Finding Unknown Measures

If you know that two triangles are congruent, you can use information about one triangle to determine the measurements of the other triangle.

- $\Delta RAF \cong \Delta SYQ$

  Since we know that these are congruent triangles, we know that their corresponding parts are congruent.

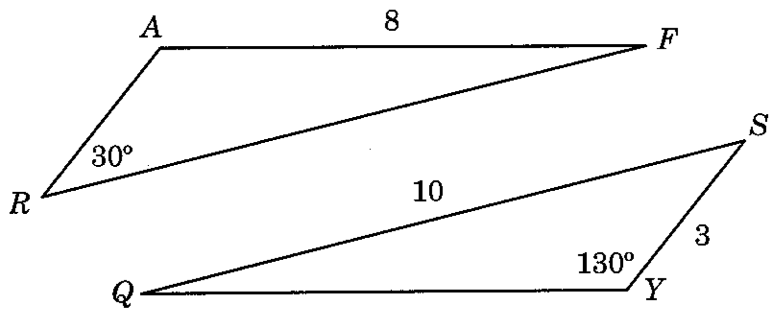
  $m\angle R = 30°$, so $m\angle S = 30°$

  $m\angle Y = 130°$, so $m\angle A = 130°$

  $AF = 8$, so $YQ = 8$

  $SY = 3$, so $RA = 3$

  $QS = 10$, so $FR = 10$

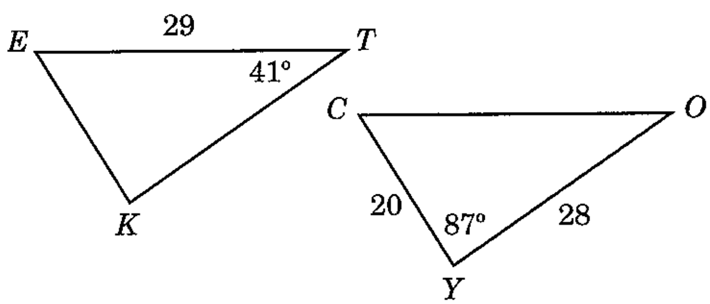


- Since the sum of the interior angles of a triangle is 180°, and since we know that two of the angles in these triangles are 30° and 130°, $m\angle F = m\angle Q = 180° - (130° + 30°) = 20°$.

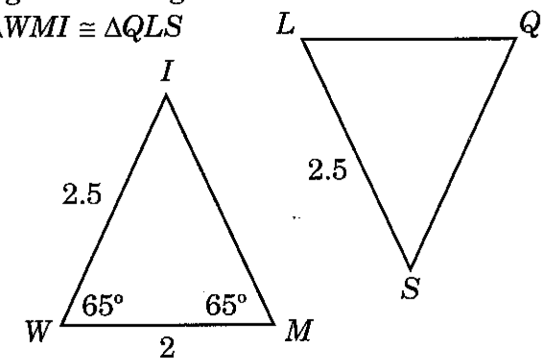
Practice

Find the missing measurements in each of these pairs of congruent triangles.

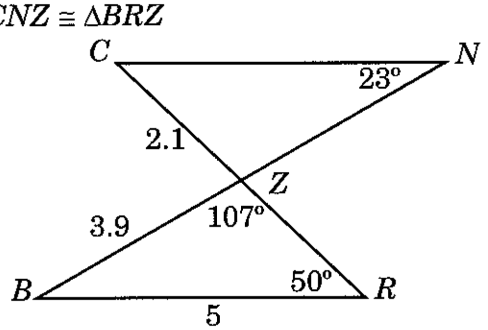
1. $\Delta KET \cong \Delta YCO$



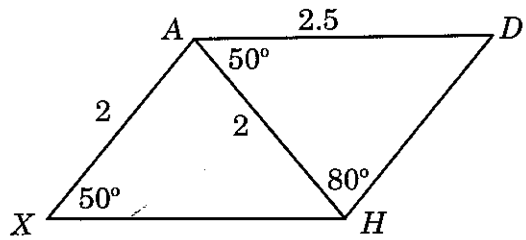
2. $\Delta WMI \cong \Delta QLS$



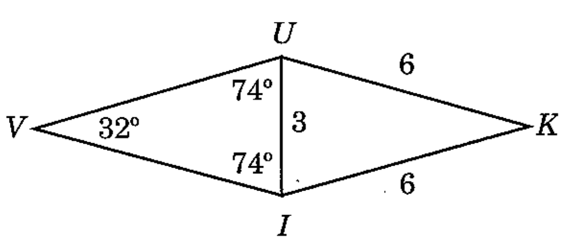
3. $\Delta CNZ \cong \Delta BRZ$



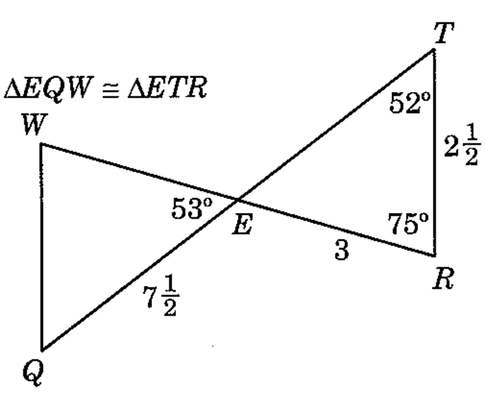
4. $\Delta DAH \cong \Delta XHA$



5. $\Delta VUI \cong \Delta KUI$



6. $\Delta EQW \cong \Delta ETR$

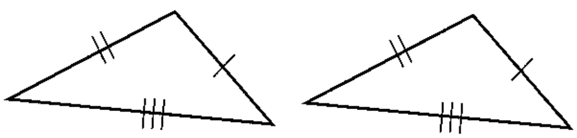


Proving Triangles Congruent 1

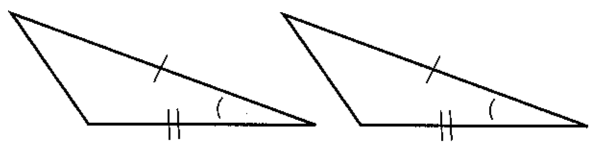
Information that can be used to prove that two triangles are congruent:

- **SSS or Side-Side-Side:** Two triangles are congruent if all 3 pairs of corresponding sides are congruent.
- **SAS or Side-Angle-Side:** Two triangles are congruent if two pairs of corresponding sides are congruent and the corresponding angles between the sides are congruent.
- **ASA or Angle-Side-Angle:** Two triangles are congruent if two pairs of corresponding angles are congruent and the corresponding sides between the angles are congruent.
- **AAS or Angle-Angle-Side:** Two triangles are congruent if two pairs of corresponding angles are congruent and a pair of corresponding sides that is not between the angles are congruent.
- **HL or Hypotenuse-Leg**: Two right triangles are congruent if they have a pair of congruent legs and their hypotenuses are congruent.

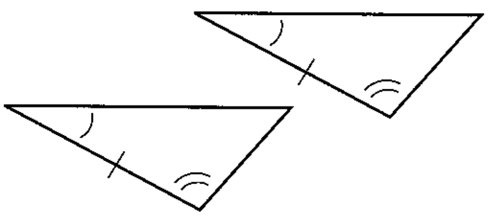
Examples of SSS, SAS, ASA, AAS, and HL



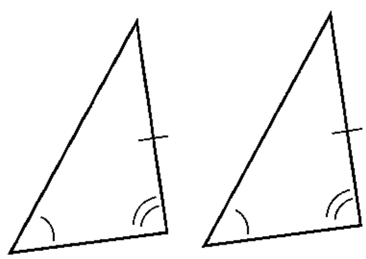
SSS



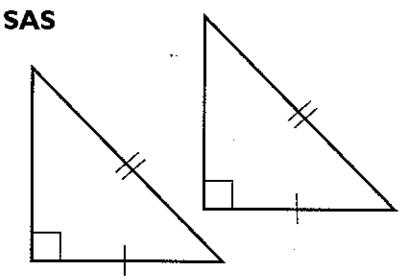
SAS



ASA



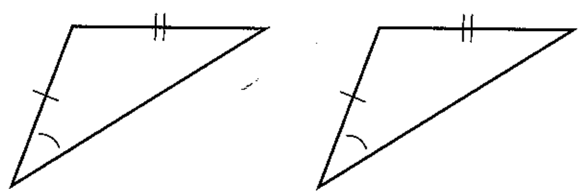
AAS



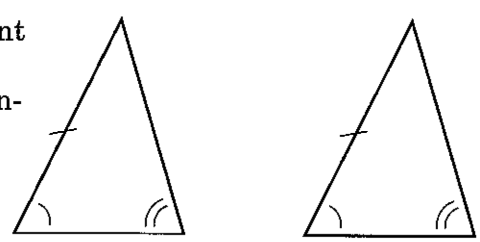
HL

Examples that often cause confusion:

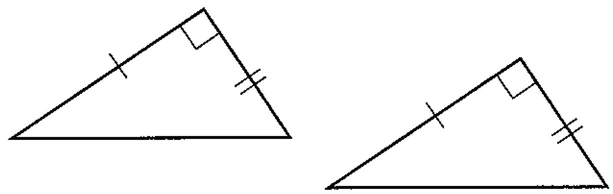
- This is not SAS because the corresponding congruent angles are not between the congruent sides.



- This is not ASA because the congruent sides are not between the congruent angles. It is not AAS because the congruent sides do not correspond (they come from different angles).



- This is not HL because the hypotenuse is not one of the congruent sides. It is SAS, however, because the angle between the congruent corresponding sides is a right angle, and therefore congruent from one triangle to the other.



Practice

State which method can be used to prove that the triangles are congruent, or state that they are not congruent. If they are congruent, write a congruency statement.

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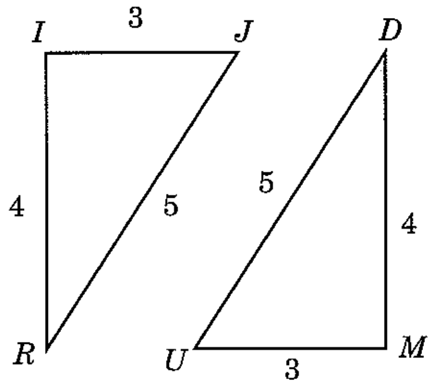

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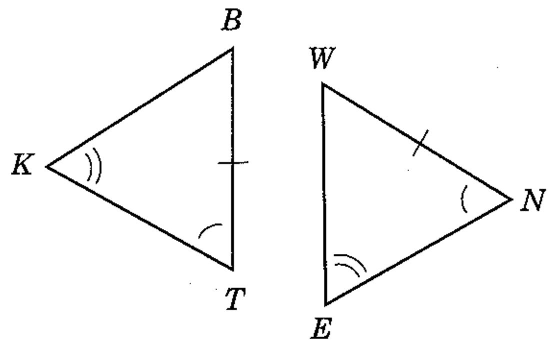

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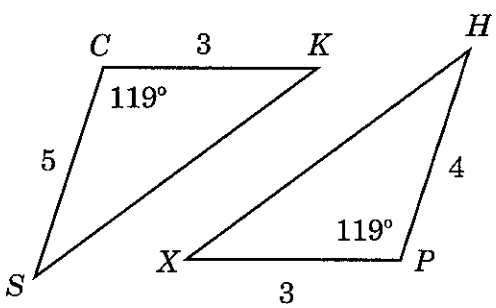

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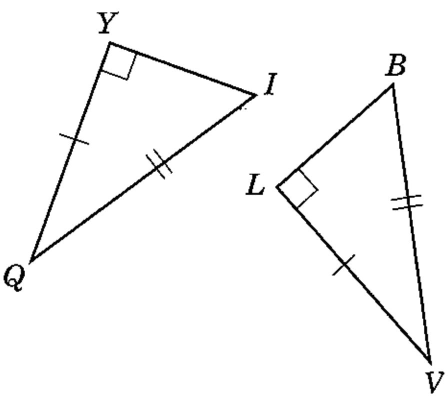

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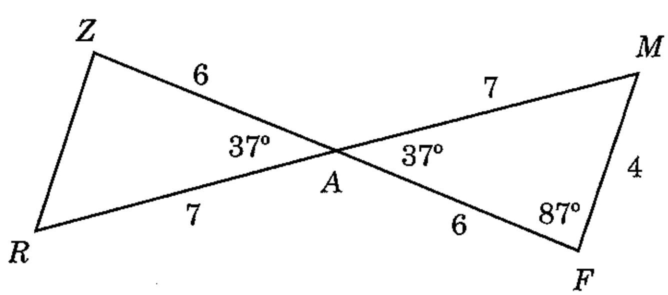

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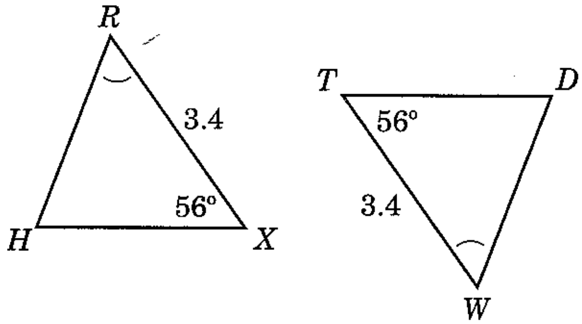
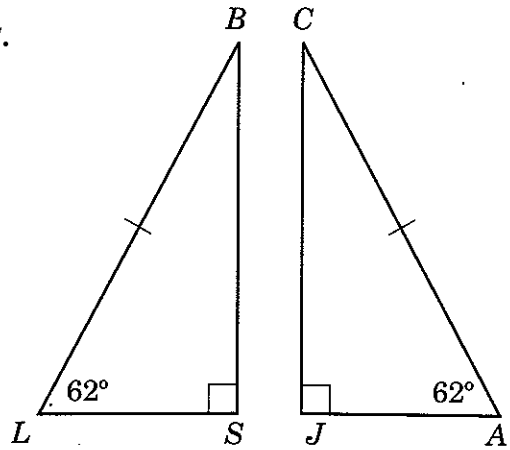
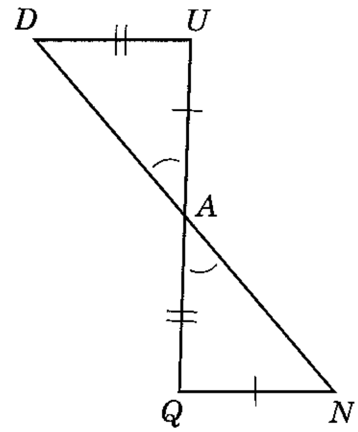


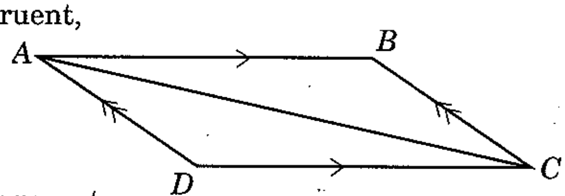
7. 

8. 

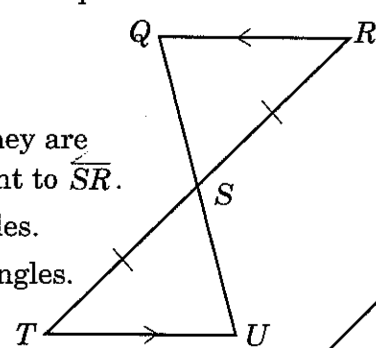
Proving Triangles Congruent 2

Sometimes you must determine which parts of the triangles are congruent based on other information in the picture before you can determine if the triangles are congruent.

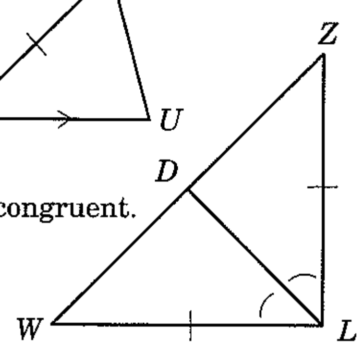
- There are three different ways to show that $\triangle BAC \cong \triangle DCA$.
  1. **SSS:** Opposite sides of a parallelogram are congruent, $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$. The two triangles share the diagonal, $\overline{AC}$, so that acts as a pair of congruent sides as well.



  2. **SAS:** In a parallelogram, opposite angles are congruent. Therefore, $\angle D \cong \angle B$. In #1 it was explained that $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$.
  3. **ASA:** Since $\overline{AC}$ is a transversal cutting two different pairs of parallel lines, it creates two sets of congruent alternate interior angles: $\angle DAC \cong \angle BCA$ and $\angle BAC \cong \angle DCA$. With all angles known to be congruent, you can pick which side from the explanation in #1 to complete the information.



- There are two different ways to show that $\triangle RQS \cong \triangle TUS$.
  1. **AAS:** $\angle QRS \cong \angle STU$ and $\angle RQS \cong \angle TUS$ because they are alternate interior angles. $\overline{TS}$ is shown to be congruent to $\overline{SR}$.
  2. **ASA:** $\angle QRS \cong \angle UST$ because they are vertical angles. $\angle QRS \cong \angle STU$ because they are alternate interior angles. $\overline{TS}$ is shown to be congruent to $\overline{SR}$.

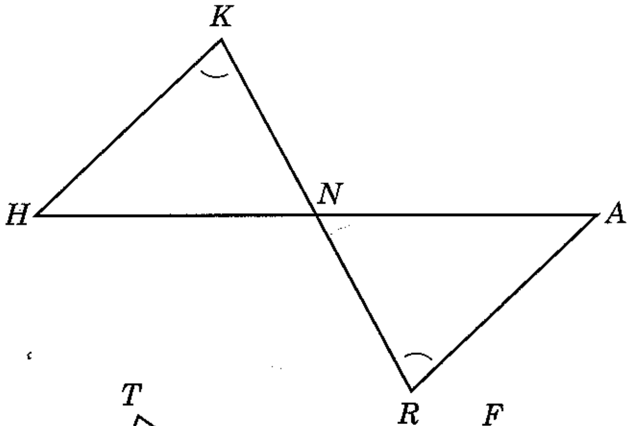


- There are two ways to prove that $\triangle ZLD \cong \triangle WLD$.
  1. **ASA:** $\triangle ZLW$ is an isosceles triangle, so its base angles are congruent. $\overline{WL} \cong \overline{ZL}$ (and $\angle DLW \cong \angle DLZ$ according to the picture).
  2. **SAS:** The two triangles share $\overline{DL}$. $\angle DLW \cong \angle DLZ$ (and $\overline{WL} \cong \overline{ZL}$ according to the picture).

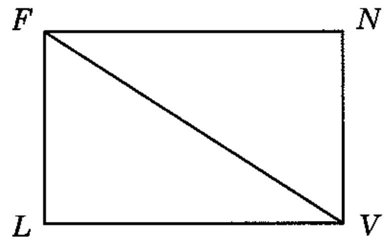
Practice

Explain whether or not each pair of triangles is congruent. Provide the method and write a congruency statement if they are congruent.

1. N is the midpoint of $\overline{AH}$.



2. $FNVL$ is a rectangle.



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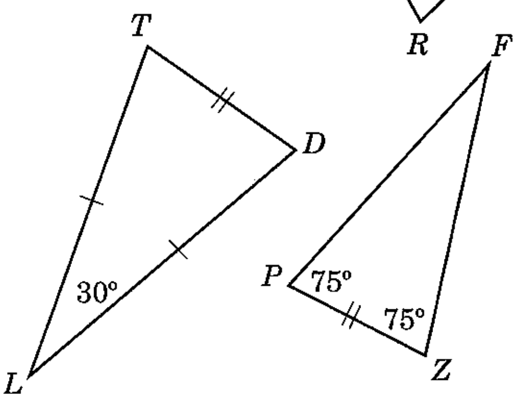

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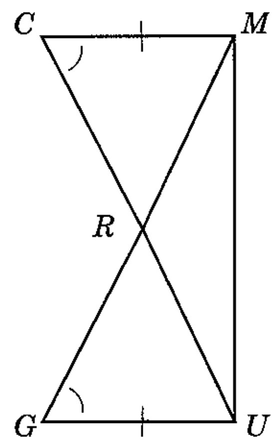

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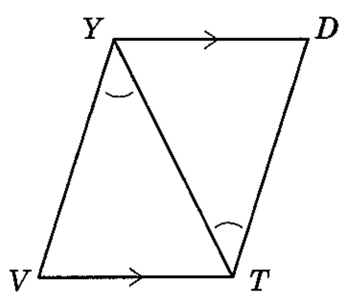

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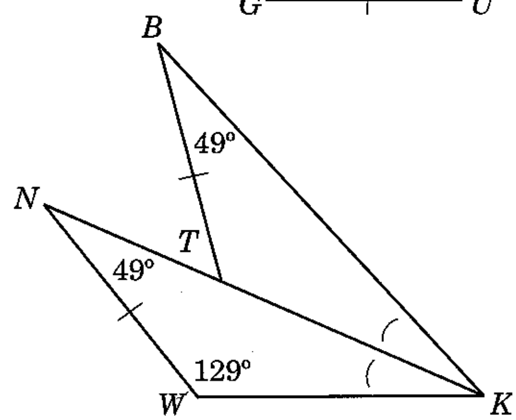

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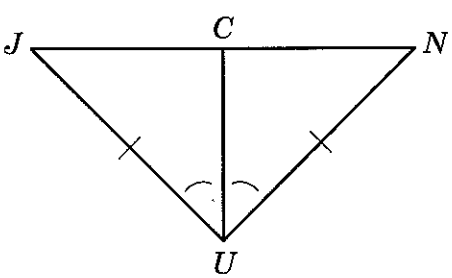

8. $EIOD$ is a square.
$EKVI$ is an isosceles trapezoid.