7.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS13 for Exs. 17, 29, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 24, 25, 32, 40, and 41

◆ = **MULTIPLE REPRESENTATIONS** Ex. 42

SKILL PRACTICE

1. **VOCABULARY** In the exponential growth model $y = 2.4(1.5)^x$, identify the initial amount, the growth factor, and the percent increase.

2. ★ **WRITING** What is an asymptote?

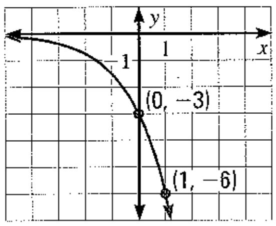
EXAMPLES 1 and 2 on pp. 478–479 for Exs. 3–14

MATCHING GRAPHS **Match the function with its graph.**

3. $y = 3 \cdot 2^x$

4. $y = -3 \cdot 2^x$

5. $y = 2 \cdot 3^x$

A.


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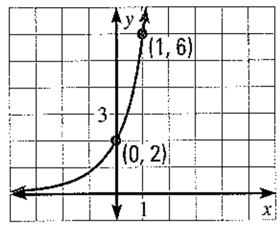

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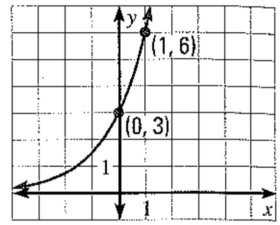


GRAPHING FUNCTIONS **Graph the function.**

6. $y = 3^x$

7. $y = -2^x$

8. $f(x) = 5 \cdot 2^x$

9. $y = 5^x$

10. $y = 2 \cdot 4^x$

11. $g(x) = -(1.5)^x$

12. $y = 3\left(\frac{4}{3}\right)^x$

13. $y = \frac{1}{2} \cdot 3^x$

14. $h(x) = -2(2.5)^x$

EXAMPLE 3 on p. 479 for Exs. 15–24

TRANSLATING GRAPHS **Graph the function. State the domain and range.**

15. $y = -3 \cdot 2^{x+2}$

16. $y = 5 \cdot 4^x + 2$

17. $y = 2^{x+1} + 3$

18. $y = 3^{x-2} - 1$

19. $y = 2 \cdot 3^{x-2} - 1$

20. $y = -3 \cdot 4^{x-1} - 2$

21. $f(x) = 6 \cdot 2^{x-3} + 3$

22. $g(x) = 5 \cdot 3^{x+2} - 4$

23. $h(x) = -2 \cdot 5^{x-1} + 1$

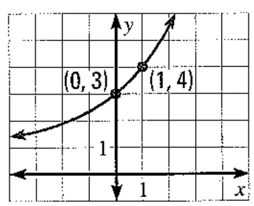
24. ★ **MULTIPLE CHOICE** The graph of which function is shown?

Ⓐ $f(x) = 2(1.5)^x - 1$

Ⓑ $f(x) = 2(1.5)^x + 1$

Ⓒ $f(x) = 3(1.5)^x - 1$

Ⓓ $f(x) = 3(1.5)^x + 1$



25. ★ **MULTIPLE CHOICE** The student enrollment E of a high school was 1310 in 1998 and has increased by 10% per year since then. Which exponential growth model gives the school's student enrollment in terms of t, where t is the number of years since 1998?

Ⓐ $E = 0.1(1310)^t$

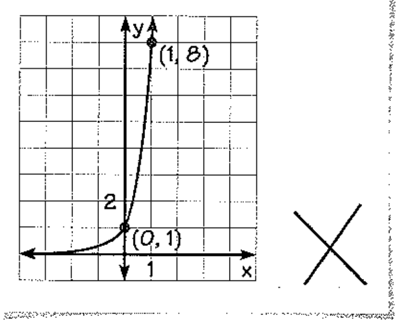
Ⓑ $E = 1310(0.1)^t$

Ⓒ $E = 1.1(1310)^t$

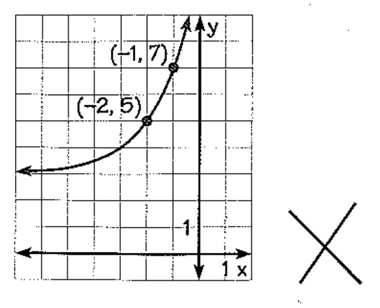
Ⓓ $E = 1310(1.1)^t$

ERROR ANALYSIS *Describe* **and correct the error in graphing the function.**

26. $y = 2 \cdot 4^x$



27. $y = 2^{x-3} + 3$



WRITING MODELS In Exercises 28–30, write an exponential growth model that describes the situation.

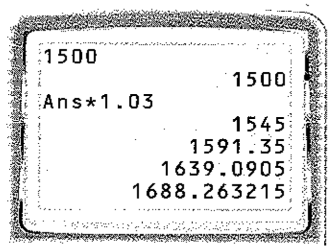
28. In 1992, 1219 monk parakeets were observed in the United States. For the next 11 years, about 12% more parakeets were observed each year.

29. You deposit \$800 in an account that pays 2% annual interest compounded daily.

30. You purchase an antique table for \$450. The value of the table increases by 6% per year.

31. **GRAPHING CALCULATOR** You deposit \$1500 in a bank account that pays 3% annual interest compounded yearly.

a. Type 1500 into a graphing calculator and press ENTER. Then enter the formula ANS * 1.03, as shown at the right. Press ENTER seven times to find your balance after 7 years.



b. Find the number of years it takes for your balance to exceed \$2500.

32. ★ **OPEN-ENDED MATH** Write an exponential function of the form $y = ab^{x-h} + k$ whose graph has a y-intercept of 5 and an asymptote of $y = 2$.

33. **GRAPHING CALCULATOR** Consider the exponential growth function $y = ab^{x-h} + k$ where $a = 2$, $b = 5$, $h = -4$, and $k = 3$. Predict the effect on the function's graph of each change in a, b, h, or k described in parts (a)–(d). Use a graphing calculator to check your prediction.

a. a changes to 1 **b.** b changes to 4 **c.** h changes to 3 **d.** k changes to −1

34. **CHALLENGE** Consider the exponential function $f(x) = ab^x$.

a. Show that $\frac{f(x+1)}{f(x)} = b$.

b. Use the result from part (a) to explain why there is no exponential function of the form $f(x) = ab^x$ whose graph passes through the points in the table below.

x	0	1	2	3	4
y	4	4	8	24	72

EXAMPLE 4
on p. 480
for Exs. 35–36

35. DVD PLAYERS From 1997 to 2002, the number n (in millions) of DVD players sold in the United States can be modeled by $n = 0.42(2.47)^t$ where t is the number of years since 1997.

a. Identify the initial amount, the growth factor, and the annual percent increase.

b. Graph the function. Estimate the number of DVD players sold in 2001.

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36. INTERNET Each March from 1998 to 2003, a website recorded the number y of referrals it received from Internet search engines. The results can be modeled by $y = 2500(1.50)^t$ where t is the number of years since 1998.

a. Identify the initial amount, the growth factor, and the annual percent increase.

b. Graph the function and state the domain and range. Estimate the number of referrals the website received from Internet search engines in March of 2002.

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EXAMPLE 5
on p. 481
for Exs. 37–38

(37.) ACCOUNT BALANCE You deposit \$2200 in a bank account. Find the balance after 4 years for each of the situations described below.

a. The account pays 3% annual interest compounded quarterly.

b. The account pays 2.25% annual interest compounded monthly.

c. The account pays 2% annual interest compounded daily.

38. DEPOSITING FUNDS You want to have \$3000 in your savings account after 3 years. Find the amount you should deposit for each of the situations described below.

a. The account pays 2.25% annual interest compounded quarterly.

b. The account pays 3.5% annual interest compounded monthly.

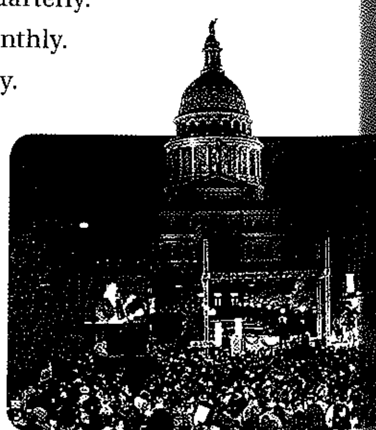
c. The account pays 4% annual interest compounded yearly.

39. MULTI-STEP PROBLEM In 1990, the population of Austin, Texas, was 494,290. During the next 10 years, the population increased by about 3% each year.

a. Write a model giving the population P (in thousands) of Austin t years after 1990. What was the population in 2000?

b. Graph the model and state the domain and range.

c. Estimate the year when the population was about 590,000.



Austin, Texas

40. ★ SHORT RESPONSE At an online auction, the opening bid for a pair of in-line skates is \$50. The price of the skates increases by 10.5% per bid during the next 6 bids.

a. Write a model giving the price p (in dollars) of the skates after n bids.

b. What was the price after 5 bids? According to the model, what will the price be after 100 bids? Is this predicted price reasonable? *Explain.*

○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

◆ = MULTIPLE REPRESENTATIONS

41. ★ **EXTENDED RESPONSE** In 2000, the average price of a football ticket for a Minnesota Vikings game was \$48.28. During the next 4 years, the price increased an average of 6% each year.

a. Write a model giving the average price p (in dollars) of a ticket t years after 2000.

b. Graph the model. Estimate the year when the average price of a ticket was about \$60.

c. *Explain* how you can use the graph of $p(t)$ to determine the minimum and maximum t-values in the domain for which the function gives meaningful results.

42. **MULTIPLE REPRESENTATIONS** In 1977, there were 41 breeding pairs of bald eagles in Maryland. Over the next 24 years, the number of breeding pairs increased by about 8.9% each year.

a. **Writing an Equation** Write a model giving the number n of breeding pairs of bald eagles in Maryland t years after 1977.

b. **Making a Table** Make a table of values for the model.

c. **Drawing a Graph** Graph the model.

d. **Using a Graph** About how many breeding pairs of bald eagles were in Maryland in 2001?

43. **REASONING** Is investing \$3000 at 6% annual interest and \$3000 at 8% annual interest equivalent to investing \$6000 (the total of the two principals) at 7% annual interest (the average of the two interest rates)? *Explain.*

44. **CHALLENGE** The yearly cost for residents to attend a state university has increased from \$5200 to \$9000 in the last 5 years.

a. To the nearest tenth of a percent, what has been the average annual growth rate in cost?

b. If this growth rate continues, what will the cost be in 5 more years?

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.2 in Exs. 45–52.

Evaluate the power.

45. $(0.6)^3$ *(p. 10)*
46. $(0.4)^2$ *(p. 10)*
47. $(0.5)^5$ *(p. 10)*
48. $(0.25)^3$ *(p. 10)*
49. $\left(\frac{1}{2}\right)^4$ *(p. 330)*
50. $\left(\frac{3}{8}\right)^3$ *(p. 330)*
51. $\left(\frac{7}{10}\right)^5$ *(p. 330)*
52. $\left(\frac{4}{5}\right)^3$ *(p. 330)*

Factor the expression.

53. $x^2 + 7x - 30$ *(p. 252)*
54. $x^2 + 15x + 54$ *(p. 252)*
55. $2x^2 - 7x - 30$ *(p. 259)*
56. $12x^2 - 5x + 25$ *(p. 259)*
57. $x^3 - 2x^2 - 3x + 6$ *(p. 353)*
58. $x^3 - 64$ *(p. 353)*

Solve the equation. *(p. 414)*

59. $x^5 = 3125$
60. $3x^3 = 1029$
61. $x^7 + 8 = -64$
62. $(x + 12)^4 = 52$
63. $-5x^6 = -1000$
64. $(x - 9)^8 = 17$

7.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS13 for Exs. 9, 19, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 27, 28, 33, and 35

SKILL PRACTICE

1. **VOCABULARY** In the exponential decay model $y = 1250(0.85)^t$, identify the initial amount, the decay factor, and the percent decrease.

2. ★ **WRITING** *Explain* how to tell whether the function $y = b^x$ represents exponential growth or exponential decay.

CLASSIFYING FUNCTIONS Tell whether the function represents *exponential growth* or *exponential decay*.

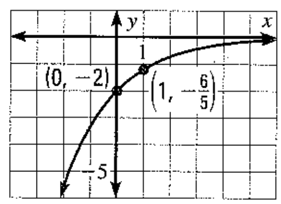
3. $f(x) = 3\left(\frac{3}{4}\right)^x$
4. $f(x) = 4\left(\frac{5}{2}\right)^x$
5. $f(x) = \frac{2}{7} \cdot 4^x$
6. $f(x) = 25(0.25)^x$

EXAMPLES 1 and 2 on pp. 486–487 for Exs. 7–15

GRAPHING FUNCTIONS Graph the function.

7. $y = \left(\frac{1}{4}\right)^x$
8. $y = \left(\frac{1}{3}\right)^x$
9. $f(x) = 2\left(\frac{1}{5}\right)^x$
10. $y = -(0.2)^x$
11. $y = -4\left(\frac{1}{3}\right)^x$
12. $g(x) = 2(0.75)^x$
13. $y = \left(\frac{3}{5}\right)^x$
14. $h(x) = -3\left(\frac{3}{8}\right)^x$

15. ★ **MULTIPLE CHOICE** The graph of which function is shown?



Ⓐ $y = 2\left(-\frac{3}{5}\right)^x$ Ⓑ $y = -2\left(\frac{3}{5}\right)^x$

Ⓒ $y = -2\left(\frac{2}{5}\right)^x$ Ⓓ $y = 2\left(-\frac{2}{5}\right)^x$

EXAMPLE 3 on p. 487 for Exs. 16–25

TRANSLATING GRAPHS Graph the function. State the domain and range.

16. $y = \left(\frac{1}{3}\right)^x + 1$
17. $y = -\left(\frac{1}{2}\right)^{x-1}$
18. $y = 2\left(\frac{1}{3}\right)^{x+1} - 3$
19. $y = \left(\frac{2}{3}\right)^{x-4} - 1$
20. $y = 3(0.25)^x + 3$
21. $y = \left(\frac{1}{3}\right)^{x-2} + 2$
22. $f(x) = -3\left(\frac{1}{4}\right)^{x-1}$
23. $g(x) = 6\left(\frac{1}{2}\right)^{x+5} - 2$
24. $h(x) = 4\left(\frac{1}{2}\right)^{x+1}$

25. **GRAPHING CALCULATOR** Consider the exponential decay function $y = ab^{x-h} + k$ where $a = 3$, $b = 0.4$, $h = 2$, and $k = -1$. Predict the effect on the function's graph of each change in a, b, h, or k described in parts (a)–(d). Use a graphing calculator to check your prediction.

 a. a changes to 4
 b. b changes to 0.2
 c. h changes to 5
 d. k changes to 3

26. **ERROR ANALYSIS** You invest $500 in the stock of a company. The value of the stock decreases 2% each year. *Describe* and correct the error in writing a model for the value of the stock after t years.

$y = (\text{Initial amount})(\text{Decay factor})^t$

$y = 500(0.02)^t$

27. ★ **MULTIPLE CHOICE** What is the asymptote of the graph of $y = \left(\frac{1}{2}\right)^{x-2} + 3$?

Ⓐ $y = -3$ Ⓑ $y = -2$ Ⓒ $y = 2$ Ⓓ $y = 3$

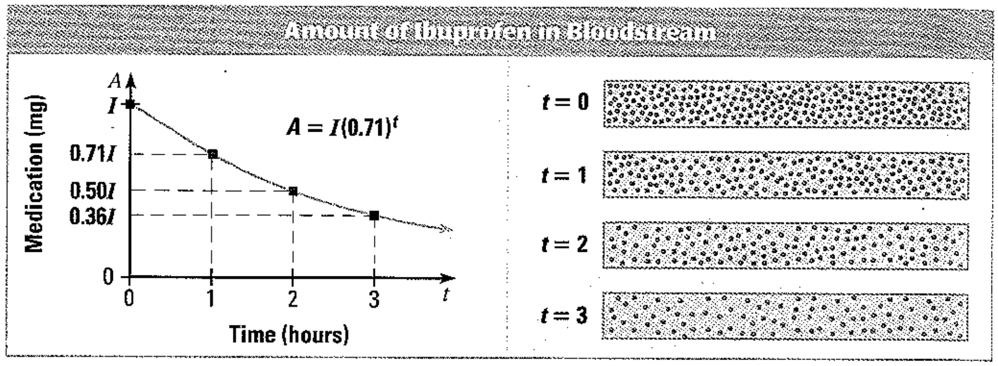
28. ★ **OPEN-ENDED MATH** Write an exponential function whose graph lies between the graphs of $y = (0.5)^x$ and $y = (0.25)^x + 3$.

29. **CHALLENGE** Do $f(x) = 5(4)^{-x}$ and $g(x) = 5(0.25)^x$ represent the same function? *Justify* your answer.

PROBLEM SOLVING

EXAMPLE 4 on p. 488 for Exs. 30–31

30. **MEDICINE** When a person takes a dosage of I milligrams of ibuprofen, the amount A (in milligrams) of medication remaining in the person's bloodstream after t hours can be modeled by the equation $A = I(0.71)^t$.



Find the amount of ibuprofen remaining in a person's bloodstream for the given dosage and elapsed time since the medication was taken.

a. Dosage: 200 mg
Time: 1.5 hours

b. Dosage: 325 mg
Time: 3.5 hours

c. Dosage: 400 mg
Time: 5 hours

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31. **BIKE COSTS** You buy a new mountain bike for \$200. The value of the bike decreases by 25% each year.

a. Write a model giving the mountain bike's value y (in dollars) after t years. Use the model to estimate the value of the bike after 3 years.

b. Graph the model.

c. Estimate when the value of the bike will be \$100.

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32. **DEPRECIATION** The table shows the amount d that a boat depreciates during each year t since it was new. Show that the ratio of depreciation amounts for consecutive years is constant. Then write an equation that gives d as a function of t.

Year, t	1	2	3	4	5
Depreciation, d	\$1906	\$1832	\$1762	\$1692	\$1627

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

33. ★ **SHORT RESPONSE** The value of a car can be modeled by the equation $y = 24{,}000(0.845)^t$ where t is the number of years since the car was purchased.

a. Graph the model. Estimate when the value of the car will be \$10,000.

b. Use the model to predict the value of the car after 50 years. Is this a reasonable value? *Explain.*

34. **MULTI-STEP PROBLEM** When a plant or animal dies, it stops acquiring carbon-14 from the atmosphere. Carbon-14 decays over time with a half-life of about 5730 years. The percent P of the original amount of carbon-14 that remains in a sample after t years is given by this equation:

$$P = 100\left(\frac{1}{2}\right)^{t/5730}$$

a. What percent of the original carbon-14 remains in a sample after 2500 years? 5000 years? 10,000 years?

b. Graph the model.

c. An archaeologist found a bison bone that contained about 37% of the carbon-14 present when the bison died. Use the graph to estimate the age of the bone when it was found.

35. ★ **EXTENDED RESPONSE** The number E of eggs a Leghorn chicken produces per year can be modeled by the equation $E = 179.2(0.89)^{w/52}$ where w is the age (in weeks) of the chicken and $w \ge 22$.

a. Interpret Identify the decay factor and the percent decrease.

b. Graph Graph the model.

c. Estimate Estimate the egg production of a chicken that is 2.5 years old.

d. Reasoning *Explain* how you can rewrite the given equation so that time is measured in years rather than in weeks.

36. **CHALLENGE** You buy a new stereo for \$1300 and are able to sell it 4 years later for \$275. Assume that the resale value of the stereo decays exponentially with time. Write an equation giving the stereo's resale value V (in dollars) as a function of the time t (in years) since you bought it.

MIXED REVIEW

PREVIEW Prepare for Lesson 7.3 in Exs. 37–45.

Graph the function. State the domain and range.

37. $y = x^3 + 5$ *(p. 337)*

38. $y = x^4 - 5x^2 + 4$ *(p. 337)*

39. $y = x^4 - 9$ *(p. 337)*

40. $y = \sqrt{x} + 3$ *(p. 446)*

41. $y = \sqrt{x - 4}$ *(p. 446)*

42. $y = -\frac{1}{2}\sqrt[3]{x} + 2$ *(p. 446)*

43. $y = 3 \cdot 2^x$ *(p. 478)*

44. $y = -2\left(\frac{5}{2}\right)^x$ *(p. 478)*

45. $y = \frac{1}{3} \cdot 5^x$ *(p. 478)*

Verify that f and g are inverse functions. *(p. 438)*

46. $f(x) = 5x - 2,\ g(x) = \frac{x + 2}{5}$

47. $f(x) = -3x + 10,\ g(x) = \frac{10 - x}{3}$

48. $f(x) = 4x^3 - 7,\ g(x) = \left(\frac{x + 7}{4}\right)^{1/3}$

49. $f(x) = \frac{x^5 + 7}{12},\ g(x) = \sqrt[5]{12x - 7}$

EXAMPLE 5 Model continuously compounded interest

FINANCE You deposit $4000 in an account that pays 6% annual interest compounded continuously. What is the balance after 1 year?

Solution

Use the formula for continuously compounded interest.

$A = Pe^{rt}$	**Write formula.**
$= 4000e^{0.06(1)}$	**Substitute 4000 for *P*, 0.06 for *r*, and 1 for *t*.**
≈ 4247.35	**Use a calculator.**

▶ The balance at the end of 1 year is $4247.35.

✓ GUIDED PRACTICE for Example 5

10. **FINANCE** You deposit $2500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time.

    a. 2 years   b. 5 years   c. 7.5 years

11. **FINANCE** Find the amount of interest earned in parts (a)–(c) of Exercise 10.

7.3 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS13 for Exs. 5, 35, and 57
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 52, 53, and 60

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The number _?_ is an irrational number approximately equal to 2.71828.

2. ★ **WRITING** Tell whether the function $f(x) = \frac{1}{3}e^{4x}$ is an example of *exponential growth* or *exponential decay. Explain.*

EXAMPLE 1 on p. 492 for Exs. 3–18

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

3. $e^3 \cdot e^4$
4. $e^{-2} \cdot e^6$
5. $(2e^{3x})^3$
6. $(2e^{-2})^{-4}$
7. $(3e^{5x})^{-1}$
8. $e^x \cdot e^{-3x} \cdot e^4$
9. $\sqrt{9e^6}$
10. $e^x \cdot 5e^{x+3}$
11. $\frac{3e}{e^x}$
12. $\frac{4e^x}{e^{4x}}$
13. $\sqrt[3]{8e^{9x}}$
14. $\frac{6e^{4x}}{8e}$

15. ★ **MULTIPLE CHOICE** What is the simplified form of $(4e^{2x})^3$?

    (A) $4e^{6x}$ (B) $4e^{8x}$ (C) $64e^{6x}$ (D) $64e^{8x}$

16. ★ **MULTIPLE CHOICE** What is the simplified form of $\sqrt{\frac{4(27e^{13}x)}{3e^7x^{-3}}}$?

    (A) $6e^{10}x$ (B) $6e^6x^4$ (C) $\frac{6e^3}{x^2}$ (D) $6e^3x^2$

ERROR ANALYSIS *Describe* and correct the error in simplifying the expression.

17. $(3e^{5x})^2 = 3e^{(5x)(2)}$

$= 3e^{10x}$ ✗

18. $\dfrac{e^{6x}}{e^{-2x}} = e^{6x - 2x}$

$= e^{4x}$ ✗

EXAMPLE 2
on p. 492
for Exs. 19–30

EVALUATING EXPRESSIONS Use a calculator to evaluate the expression.

19. e^3 **20.** $e^{-3/4}$ **21.** $e^{2.2}$ **22.** $e^{1/2}$

23. $e^{-2/5}$ **24.** $e^{4.3}$ **25.** e^7 **26.** e^{-4}

27. $2e^{-0.3}$ **28.** $5e^{2/3}$ **29.** $-6e^{2.4}$ **30.** $0.4e^{4.1}$

GROWTH OR DECAY Tell whether the function is an example of *exponential growth* or *exponential decay.*

31. $f(x) = 3e^{-x}$ **32.** $f(x) = \frac{1}{3}e^{4x}$ **33.** $f(x) = e^{-4x}$ **34.** $f(x) = \frac{3}{5}e^{x}$

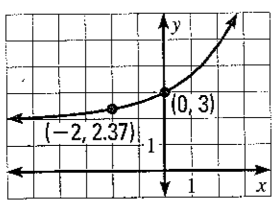
35. $f(x) = \frac{1}{4}e^{-5x}$ **36.** $f(x) = e^{3x}$ **37.** $f(x) = 2e^{4x}$ **38.** $f(x) = 4e^{-2x}$

EXAMPLE 3
on p. 493
for Exs. 39–50

MATCHING GRAPHS Match the function with its graph.

39. $y = 0.5e^{0.5x}$ **40.** $y = 2e^{0.5x}$ **41.** $y = e^{0.5x} + 2$

A.



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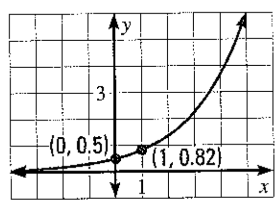

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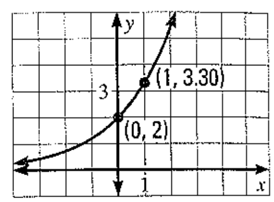

GRAPHING FUNCTIONS Graph the function. State the domain and range.

42. $y = e^{-2x}$ **43.** $y = 3e^{x}$ **44.** $y = 0.5e^{x}$

45. $y = 2e^{-3x} - 1$ **46.** $y = 2.5e^{-0.5x} + 2$ **47.** $y = 0.6e^{x-2}$

48. $f(x) = \frac{1}{2}e^{x+3} - 2$ **49.** $g(x) = \frac{4}{3}e^{x-1} + 1$ **50.** $h(x) = e^{-2(x+1)} - 3$

51. **GRAPHING CALCULATOR** Use the *table* feature of a graphing calculator to find the value of n for which $\left(1 + \frac{1}{n}\right)^n$ gives the value of e correct to 9 decimal places. *Explain* the process you used to find your answer.

52. ★ **SHORT RESPONSE** Can e be expressed as a ratio of two integers? *Explain* your reasoning.

53. ★ **OPEN-ENDED MATH** Find values of a, b, r, and q such that $f(x) = ae^{rx}$ and $g(x) = be^{qx}$ are exponential *decay* functions and $\frac{f(x)}{g(x)}$ is an exponential *growth* function.

54. **CHALLENGE** *Explain* why $A = P\left(1 + \frac{r}{n}\right)^{nt}$ approximates $A = Pe^{rt}$ as n approaches positive infinity. (*Hint:* Let $m = \frac{n}{r}$.)

PROBLEM SOLVING

EXAMPLE 4
on p. 494
for Exs. 55–56

55. CAMERA PHONES The number of camera phones shipped globally can be modeled by the function $y = 1.28e^{1.31x}$ where x is the number of years since 1997 and y is the number of camera phones shipped (in millions). How many camera phones were shipped in 2002?

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56. BIOLOGY Scientists used traps to study the Formosan subterranean termite population in New Orleans. The mean number y of termites collected annually can be modeled by $y = 738e^{0.345t}$ where t is the number of years since 1989. What was the mean number of termites collected in 1999?

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EXAMPLE 5
on p. 495
for Exs. 57–58

57. FINANCE You deposit \$2000 in an account that pays 4% annual interest compounded continuously. What is the balance after 5 years?

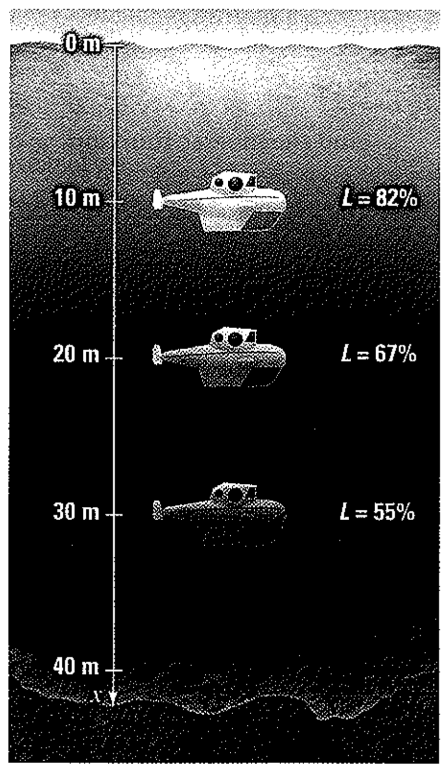
58. FINANCE You deposit \$800 in an account that pays 2.65% annual interest compounded continuously. What is the balance after 12.5 years?

59. MULTI-STEP PROBLEM The percent L of surface light that filters down through bodies of water can be modeled by the exponential function $L(x) = 100e^{kx}$ where k is a measure of the murkiness of the water and x is the depth below the surface (in meters).

a. A recreational submersible is traveling in clear water with a k-value of about -0.02. Write and graph an equation giving the percent of surface light that filters down through clear water as a function of depth.

b. Use your graph to estimate the percent of surface light available at a depth of 40 meters.

c. Use your graph to estimate how deep the submersible can descend in clear water before only 50% of surface light is available.



60. ★ EXTENDED RESPONSE The growth of the bacteria *mycobacterium tuberculosis* can be modeled by the function $P(t) = P_0e^{0.116t}$ where $P(t)$ is the population after t hours and P_0 is the population when $t = 0$.

a. Model At 1:00 P.M., there are 30 *mycobacterium tuberculosis* bacteria in a sample. Write a function for the number of bacteria after 1:00 P.M.

b. Graph Graph the function from part (a).

c. Estimate What is the population at 5:00 P.M.?

d. Reasoning *Describe* how to find the population at 3:45 P.M.

61. **RATE OF HEALING** The area of a wound decreases exponentially with time. The area A of a wound after t days can be modeled by $A = A_0e^{-0.05t}$ where A_0 is the initial wound area. If the initial wound area is 4 square centimeters, what is the area after 14 days?

62. **CHALLENGE** The height y (in feet) of the Gateway Arch in St. Louis, Missouri, can be modeled by the function $y = 757.7 - 63.85(e^{x/127.7} + e^{-x/127.7})$ where x is the horizontal distance (in feet) from the center of the arch.

   a. Use a graphing calculator to graph the function. How tall is the arch at its highest point?

   b. About how far apart are the ends of the arch?

MIXED REVIEW

Solve the equation.

63. $|x + 8| = 13$ *(p. 51)*

64. $|3x + 17| = 16$ *(p. 51)*

65. $2x^2 - 4x + 9 = 0$ *(p. 292)*

66. $x^2 + 12x - 3 = 0$ *(p. 292)*

67. $\sqrt{5x + 9} = 7$ *(p. 452)*

68. $\sqrt{15x + 34} = x + 6$ *(p. 452)*

PREVIEW
Prepare for Lesson 7.4 in Exs. 69–74.

Find the inverse function. *(p. 437)*

69. $f(x) = 2x$

70. $f(x) = 5x - 3$

71. $f(x) = -4x + 14$

72. $f(x) = \frac{1}{3}x + 4$

73. $f(x) = -12x - 6$

74. $f(x) = -\frac{1}{4}x + 7$

QUIZ *for Lessons 7.1–7.3*

Graph the function. State the domain and range.

1. $y = 2 \cdot 3^{x-2}$ *(p. 478)*

2. $y = \left(\frac{2}{5}\right)^x$ *(p. 486)*

3. $f(x) = \left(\frac{3}{8}\right)^x + 2$ *(p. 486)*

Simplify the expression. *(p. 492)*

4. $3e^4 \cdot e^3$

5. $(-5e^{3x})^3$

6. $\frac{e^{4x}}{5e}$

7. $\frac{8e^{5x}}{6e^{2x}}$

Graph the function. State the domain and range. *(p. 492)*

8. $y = 2e^x$

9. $y = 3e^{-2x}$

10. $y = e^{x+1} - 2$

11. $g(x) = 4e^{-3x} + 1$

12. **TV SALES** From 1997 to 2001, the number n (in millions) of black-and-white TVs sold in the United States can be modeled by $n = 26.8(0.85)^t$ where t is the number of years since 1997. Identify the decay factor and the percent decrease. Graph the model and state the domain and range. Estimate the number of black-and-white TVs sold in 1999. *(p. 486)*

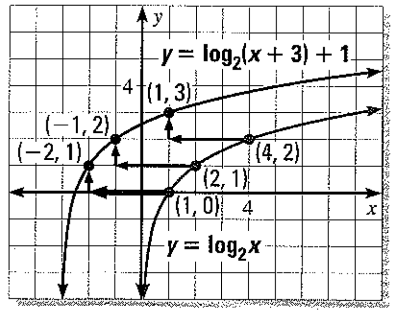
13. **FINANCE** You deposit \$1200 in an account that pays 4.5% annual interest compounded continuously. What is the balance after 5 years? *(p. 492)*

TRANSLATIONS You can graph a logarithmic function of the form $y = \log_b (x - h) + k$ by translating the graph of the parent function $y = \log_b x$.

EXAMPLE 8 Translate a logarithmic graph

Graph $y = \log_2 (x + 3) + 1$. State the domain and range.

Solution



STEP 1 **Sketch** the graph of the parent function $y = \log_2 x$, which passes through (1, 0), (2, 1), and (4, 2).

STEP 2 **Translate** the parent graph left 3 units and up 1 unit. The translated graph passes through (−2, 1), (−1, 2), and (1, 3). The graph's asymptote is $x = -3$. The domain is $x > -3$, and the range is all real numbers.

✓ **GUIDED PRACTICE** for Examples 7 and 8

Graph the function. State the domain and range.

16. $y = \log_5 x$ 17. $y = \log_{1/3} (x - 3)$ 18. $f(x) = \log_4 (x + 1) - 2$

7.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS13 for Exs. 13, 33, and 61
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 36, 61, and 62

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A logarithm with base 10 is called a(n) __?__ logarithm.

2. ★ **WRITING** *Describe* the relationship between $y = 5^x$ and $y = \log_5 x$.

EXAMPLE 1 on p. 499 for Exs. 3–7

EXPONENTIAL FORM Rewrite the equation in exponential form.

3. $\log_4 16 = 2$ 4. $\log_7 343 = 3$ 5. $\log_6 \frac{1}{36} = -2$ 6. $\log_{64} 1 = 0$

7. **ERROR ANALYSIS** *Describe* and correct the error in rewriting the equation $2^{-3} = \frac{1}{8}$ in logarithmic form.

$\log_2 -3 = \frac{1}{8}$ ✗

EXAMPLE 2 on p. 500 for Exs. 8–19

EVALUATING LOGARITHMS Evaluate the logarithm without using a calculator.

8. $\log_{15} 15$ 9. $\log_7 49$ 10. $\log_6 216$ 11. $\log_2 64$

12. $\log_9 1$ (13.) $\log_{1/2} 8$ 14. $\log_3 \frac{1}{27}$ 15. $\log_{16} \frac{1}{4}$

16. $\log_{1/4} 16$ 17. $\log_8 512$ 18. $\log_5 625$ 19. $\log_{11} 121$

EXAMPLE 3
on p. 500
for Exs. 20–27

CALCULATING LOGARITHMS Use a calculator to evaluate the logarithm.

20. $\log 14$ **21.** $\ln 6$ **22.** $\ln 0.43$ **23.** $\log 6.213$

24. $\log 27$ **25.** $\ln 5.38$ **26.** $\log 0.746$ **27.** $\ln 110$

EXAMPLE 5
on p. 501
for Exs. 28–36

USING INVERSE PROPERTIES Simplify the expression.

28. $7^{\log_7 x}$ **29.** $\log_5 5^x$ **30.** $30^{\log_{30} 4}$ **31.** $10^{\log 8}$

32. $\log_6 36^x$ **(33.)** $\log_3 81^x$ **34.** $\log_5 125^x$ **35.** $\log_2 32^x$

36. ★ **MULTIPLE CHOICE** Which expression is equivalent to $\log 100^x$?

(A) x (B) $2x$ (C) $10x$ (D) $100x$

EXAMPLE 6
on p. 501
for Exs. 37–44

FINDING INVERSES Find the inverse of the function.

37. $y = \log_8 x$ **38.** $y = 7^x$ **39.** $y = (0.4)^x$ **40.** $y = \log_{1/2} x$

41. $y = e^{x+2}$ **42.** $y = 2^x - 3$ **43.** $y = \ln (x + 1)$ **44.** $y = 6 + \log x$

EXAMPLES 7 and 8
on pp. 502–503
for Exs. 45–53

GRAPHING FUNCTIONS Graph the function. State the domain and range.

45. $y = \log_4 x$ **46.** $y = \log_6 x$ **47.** $y = \log_{1/3} x$

48. $y = \log_{1/5} x$ **49.** $y = \log_2 (x - 3)$ **50.** $y = \log_3 x + 4$

51. $f(x) = \log_4 (x + 2) - 1$ **52.** $g(x) = \log_6 (x - 4) + 2$ **53.** $h(x) = \log_5 (x + 1) - 3$

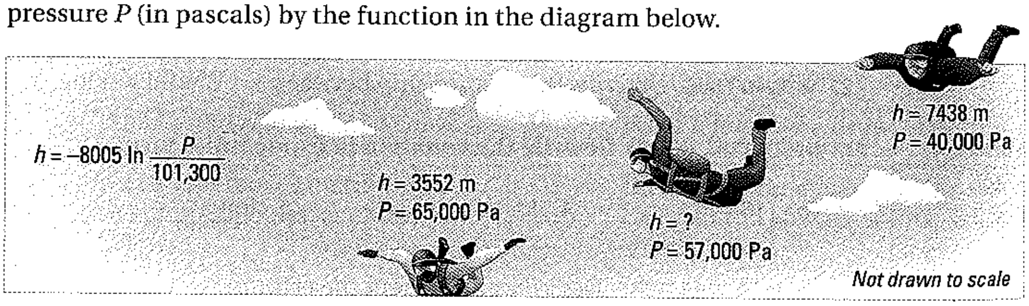
CHALLENGE Evaluate the logarithm. (*Hint:* For each logarithm $\log_b x$, rewrite b and x as powers of the same number.)

54. $\log_{27} 9$ **55.** $\log_8 32$ **56.** $\log_{125} 625$ **57.** $\log_4 128$

PROBLEM SOLVING

EXAMPLE 4
on p. 500
for Exs. 58–59

58. ALTIMETER Skydivers use an instrument called an altimeter to track their altitude as they fall. The altimeter determines altitude by measuring air pressure. The altitude h (in meters) above sea level is related to the air pressure P (in pascals) by the function in the diagram below.



What is the altitude above sea level when the air pressure is 57,000 pascals?

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59. CHEMISTRY The pH value for a substance measures how acidic or alkaline the substance is. It is given by the formula $\text{pH} = -\log [H^+]$ where H^+ is the hydrogen ion concentration (in moles per liter). Lemon juice has a hydrogen ion concentration of $10^{-2.3}$ moles per liter. What is its pH value?

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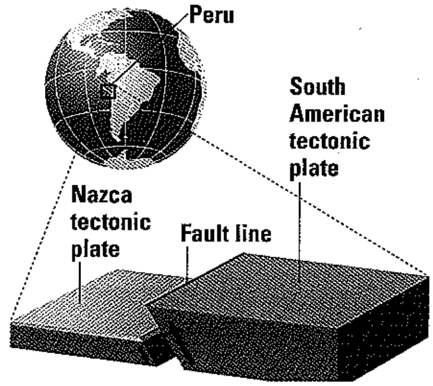
○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE

60. **MULTI-STEP PROBLEM** Biologists have found that an alligator's length ℓ (in inches) and weight w (in pounds) are related by the function $\ell = 27.1 \ln w - 32.8$. Graph the function. Use your graph to estimate the weight of an alligator that is 10 feet long.

61. ★ **SHORT RESPONSE** The energy magnitude M of an earthquake can be modeled by

$$M = 0.29(\ln E) - 9.9$$

where E is the amount of energy released (in ergs).



a. In 2001, a powerful earthquake in Peru, caused by the slippage of two tectonic plates along a fault, released 2.5×10^{24} ergs. What was the energy magnitude of the earthquake?

b. Find the inverse of the given function. *Describe* what it represents.

62. ★ **EXTENDED RESPONSE** A study in Florida found that the number of fish species s in a pool or lake can be modeled by the function

$$s = 30.6 - 20.5(\log A) + 3.8(\log A)^2$$

where A is the area (in square meters) of the pool or lake.

a. **Graph** Use a graphing calculator to graph the function on the domain $200 \le A \le 35{,}000$.

b. **Estimate** Use your graph to estimate the number of fish species in a lake with an area of 30,000 square meters.

c. **Estimate** Use your graph to estimate the area of a lake that contains 6 species of fish.

d. **Reasoning** *Describe* what happens to the number of fish species as the area of a pool or lake increases. *Explain* why your answer makes sense.

63. **CHALLENGE** The function $s = 0.159 + 0.118(\log d)$ gives the slope s of a beach in terms of the average diameter d (in millimeters) of sand particles on the beach. Find the inverse of this function. Then use the inverse to estimate the average diameter of the sand particles on a beach with a slope of 0.2.

MIXED REVIEW

PREVIEW
Prepare for Lesson 7.5 in Exs. 64–71.

Evaluate the expression. *(p. 330)*

64. $2^3 \cdot 2^5$

65. $(5^{-3})^2$

66. $8^1 \cdot 8^3 \cdot 8^{-5}$

67. $\left(\frac{5}{3}\right)^{-3}$

68. $\frac{10^6}{10^4}$

69. $(6^{-2})^{-1}$

70. $\frac{4^2}{4^5}$

71. $\left(\frac{7^8}{7^9}\right)^{-2}$

Simplify the expression. Assume all variables are positive. *(p. 420)*

72. $x^{1/2} \cdot x^{2/3}$

73. $(m^9)^{-1/6}$

74. $\sqrt[3]{54x^6y^3}$

75. $(n^{4/3} \cdot n^{2/5})^{1/6}$

76. $\frac{x^{1/4}y^3}{x^{5/2}y^{1/2}}$

77. $\sqrt[4]{\frac{x^{16}}{y^{12}}}$

78. $(\sqrt[5]{x^{10}} \cdot \sqrt[3]{x^9})^2$

79. $\frac{5\sqrt{x} \cdot \sqrt{x^7}}{\sqrt[3]{250x^{16}}}$

7.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS13 for Exs. 11, 17, and 71
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 43, 44, 64, 71, and 73

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: To condense the expression $\log_3 2x + \log_3 y$, you need to use the __?__ property of logarithms.

2. ★ **WRITING** *Describe* two ways to evaluate $\log_7 12$ using a calculator.

EXAMPLE 1 on p. 507 for Exs. 3–14

MATCHING EXPRESSIONS Match the expression with the logarithm that has the same value.

3. $\ln 6 - \ln 2$ 4. $2 \ln 6$ 5. $6 \ln 2$ 6. $\ln 6 + \ln 2$

A. $\ln 64$ B. $\ln 3$ C. $\ln 12$ D. $\ln 36$

APPROXIMATING EXPRESSIONS Use $\log 4 \approx 0.602$ and $\log 12 \approx 1.079$ to evaluate the logarithm.

7. $\log 3$ 8. $\log 48$ 9. $\log 16$ 10. $\log 64$

11. $\log 144$ 12. $\log \frac{1}{3}$ 13. $\log \frac{1}{4}$ 14. $\log \frac{1}{12}$

EXAMPLE 2 on p. 508 for Exs. 15–32

EXPANDING EXPRESSIONS Expand the expression.

15. $\log_3 4x$ 16. $\ln 15x$ 17. $\log 3x^4$ 18. $\log_5 x^5$

19. $\log_2 \frac{2}{5}$ 20. $\ln \frac{12}{5}$ 21. $\log_4 \frac{x}{3y}$ 22. $\ln 4x^2y$

23. $\log_7 5x^3yz^2$ 24. $\log_6 36x^2$ 25. $\ln x^2y^{1/3}$ 26. $\log 10x^3$

27. $\log_2 \sqrt{x}$ 28. $\ln \frac{6x^2}{y^4}$ 29. $\ln \sqrt[4]{x^3}$ 30. $\log_3 \sqrt{9x}$

ERROR ANALYSIS *Describe* **and correct the error in expanding the logarithmic expression.**

31. $\log_2 5x = (\log_2 5)(\log_2 x)$ ✗

32. $\ln 8x^3 = 3 \ln 8 + \ln x$ ✗

EXAMPLE 3 on p. 508 for Exs. 33–43

CONDENSING EXPRESSIONS Condense the expression.

33. $\log_4 7 - \log_4 10$
34. $\ln 12 - \ln 4$
35. $2 \log x + \log 11$
36. $6 \ln x + 4 \ln y$
37. $5 \log x - 4 \log y$
38. $5 \log_4 2 + 7 \log_4 x + 4 \log_4 y$
39. $\ln 40 + 2 \ln \frac{1}{2} + \ln x$
40. $\log_5 4 + \frac{1}{3} \log_5 x$
41. $6 \ln 2 - 4 \ln y$
42. $2(\log_3 20 - \log_3 4) + 0.5 \log_3 4$

43. ★ **MULTIPLE CHOICE** Which of the following is equivalent to $3 \log_4 6$?

Ⓐ $\log_4 18$ Ⓑ $\log_4 72$ Ⓒ $\log_4 216$ Ⓓ $\log_4 256$

44. ★ **MULTIPLE CHOICE** Which of the following statements is *not* correct?

Ⓐ $\log_3 48 = \log_3 16 + \log_3 3$　　Ⓑ $\log_3 48 = 3\log_3 2 + \log_3 6$

Ⓒ $\log_3 48 = 2\log_3 4 + \log_3 3$　　Ⓓ $\log_3 48 = \log_3 8 + 2\log_3 3$

EXAMPLE 4 on p. 509 for Exs. 45–61

CHANGE-OF-BASE FORMULA **Use the change-of-base formula to evaluate the logarithm.**

45. $\log_4 7$　46. $\log_5 13$　47. $\log_3 15$　48. $\log_8 22$

49. $\log_3 6$　50. $\log_5 14$　51. $\log_6 17$　52. $\log_2 28$

53. $\log_7 19$　54. $\log_4 48$　55. $\log_9 27$　56. $\log_8 32$

57. $\log_6 \frac{24}{5}$　58. $\log_2 \frac{15}{7}$　59. $\log_3 \frac{9}{40}$　60. $\log_7 \frac{3}{16}$

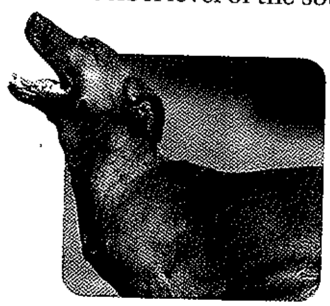
61. **ERROR ANALYSIS** *Describe* and correct the error in using the change-of-base formula.

$$\log_3 7 = \frac{\log 3}{\log 7}$$ ✗

EXAMPLE 5 on p. 509 for Exs. 62–63

SOUND INTENSITY **In Exercises 62 and 63, use the function in Example 5.**

62. Find the decibel level of the sound made by each object shown below.

a. 

Barking dog: $I = 10^{-4}$ W/m^2

b. 

Ambulance siren: $I = 10^{0}$ W/m^2

c. 

Bee: $I = 10^{-6.5}$ W/m^2

63. The intensity of the sound of a trumpet is 10^3 watts per square meter. Find the decibel level of a trumpet.

64. ★ **OPEN-ENDED MATH** For each statement, find positive numbers M, N, and b (with $b \neq 1$) that show the statement is false in general.

a. $\log_b (M + N) = \log_b M + \log_b N$　　b. $\log_b (M - N) = \log_b M - \log_b N$

CHALLENGE **In Exercises 65–68, use the given hint and properties of exponents to prove the property of logarithms.**

65. **Product property** $\log_b mn = \log_b m + \log_b n$
(*Hint:* Let $x = \log_b m$ and let $y = \log_b n$. Then $m = b^x$ and $n = b^y$.)

66. **Quotient property** $\log_b \frac{m}{n} = \log_b m - \log_b n$
(*Hint:* Let $x = \log_b m$ and let $y = \log_b n$. Then $m = b^x$ and $n = b^y$.)

67. **Power property** $\log_b m^n = n \log_b m$
(*Hint:* Let $x = \log_b m$. Then $m = b^x$ and $m^n = b^{nx}$.)

68. **Change-of-base formula** $\log_c a = \frac{\log_b a}{\log_b c}$
(*Hint:* Let $x = \log_b a$, $y = \log_b c$, and $z = \log_c a$. Then $a = b^x$, $c = b^y$, and $a = c^z$, so that $b^x = c^z$.)

PROBLEM SOLVING

EXAMPLE 5
on p. 509
for Exs. 69–72

69. CONVERSATION Three groups of people are having separate conversations in a room. The sound of each conversation has an intensity of 1.4×10^{-5} watts per square meter. What is the decibel level of the combined conversations in the room?

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70. PARKING GARAGE The sound made by each of five cars in a parking garage has an intensity of 3.2×10^{-4} watts per square meter. What is the decibel level of the sound made by all five cars in the parking garage?

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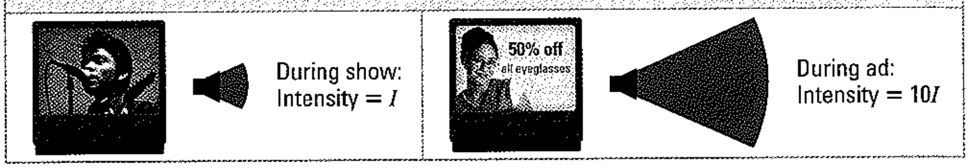
(71.) ★ **SHORT RESPONSE** The intensity of the sound TV ads make is ten times as great as the intensity for an average TV show. How many decibels louder is a TV ad? *Justify* your answer using properties of logarithms.

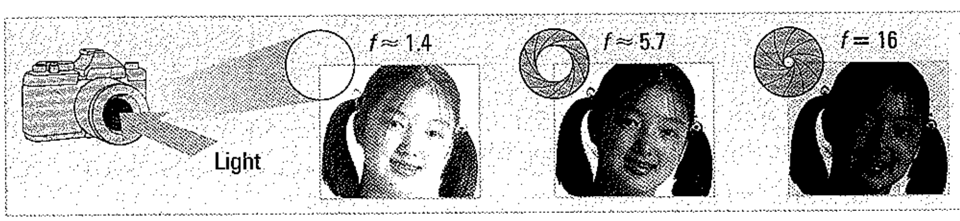
Intensity of Television Sound



72. BIOLOGY The loudest animal on Earth is the blue whale. It can produce a sound with an intensity of $10^{6.8}$ watts per square meter. The loudest sound a human can make has an intensity of $10^{0.8}$ watts per square meter. *Compare* the decibel levels of the sounds made by a blue whale and a human.

73. ★ **EXTENDED RESPONSE** The f-stops on a 35 millimeter camera control the amount of light that enters the camera. Let s be a measure of the amount of light that strikes the film and let f be the f-stop. Then s and f are related by the equation:

$$s = \log_2 f^2$$



a. **Use Properties** Expand the expression for s.

b. **Calculate** The table shows the first eight f-stops on a 35 millimeter camera. Copy and complete the table. *Describe* the pattern you observe.

f	1.414	2.000	2.828	4.000	5.657	8.000	11.314	16.000
s	?	?	?	?	?	?	?	?

c. **Reasoning** Many 35 millimeter cameras have nine f-stops. What do you think the ninth f-stop is? *Explain* your reasoning.

○ = **WORKED-OUT SOLUTIONS** on p. WS1

★ = **STANDARDIZED TEST PRACTICE**

74. **CHALLENGE** Under certain conditions, the wind speed s (in knots) at an altitude of h meters above a grassy plain can be modeled by this function:

$$s(h) = 2 \ln (100h)$$

a. By what amount does the wind speed increase when the altitude doubles?

b. Show that the given function can be written in terms of common logarithms as $s(h) = \frac{2}{\log e}(\log h + 2)$.

MIXED REVIEW

Perform the indicated operation. ***(p. 187)***

75. $\begin{bmatrix} 5 & -8 \\ 12 & 20 \end{bmatrix} + \begin{bmatrix} 6 & 7 \\ -2 & -19 \end{bmatrix}$ **76.** $\begin{bmatrix} -7 & 11 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} -9 & 17 \\ -13 & 1 \end{bmatrix}$ **77.** $3\begin{bmatrix} 1.7 & 2.4 & 6.8 \\ 9.2 & 5.3 & 7.2 \end{bmatrix}$

PREVIEW Prepare for Lesson 7.6 in Exs. 78–83.

Solve the equation. Check for extraneous solutions. ***(p. 452)***

78. $\sqrt{x+12} + 4 = 11$ **79.** $\sqrt[3]{x+10} + 6 = 4$ **80.** $\sqrt{x+6} = \sqrt{3x-14}$

81. $\sqrt[3]{2x-7} = \sqrt[3]{8-x}$ **82.** $\sqrt{x-1} = x - 3$ **83.** $x + 2 = \sqrt{9x+28}$

Use a calculator to evaluate the expression.

84. e^8 ***(p. 492)*** **85.** e^{-6} ***(p. 492)*** **86.** $e^{3.5}$ ***(p. 492)*** **87.** $e^{-0.4}$ ***(p. 492)***

88. $\log 12$ ***(p. 499)*** **89.** $\log 1.8$ ***(p. 499)*** **90.** $\ln 24$ ***(p. 499)*** **91.** $\ln 8.49$ ***(p. 499)***

QUIZ *for Lessons 7.4–7.5*

Evaluate the logarithm without using a calculator. ***(p. 499)***

1. $\log_4 16$ **2.** $\log_5 1$ **3.** $\log_8 8$ **4.** $\log_{1/2} 32$

Graph the function. State the domain and range. ***(p. 499)***

5. $y = \log_2 x$ **6.** $y = \ln x + 2$ **7.** $y = \log_3 (x+4) - 1$

Expand the expression. ***(p. 507)***

8. $\log_2 5x$ **9.** $\log_5 x^7$ **10.** $\ln 5xy^3$ **11.** $\log_3 \frac{6y^4}{x^8}$

Condense the expression. ***(p. 507)***

12. $\log_3 5 - \log_3 20$ **13.** $\ln 6 + \ln 4x$ **14.** $\log_6 5 + 3 \log_6 2$ **15.** $4 \ln x - 5 \ln x$

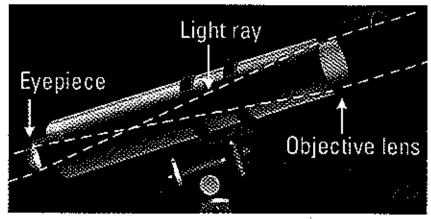
Use the change-of-base formula to evaluate the logarithm. ***(p. 507)***

16. $\log_3 10$ **17.** $\log_7 14$ **18.** $\log_5 24$ **19.** $\log_8 40$

20. SOUND INTENSITY The sound of an alarm clock has an intensity of $I = 10^{-4}$ watts per square meter. Use the model $L(I) = 10 \log \frac{I}{I_0}$, where $I_0 = 10^{-12}$ watts per square meter, to find the alarm clock's loudness $L(I)$. ***(p. 507)***

EXAMPLE 7 Use a logarithmic model

ASTRONOMY The *apparent magnitude* of a star is a measure of the brightness of the star as it appears to observers on Earth. The apparent magnitude M of the dimmest star that can be seen with a telescope is given by the function



$$M = 5 \log D + 2$$

where D is the diameter (in millimeters) of the telescope's objective lens. If a telescope can reveal stars with a magnitude of 12, what is the diameter of its objective lens?

ANOTHER WAY
For an alternative method for solving the problem in Example 7, turn to page 523 for the **Problem Solving Workshop**.

Solution

$M = 5 \log D + 2$	**Write original equation.**
$12 = 5 \log D + 2$	**Substitute 12 for *M*.**
$10 = 5 \log D$	**Subtract 2 from each side.**
$2 = \log D$	**Divide each side by 5.**
$10^2 = 10^{\log D}$	**Exponentiate each side using base 10.**
$100 = D$	**Simplify.**

▶ The diameter is 100 millimeters.

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✓ GUIDED PRACTICE for Example 7

11. **WHAT IF?** Use the information from Example 7 to find the diameter of the objective lens of a telescope that can reveal stars with a magnitude of 7.

7.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS14 for Exs. 15, 35, and 57
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 44, 47, 58, and 60
◆ = **MULTIPLE REPRESENTATIONS** Ex. 59

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The equation $5^x = 8$ is an example of a(n) __?__ equation.

2. ★ **WRITING** When do logarithmic equations have extraneous solutions?

EXAMPLE 1
on p. 515
for Exs. 3–11

SOLVING EXPONENTIAL EQUATIONS Solve the equation.

3. $5^{x-4} = 25^{x-6}$
4. $7^{3x+4} = 49^{2x+1}$
5. $8^{x-1} = 32^{3x-2}$
6. $27^{4x-1} = 9^{3x+8}$
7. $4^{2x-5} = 64^{3x}$
8. $3^{3x-7} = 81^{12-3x}$
9. $36^{5x+2} = \left(\frac{1}{6}\right)^{11-x}$
10. $10^{3x-10} = \left(\frac{1}{100}\right)^{6x-1}$
11. $25^{10x+8} = \left(\frac{1}{125}\right)^{4-2x}$

EXAMPLE 2
on p. 516
for Exs. 12–23

SOLVING EXPONENTIAL EQUATIONS **Solve the equation.**

12. $8^x = 20$
13. $e^{-x} = 5$
14. $7^{3x} = 18$
15. $11^{5x} = 33$
16. $7^{6x} = 12$
17. $4e^{-2x} = 17$
18. $10^{3x} + 4 = 9$
19. $-3e^{2x} + 16 = 5$
20. $0.5^x - 0.25 = 4$
21. $\frac{1}{3}(6)^{-4x} + 1 = 6$
22. $2^{0.1x} - 5 = 7$
23. $\frac{3}{4}e^{2x} + \frac{7}{2} = 4$

EXAMPLE 4
on p. 517
for Exs. 24–31

SOLVING LOGARITHMIC EQUATIONS **Solve the equation. Check for extraneous solutions.**

24. $\log_5(5x + 9) = \log_5 6x$
25. $\ln(4x - 7) = \ln(x + 11)$
26. $\ln(x + 19) = \ln(7x - 8)$
27. $\log_5(2x - 7) = \log_5(3x - 9)$
28. $\log(12x - 11) = \log(3x + 13)$
29. $\log_3(18x + 7) = \log_3(3x + 38)$
30. $\log_6(3x - 10) = \log_6(14 - 5x)$
31. $\log_8(5 - 12x) = \log_8(6x - 1)$

EXAMPLES 5 and 6
on pp. 517–518
for Exs. 32–44

EXPONENTIATING TO SOLVE EQUATIONS **Solve the equation. Check for extraneous solutions.**

32. $\log_4 x = -1$
33. $5 \ln x = 35$
34. $\frac{1}{3}\log_5 12x = 2$
35. $5.2 \log_4 2x = 16$
36. $\log_2(x - 4) = 6$
37. $\log_2 x + \log_2(x - 2) = 3$
38. $\log_4(-x) + \log_4(x + 10) = 2$
39. $\ln(x + 3) + \ln x = 1$
40. $4 \ln(-x) + 3 = 21$
41. $\log_5(x + 4) + \log_5(x + 1) = 2$
42. $\log_6 3x + \log_6(x - 1) = 3$
43. $\log_3(x - 9) + \log_3(x - 3) = 2$

44. ★ **MULTIPLE CHOICE** What is the solution of $3 \log_8(2x + 7) + 8 = 10$?

   (A) -1.5 (B) -1.179 (C) 4 (D) 4.642

ERROR ANALYSIS ***Describe*** **and correct the error in solving the equation.**

45.

$$3^{x+1} = 6^x$$
$$\log_3 3^{x+1} = \log_3 6^x$$
$$x + 1 = x \log_3 6$$
$$x + 1 = 2x$$
$$1 = x$$

46.

$$\log_3 10x = 5$$
$$e^{\log_3 10x} = e^5$$
$$10x = e^5$$
$$x = \frac{e^5}{10}$$

47. ★ **OPEN-ENDED MATH** Give an example of an exponential equation whose only solution is 4 and an example of a logarithmic equation whose only solution is -3.

CHALLENGE **Solve the equation.**

48. $3^{x+4} = 6^{2x-5}$
49. $10^{3x-8} = 2^{5-x}$
50. $\log_2(x + 1) = \log_8 3x$
51. $\log_3 x = \log_9 6x$
52. $2^{2x} - 12 \cdot 2^x + 32 = 0$
53. $5^{2x} + 20 \cdot 5^x - 125 = 0$

PROBLEM SOLVING

EXAMPLE 3
on p. 516
for Exs. 54–58

54. COOKING You are cooking beef stew. When you take the beef stew off the stove, it has a temperature of 200°F. The room temperature is 75°F and the cooling rate of the beef stew is $r = 0.054$. How long (in minutes) will it take to cool the beef stew to a serving temperature of 100°F?

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55. THERMOMETER As you are hanging an outdoor thermometer, its reading drops from the indoor temperature of 75°F to 37°F in one minute. If the cooling rate is $r = 1.37$, what is the outdoor temperature?

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56. COMPOUND INTEREST You deposit $100 in an account that pays 6% annual interest. How long will it take for the balance to reach $1000 for each given frequency of compounding?

a. Annual **b.** Quarterly **c.** Daily

57. RADIOACTIVE DECAY One hundred grams of radium are stored in a container. The amount R (in grams) of radium present after t years can be modeled by $R = 100e^{-0.00043t}$. After how many years will only 5 grams of radium be present?

58. ★ MULTIPLE CHOICE You deposit $800 in an account that pays 2.25% annual interest compounded continuously. About how long will it take for the balance to triple?

Ⓐ 24 years Ⓑ 36 years

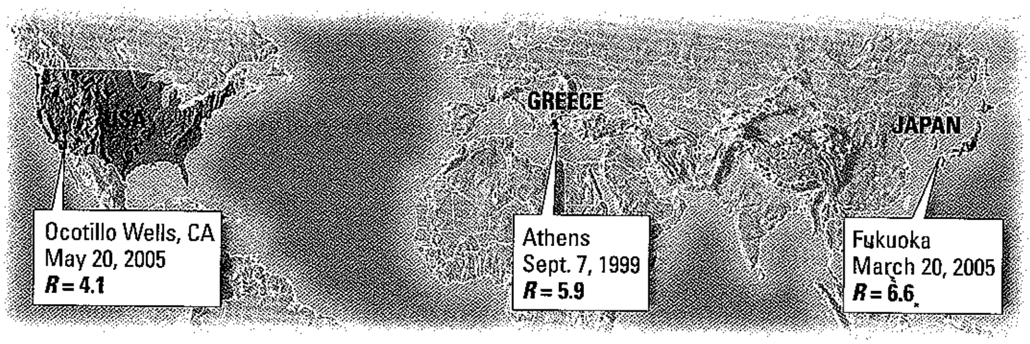
Ⓒ 48.8 years Ⓓ 52.6 years

EXAMPLE 7
on p. 519
for Ex. 59

59. ◆ MULTIPLE REPRESENTATIONS The Richter scale is used for measuring the magnitude of an earthquake. The Richter magnitude R is given by the function

$$R = 0.67 \log (0.37E) + 1.46$$

where E is the energy (in kilowatt-hours) released by the earthquake.



a. Making a Graph Graph the function using a graphing calculator. Use your graph to approximate the amount of energy released by each earthquake indicated in the diagram above.

b. Solving Equations Write and solve a logarithmic equation to find the amount of energy released by each earthquake in the diagram.

60. ★ **EXTENDED RESPONSE** If X-rays of a fixed wavelength strike a material x centimeters thick, then the intensity $I(x)$ of the X-rays transmitted through the material is given by $I(x) = I_0e^{-\mu x}$, where I_0 is the initial intensity and μ is a number that depends on the type of material and the wavelength of the X-rays. The table shows the values of μ for various materials. These μ-values apply to X-rays of medium wavelength.

Material	Aluminum	Copper	Lead
Value of μ	0.43	3.2	43

a. Find the thickness of aluminum shielding that reduces the intensity of X-rays to 30% of their initial intensity. (*Hint:* Find the value of x for which $I(x) = 0.3I_0$.)

b. Repeat part (a) for copper shielding.

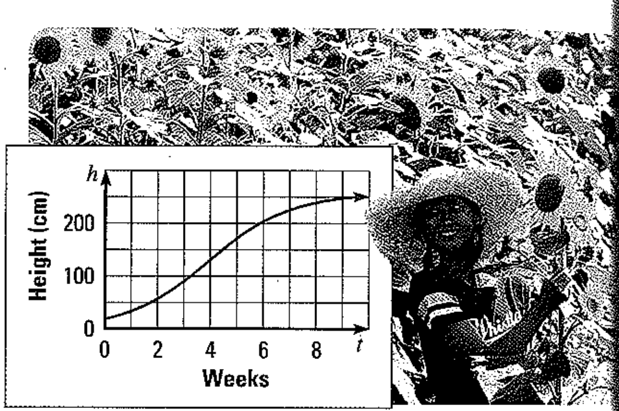
c. Repeat part (a) for lead shielding.

d. **Reasoning** Your dentist puts a lead apron on you before taking X-rays of your teeth to protect you from harmful radiation. Based on your results from parts (a)–(c), explain why lead is a better material to use than aluminum or copper.

61. **CHALLENGE** You plant a sunflower seedling in your garden. The seedling's height h (in centimeters) after t weeks can be modeled by the function below, which is called a *logistic function.*

$$h(t) = \frac{256}{1 + 13e^{-0.65t}}$$

Find the time it takes the sunflower seedling to reach a height of 200 centimeters.



MIXED REVIEW

PREVIEW
Prepare for Lesson 7.7 in Exs. 62–64.

Solve the system using any algebraic method. *(p. 160)*

62. $3x - y = 7$
$x + 2y = 14$

63. $5x - y = 7$
$2x + 5y = -3$

64. $x + 4y = -6$
$-2x + y = 12$

Determine the possible numbers of positive real zeros, negative real zeros, and imaginary zeros for the function. *(p. 379)*

65. $f(x) = x^3 - 2x^2 + 5$

66. $f(x) = x^4 + 6x^3 - x^2 + 7x - 8$

67. $f(x) = x^5 - 3x^3 + 7x^2 + 6x + 9$

68. $f(x) = x^7 + 10x^6 - 5x^4 + 12x^3 - 17$

Use finite differences and a system of equations to find a polynomial function that fits the data. You may want to use a graphing calculator to solve the system. *(p. 393)*

69.

x	1	2	3	4	5	6
$f(x)$	19	28	27	16	−5	−36

70.

x	1	2	3	4	5	6
$f(x)$	0	2	12	36	80	150