs. 15–22

PROPERTIES OF RADICALS Simplify the expression.

15. $\sqrt{20} \cdot \sqrt{5}$
16. $\sqrt[3]{16} \cdot \sqrt[3]{4}$
17. $\sqrt[4]{8} \cdot \sqrt[4]{8}$
18. $(\sqrt[3]{3} \cdot \sqrt[4]{3})^{12}$
19. $\frac{\sqrt[5]{64}}{\sqrt[5]{2}}$
20. $\frac{\sqrt{3}}{\sqrt{75}}$
21. $\frac{\sqrt[4]{36} \cdot \sqrt[4]{9}}{\sqrt[4]{4}}$
22. $\frac{\sqrt[4]{8} \cdot \sqrt[4]{16}}{\sqrt[8]{2} \cdot \sqrt[8]{3}}$

EXAMPLE 4 on p. 422 for Exs. 23–31

23. ★ **MULTIPLE CHOICE** What is the simplest form of the expression $3\sqrt[4]{32} \cdot (-6\sqrt[4]{5})$?

(A) $\sqrt[4]{10}$ (B) $-18\sqrt[4]{10}$ (C) $-36\sqrt[4]{10}$ (D) $36\sqrt[8]{10}$

SIMPLEST FORM Write the expression in simplest form.

24. $\sqrt{72}$
25. $\sqrt[6]{256}$
26. $\sqrt[3]{108} \cdot \sqrt[3]{4}$
27. $5\sqrt[4]{64} \cdot 2\sqrt[4]{8}$
28. $\sqrt[3]{\frac{1}{6}}$
29. $\frac{3}{\sqrt[4]{144}}$
30. $\sqrt[6]{\frac{81}{4}}$
31. $\frac{\sqrt[3]{9}}{\sqrt[5]{27}}$

EXAMPLE 5 on p. 422 for Exs. 32–41

COMBINING RADICALS AND ROOTS Simplify the expression.

32. $2\sqrt[6]{3} + 7\sqrt[6]{3}$
33. $\frac{3}{5}\sqrt[3]{5} - \frac{1}{5}\sqrt[3]{5}$
34. $25\sqrt[5]{2} - 15\sqrt[5]{2}$
35. $\frac{1}{8}\sqrt[4]{7} + \frac{3}{8}\sqrt[4]{7}$
36. $6\sqrt[3]{5} + 4\sqrt[3]{625}$
37. $-6\sqrt[7]{2} + 2\sqrt[7]{256}$
38. $12\sqrt[4]{2} - 7\sqrt[4]{512}$
39. $2\sqrt[4]{1250} - 8\sqrt[4]{32}$
40. $5\sqrt[3]{48} - \sqrt[3]{750}$

ERROR ANALYSIS *Describe* and correct the error in simplifying the expression.

41.
$$2\sqrt[3]{10} + 6\sqrt[3]{5} = (2 + 6)\sqrt[3]{15}$$
$$= 8\sqrt[3]{15}$$

42.
$$\sqrt[3]{\frac{x}{y^2}} = \sqrt[3]{\frac{x}{y^2 \cdot y}} = \sqrt[3]{\frac{x}{y^3}}$$
$$= \frac{\sqrt[3]{x}}{y}$$

EXAMPLE 6
on p. 423
for Exs. 43–51

VARIABLE EXPRESSIONS **Simplify the expression. Assume all variables are positive.**

43. $x^{1/4} \cdot x^{1/3}$ 44. $(y^4)^{1/6}$ 45. $\sqrt[4]{81x^4}$ 46. $\dfrac{2}{x^{-3/2}}$

47. $\dfrac{x^{2/5}y}{xy^{-1/3}}$ 48. $\sqrt[3]{\dfrac{x^{15}}{y^6}}$ 49. $\left(\sqrt[3]{x^2} \cdot \sqrt[6]{x^4}\right)^{-3}$ 50. $\dfrac{\sqrt[3]{x} \cdot \sqrt{x^5}}{\sqrt{25x^{16}}}$

51. ★ **OPEN-ENDED MATH** Write two variable expressions with noninteger exponents whose quotient is $x^{3/4}$.

EXAMPLE 7
on p. 423
for Exs. 52–59

SIMPLEST FORM **Write the expression in simplest form. Assume all variables are positive.**

52. $\sqrt{49x^5}$ 53. $\sqrt[4]{12x^2y^6z^{12}}$ 54. $\sqrt[3]{4x^3y^5} \cdot \sqrt[3]{12y^2}$ 55. $\sqrt{x^2yz^3} \cdot \sqrt{x^3z^5}$

56. $\dfrac{-3}{\sqrt[5]{x^6}}$ 57. $\sqrt[3]{\dfrac{x^3}{y^4}}$ 58. $\sqrt{\dfrac{20x^3y^2}{9xz^3}}$ 59. $\dfrac{\sqrt[4]{x^6}}{\sqrt[7]{x^5}}$

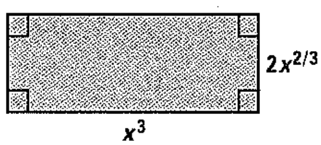
EXAMPLE 8
on p. 423
for Exs. 60–65

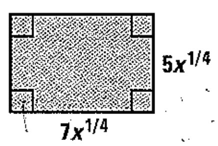
COMBINING VARIABLE EXPRESSIONS **Perform the indicated operation. Assume all variables are positive.**

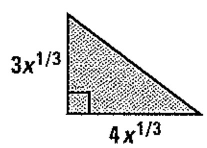
60. $3\sqrt[5]{x} + 9\sqrt[5]{x}$ 61. $\frac{3}{4}y^{3/2} - \frac{1}{4}y^{3/2}$ 62. $-7\sqrt[3]{y} + 16\sqrt[3]{y}$

63. $(x^4y)^{1/2} + (xy^{1/4})^2$ 64. $x\sqrt{9x^3} - 2\sqrt{x^5}$ 65. $y\sqrt[4]{32x^6} + \sqrt[4]{162x^2y^4}$

GEOMETRY **Find simplified expressions for the perimeter and area of the given figure.**

66.


67.


68.


69. ★ **MULTIPLE CHOICE** What is the simplified form of $-\frac{1}{6}\sqrt{4x} - \frac{1}{6}\sqrt{9x}$?

Ⓐ $-\frac{1}{3}\sqrt{x}$ Ⓑ $-\frac{1}{3}\sqrt{36x}$ Ⓒ $-\frac{5}{6}\sqrt{x}$ Ⓓ $-\frac{5}{6}\sqrt{36x}$

DECIMAL EXPONENTS **Simplify the expression. Assume all variables are positive.**

70. $x^{0.5} \cdot x^2$ 71. $y^{-0.6} \cdot y^{-6}$ 72. $(x^6y^2)^{-0.75}$ 73. $\dfrac{x^{0.3}}{x^{1.5}}$

74. $(x^5y^{-3})^{-0.25}$ 75. $\dfrac{y^{-0.5}}{y^{0.8}}$ 76. $10x^{0.6} + (4x^{0.3})^2$ 77. $15z^{0.3} - (2z^{0.1})^3$

IRRATIONAL EXPONENTS **The properties in this lesson can also be applied to irrational exponents. Simplify the expression. Assume all variables are positive.**

78. $\dfrac{x^{5\sqrt{3}}}{x^{2\sqrt{3}}}$ 79. $(x^{\sqrt{2}})^{\sqrt{3}}$ 80. $\left(\dfrac{x^{\pi}}{x^{\pi/3}}\right)^2$ 81. $x^2y^{\sqrt{2}} + 3x^2y^{\sqrt{2}}$

82. **CHALLENGE** Solve the equation using the properties of rational exponents.

a. $\dfrac{3}{9^x} = 243$ b. $2^x \cdot 2^{x+1} = \dfrac{1}{16}$ c. $(4^x)^{x+2} = 64$

PROBLEM SOLVING

EXAMPLE 2
on p. 421
for Exs. 83–84

83. BIOLOGY Look back at Example 2 on page 421. Use the model $S = km^{2/3}$ to approximate the surface area of the mammal given its mass.

a. Bat: 32 grams

b. Human: 59 kilograms

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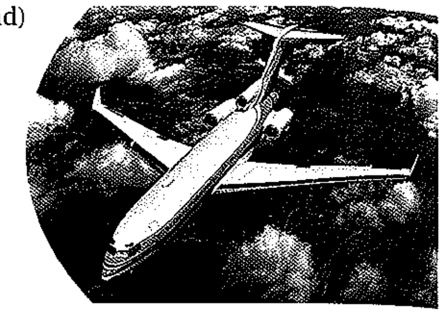
84. AIRPLANE VELOCITY The velocity v (in feet per second) of a jet can be approximated by the model

$$v = 8.8\sqrt{\frac{L}{A}}$$

where A is the area of the wings (in square feet) and L is the lift (in Newtons). Find the velocity of a jet with a wing area of 5.5×10^3 square feet and a lift of 1.4×10^7 Newtons.



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85. PINHOLE CAMERA The optimum diameter d (in millimeters) of the pinhole in a pinhole camera can be modeled by

$$d = 1.9\left[(5.5 \times 10^{-4})\ell\right]^{1/2}$$

where ℓ is the length of the camera box (in millimeters). Find the optimum pinhole diameter for a camera box with a length of 10 centimeters.

86. ★ SHORT RESPONSE Show that the hypotenuse of an isosceles right triangle with legs of length x is $x\sqrt{2}$.

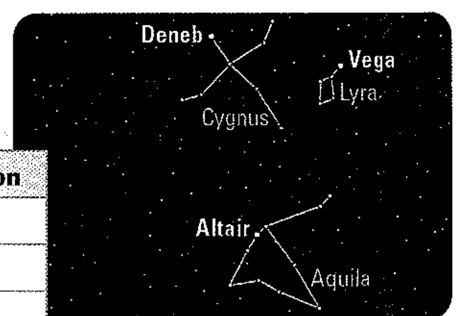
87. STAR MAGNITUDE The *apparent magnitude* of a star is a number that indicates how faint the star is in relation to other stars. The expression $\frac{2.512^{m_1}}{2.512^{m_2}}$ tells how many times fainter a star with magnitude m_1 is than a star with magnitude m_2.

a. How many times fainter is Altair than Vega?

b. How many times fainter is Deneb than Altair?

c. How many times fainter is Deneb than Vega?

Star	Apparent magnitude	Constellation
Vega	0.03	Lyra
Altair	0.77	Aquila
Deneb	1.25	Cygnus



88. PHYSICAL SCIENCE The maximum horizontal distance d that an object can travel when launched at an optimum angle of projection is given by

$$d = \frac{v_0\sqrt{(v_0)^2 + 2gh_0}}{g}$$

where h_0 is the object's initial height, v_0 is its initial speed, and g is the acceleration due to gravity. Simplify the model when $h_0 = 0$.

89. ★ **EXTENDED RESPONSE** You have filled two round balloons with water. One balloon contains twice as much water as the other balloon.

a. Solve the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, for r.

b. Substitute the expression for r from part (a) into the formula for the surface area of a sphere, $S = 4\pi r^2$. Simplify to show that $S = (4\pi)^{1/3}(3V)^{2/3}$.

c. *Compare* the surface areas of the two water balloons using the formula from part (b).

90. **CHALLENGE** Substitute different combinations of odd and even positive integers for m and n in the expression $\sqrt[n]{x^m}$. If x is not always positive, when is absolute value needed in simplifying the expression?

MIXED REVIEW

PREVIEW
Prepare for Lesson 6.3 in Exs. 101–106.

Solve the inequality.

91. $x - 7 \geq 15$ ***(p. 41)***
92. $10x + 7 < -4x + 9$ ***(p. 41)***
93. $3x \leq -6x - 20$ ***(p. 41)***
94. $x^2 + 7x + 10 > 0$ ***(p. 300)***
95. $-x^2 + 4x \geq -32$ ***(p. 300)***
96. $6x^2 + x - 7 < 5$ ***(p. 300)***

Let $f(x) = x^3 - 2x^2 - x - 3$. Evaluate the function at the given value. ***(p. 337)***

97. $f(3)$
98. $f(-3)$
99. $f(5)$
100. $f(-4)$

Perform the indicated operation.

101. $(12x^2 + 2x) - (-8x^3 + 5x^2 - 9x)$ ***(p. 346)***
102. $(35x^3 - 14) + (-15x^2 + 7x + 20)$ ***(p. 346)***
103. $18x^2(x + 4)$ ***(p. 346)***
104. $(8x - 3)^2$ ***(p. 346)***
105. $(x - 4)(x + 1)(x + 2)$ ***(p. 346)***
106. $(x^3 + x^2 - 7x - 15) \div (x - 3)$ ***(p. 362)***

QUIZ *for Lessons 6.1–6.2*

Evaluate the expression without using a calculator. ***(p. 414)***

1. $36^{3/2}$
2. $64^{-2/3}$
3. $-(625^{3/4})$
4. $(-32)^{2/5}$

Solve the equation. Round your answer to two decimal places when appropriate. ***(p. 414)***

5. $x^4 = 20$
6. $x^5 = -10$
7. $x^6 + 5 = 26$
8. $(x + 3)^3 = -16$

Simplify the expression. Assume all variables are positive. ***(p. 420)***

9. $\sqrt[4]{32} \cdot \sqrt[4]{8}$
10. $(\sqrt{10} \cdot \sqrt[3]{10})^8$
11. $(x^6y^4)^{1/8} + 2(x^{1/3}y^{1/4})^2$
12. $\dfrac{3\sqrt{7^3} + 4\sqrt{7^3}}{\sqrt{7^5}}$
13. $\dfrac{2\sqrt{x} \cdot \sqrt{x^3}}{\sqrt{64x^{15}}}$
14. $y^2\sqrt[5]{64x^6} - 6\sqrt[5]{2x^6y^{10}}$

15. **GEOMETRY** Find a radical expression for the perimeter of the red triangle inscribed in the square shown to the right. Simplify the expression. ***(p. 420)***