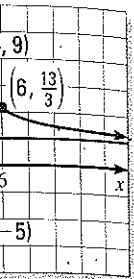


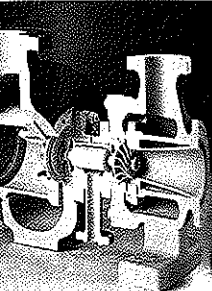
$$\frac{ax + b}{cx + d}$$



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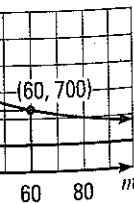
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GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

4. $y = \frac{x-1}{x+3}$

5. $y = \frac{2x+1}{4x-2}$

6. $f(x) = \frac{-3x+2}{-x-1}$

7. **WHAT IF?** In Example 4, how do the function and graph change if the cost of the 3-D printer is \$21,000?

8.2 EXERCISES

HOMWORK KEY

\bigcirc = WORKED-OUT SOLUTIONS on p. WS14 for Exs. 5, 21, and 39

\star = STANDARDIZED TEST PRACTICE Exs. 2, 23, 35, 40, and 41

\blacklozenge = MULTIPLE REPRESENTATIONS Ex. 39

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The function $y = \frac{7}{x+4} + 3$ has a(n) ? of all real numbers except 3 and a(n) ? of all real numbers except -4.

2. \star **WRITING** Is $f(x) = \frac{-3x+5}{2^x+1}$ a rational function? Explain your answer.

EXAMPLE 1

on p. 558
for Exs. 3–10

GRAPHING FUNCTIONS Graph the function. Compare the graph with the graph of $y = \frac{1}{x}$.

3. $y = \frac{3}{x}$

4. $y = \frac{10}{x}$

5. $y = \frac{-5}{x}$

6. $y = \frac{-0.5}{x}$

7. $y = \frac{0.1}{x}$

8. $f(x) = \frac{15}{x}$

9. $g(x) = \frac{-6}{x}$

10. $h(x) = \frac{-3}{x}$

EXAMPLE 2

on p. 559
for Exs. 11–23

GRAPHING FUNCTIONS Graph the function. State the domain and range.

11. $y = \frac{4}{x} + 3$

12. $y = \frac{3}{x} - 2$

13. $y = \frac{6}{x-1}$

14. $f(x) = \frac{1}{x+2}$

15. $y = \frac{-5}{x} - 7$

16. $y = \frac{-6}{x} + 4$

17. $y = \frac{-3}{x+2}$

18. $g(x) = \frac{-2}{x-7}$

19. $y = \frac{-4}{x+4} + 3$

20. $y = \frac{10}{x+7} - 5$

21. $y = \frac{-3}{x-4} - 1$

22. $h(x) = \frac{11}{x-9} + 9$

23. \star **MULTIPLE CHOICE** What are the asymptotes of the graph of $y = \frac{3}{x+8} - 3$?

(A) $x = 8, y = 3$

(B) $x = 8, y = -3$

(C) $x = -8, y = 3$

(D) $x = -8, y = -3$

24. **GRAPHING CALCULATOR** Consider the function $y = \frac{a}{x-h} + k$ where $a = 1$, $h = 3$, and $k = -2$. Predict the effect on the function's graph of each change in a , h , or k described in parts (a)–(c). Use a graphing calculator to check your prediction by graphing the original and revised functions in the same coordinate plane.

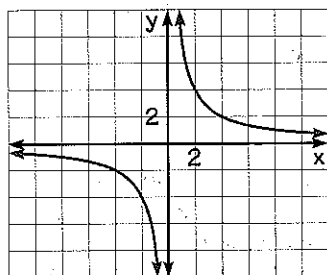
a. a changes to -3

b. h changes to -1

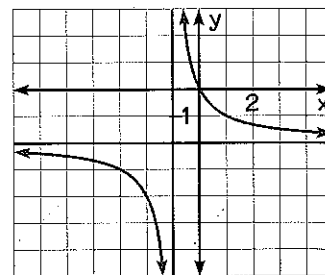
c. k changes to 2

ERROR ANALYSIS Describe and correct the error in the graph.

25. $y = \frac{-8}{x}$



26. $y = \frac{2}{x-1} - 2$



EXAMPLE 3

on p. 560
for Exs. 27–34

GRAPHING FUNCTIONS Graph the function. State the domain and range.

27. $y = \frac{x+4}{x-3}$

28. $y = \frac{x-1}{x+5}$

29. $y = \frac{x+6}{4x-8}$

30. $y = \frac{8x+3}{2x-6}$

31. $y = \frac{-5x+2}{4x+5}$

32. $f(x) = \frac{6x-1}{3x-1}$

33. $g(x) = \frac{5x}{2x+3}$

34. $h(x) = \frac{5x+3}{-x+10}$

35. **★ OPEN-ENDED MATH** Write a rational function such that the domain is all real numbers except -8 and the range is all real numbers except 3 .

36. **CHALLENGE** Show that the equation $f(x) = \frac{a}{x-h} + k$ represents a rational function by writing the right side as a quotient of polynomials.

PROBLEM SOLVING

EXAMPLE 4

on p. 560
for Exs. 37–38

37. **INTERNET SERVICE** An Internet service provider charges a \$50 installation fee and a monthly fee of \$43. Write and graph an equation that gives the average cost per month as a function of the number of months of service. After how many months will the average cost be \$53?

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38. **ROCK CLIMBING GYM** To join a rock climbing gym, you must pay an initial fee of \$100 and a monthly fee of \$59. Write and graph an equation that gives the average cost per month as a function of the number of months of membership. After how many months will the average cost be \$69?

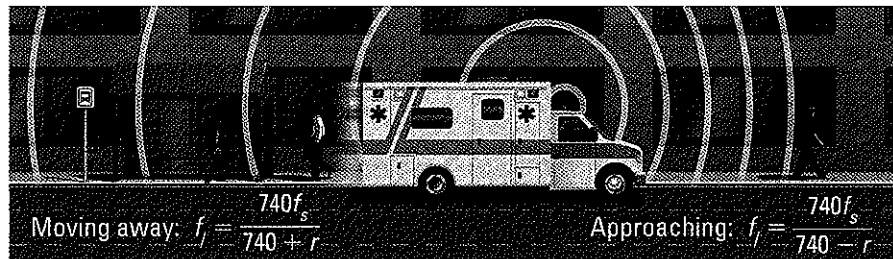
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39. **◆ MULTIPLE REPRESENTATIONS** The time t (in seconds) it takes for sound to travel 1 kilometer can be modeled by $t = \frac{1000}{0.6T + 331}$ where T is the air temperature (in degrees Celsius).

a. **Evaluating a Function** How long does it take for sound to travel 5 kilometers when the air temperature is 25°C ? *Explain.*

b. **Drawing a Graph** Suppose you are 1 kilometer from a lightning strike, and it takes 3 seconds to hear the thunder. Graph the given function, and use the graph to estimate the air temperature.

40. ★ **SHORT RESPONSE** A business is studying the cost to remove a pollutant from the ground at its site. The function $y = \frac{15x}{1.1 - x}$ models the estimated cost y (in thousands of dollars) to remove x percent (expressed as a decimal) of the pollutant.
- Graph the function. *Describe* a reasonable domain and range.
 - How much does it cost to remove 20% of the pollutant? 40% of the pollutant? 80% of the pollutant? Does doubling the percent of the pollutant removed double the cost? *Explain*.
41. ★ **EXTENDED RESPONSE** The *Doppler effect* occurs when the source of a sound is moving relative to a listener, so that the frequency f_l (in hertz) heard by the listener is different from the frequency f_s (in hertz) at the source. The frequency heard depends on whether the sound source is approaching or moving away from the listener. In both equations below, r is the speed (in miles per hour) of the sound source.



- An ambulance siren has a frequency of 2000 hertz. Write two equations modeling the frequencies you hear when the ambulance is approaching and when the ambulance is moving away.
 - Graph the equations from part (a) using the domain $0 \leq r \leq 60$.
 - For any speed r , how does the frequency heard for an approaching sound source compare with the frequency heard when the source moves away?
42. **CHALLENGE** A sailboat travels at a speed of 10 knots for 3 hours. It then uses a motor for power, which increases its speed to 15 knots. Write and graph an equation giving the boat's average speed s (in knots) for the entire trip as a function of the time t (in hours) that it uses the motor for power.

MIXED REVIEW

PREVIEW

Prepare for
Lesson 8.3
in Exs. 43–48.

Factor the expression. (p. 252)

43. $m^2 + 18m + 65$

44. $p^2 + 15p + 56$

45. $q^2 - 49$

46. $r^2 - 20r + 100$

47. $x^2 - 4x - 21$

48. $z^2 - 9z + 20$

Graph the polynomial function. (p. 337)

49. $f(x) = 2x^4$

50. $f(x) = x^3 + 5$

51. $f(x) = x^5 - 1$

52. $f(x) = 3x^4 + 2$

53. $f(x) = -x^6$

54. $f(x) = -2x^3 + 3$

Simplify the expression. (p. 492)

55. $e^5 \cdot e^{-9}$

56. $3e^6 \cdot e^x$

57. $e^x \cdot e^{3x+2}$

8.3 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS15 for Exs. 7, 15, and 33
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 6, 14, 24, and 35
- ◆ = MULTIPLE REPRESENTATIONS Ex. 33

SKILL PRACTICE

- VOCABULARY** Copy and complete: The graph of a rational function f has no ? when the degree of the function's numerator is greater than the degree of its denominator.
- ★ **WRITING** Let $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials with no common factors other than ± 1 . Describe how to find the x -intercepts and the vertical asymptotes of the graph of f .

EXAMPLES

1, 2, and 3

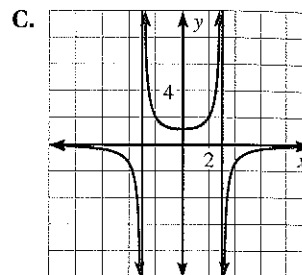
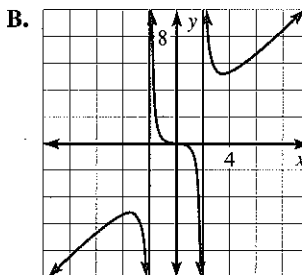
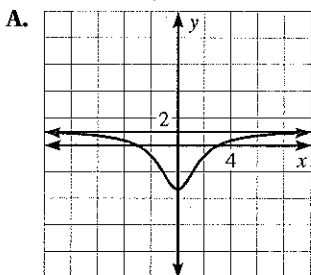
on pp. 565–566
for Exs. 3–23

MATCHING GRAPHS Match the function with its graph.

3. $y = \frac{-10}{x^2 - 9}$

4. $y = \frac{x^2 - 10}{x^2 + 3}$

5. $y = \frac{x^3}{x^2 - 4}$



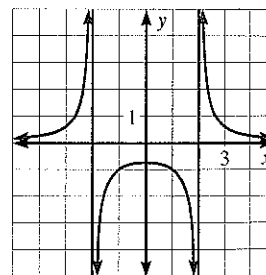
6. ★ **MULTIPLE CHOICE** The graph of which function is shown?

(A) $y = \frac{3}{x^2 - 4}$

(B) $y = \frac{3x^2}{x^2 - 4}$

(C) $y = \frac{x^2 - 4}{3x^2}$

(D) $y = \frac{x^3}{x^2 - 4}$



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ANALYZING GRAPHS Identify the x -intercept(s) and vertical asymptote(s) of the graph of the function.

7. $y = \frac{5}{x^2 - 1}$

8. $y = \frac{x + 1}{x^2 + 5}$

9. $f(x) = \frac{x^2 + 9}{x^2 - 2x - 15}$

10. $y = \frac{x^2 - 7x - 60}{x + 3}$

11. $y = \frac{x^3 + 27}{3x^2 + x}$

12. $g(x) = \frac{2x^2 - 3x - 20}{x^2 + 1}$

13. **ERROR ANALYSIS** Describe and correct the error in finding the vertical asymptote(s) of $f(x) = \frac{x - 2}{x^2 - 8x + 7}$.

The vertical asymptote occurs at the zero of the numerator $x - 2$. So, the vertical asymptote is $x = 2$.



14. ★ **MULTIPLE CHOICE** What is the horizontal asymptote of the graph of the function $y = \frac{4x^2 - 21x + 5}{x^2 - 12}$?

- (A) $y = 0$ (B) $y = \frac{1}{4}$ (C) $y = 4$ (D) $y = 4x$

GRAPHING FUNCTIONS Graph the function.

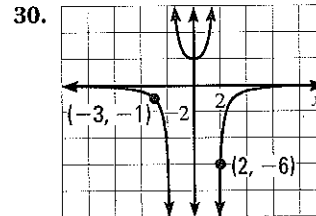
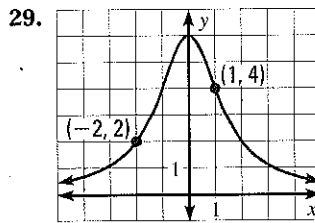
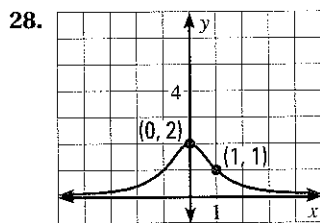
15. $y = \frac{2x}{x^2 - 1}$ 16. $y = \frac{8}{x^2 - x - 6}$ 17. $f(x) = \frac{x^2 - 9}{2x^2 + 1}$
 18. $y = \frac{x - 4}{x^2 - 3x}$ 19. $y = \frac{x^2 + 11x + 18}{2x + 1}$ 20. $g(x) = \frac{x^3 - 8}{6 - x^2}$
 21. $y = \frac{x^2 + 3}{2x^3}$ 22. $y = \frac{x^2 - 5x - 36}{3x}$ 23. $h(x) = \frac{3x^2 + 10x - 8}{x^2 + 4}$

24. ★ **OPEN-ENDED MATH** Write two different rational functions whose graphs have the same end behavior as the graph of $y = 3x^2$.

GRAPHING CALCULATOR Use a graphing calculator to find the range of the rational function.

25. $y = \frac{15}{x^2 + 2}$ 26. $y = \frac{3x^2}{x^2 - 9}$ 27. $y = \frac{x^2 - 2x}{2x + 3}$

CHALLENGE The graph of a function of the form $f(x) = \frac{a}{x^2 + b}$ is shown. Find the values of a and b .

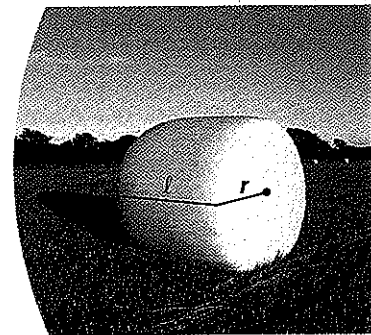


PROBLEM SOLVING

EXAMPLE 4
on p. 567
for Exs. 31–32

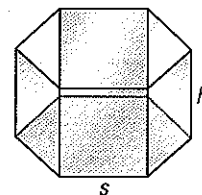
GRAPHING CALCULATOR You may wish to use a graphing calculator to complete the following Problem Solving exercises.

31. **AGRICULTURE** A farmer makes cylindrical bales of hay that have a volume of 100 cubic feet. Each bale is to be wrapped in plastic to keep the hay dry.
- Using the formula for the volume of a cylinder, write an equation that gives the length ℓ of a bale in terms of the radius r .
 - Write a function that gives the surface area of a bale in terms of only the radius r .
 - Find the dimensions of a bale that has the given volume and uses the least amount of plastic possible when the bale is wrapped.



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32. **AQUARIUM DESIGN** A manufacturer is designing an aquarium whose base is a regular hexagon. The aquarium should have a volume of 24 cubic feet and use the least amount of material possible. Let s be the length (in feet) of a side of the base, and let h be the height (in feet).



- a. Write an equation that gives h in terms of s . (*Hint:* The volume of the aquarium is given by $V = \frac{3\sqrt{3}}{2}s^2h$.)
- b. Find the dimensions s and h that minimize the amount of material used. (*Hint:* The surface area of the aquarium is given by $S = \frac{3\sqrt{3}}{2}s^2 + 6sh$.)

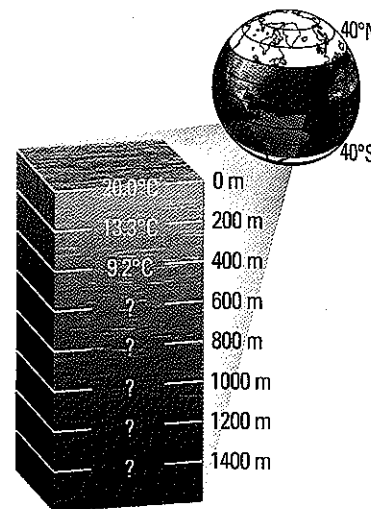
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33. **MULTIPLE REPRESENTATIONS** The mean temperature T (in degrees Celsius) of the Atlantic Ocean between latitudes 40°N and 40°S can be modeled by

$$T = \frac{17,800d + 20,000}{3d^2 + 740d + 1000}$$

where d is the depth (in meters).

- a. **Making a Table** Make a table of values showing the mean temperature for depths from 1000 meters to 1300 meters in 50 meter intervals.
- b. **Using a Graph** Graph the model. Use your graph to estimate the depth at which the mean temperature is 4°C .



34. **MULTI-STEP PROBLEM** From 1993 to 2002, the number n (in billions) of shares of stock sold on the New York Stock Exchange can be modeled by

$$n = \frac{1054t + 6204}{-6.62t + 100}$$

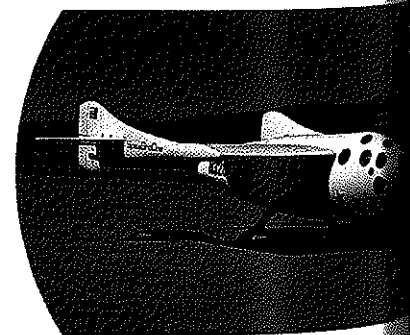
where t is the number of years since 1993.

- a. Graph the model.
- b. *Describe* the general trends shown by the graph.
- c. Estimate the year when the number of shares of stock sold was first greater than 100 billion.
35. **★ EXTENDED RESPONSE** The acceleration due to gravity g (in meters per second squared) changes as altitude changes and is given by the function

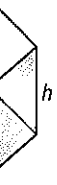
$$g = \frac{3.99 \times 10^{14}}{h^2 + (1.28 \times 10^7)h + (4.07 \times 10^{13})}$$

where h is the altitude (in meters) above sea level.

- a. **Graph** Graph the function.
- b. **Apply** A mountaineer is climbing to a height of 8000 meters. What is the value of g at this altitude?
- c. **Apply** A spacecraft reaches an altitude of 112 kilometers above Earth. What is the value of g at this altitude?
- d. **Explain Describe** what happens to the value of g as altitude increases.

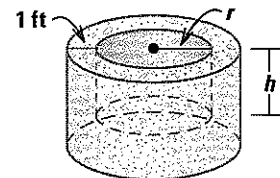


This spacecraft reached an altitude of 112 km in 2004.



36. CHALLENGE You need to build a cylindrical water tank using 100 cubic feet of concrete. The sides and the base of the tank must be 1 foot thick.

- Write an equation that gives the tank's inner height h in terms of its inner radius r .
- Write an equation that gives the volume V of water that the tank can hold as a function of r .
- Graph the equation from part (b). What values of r and h maximize the tank's capacity?



MIXED REVIEW

PREVIEW

Prepare for
Lesson 8.4
in Exs. 37–45.

Factor the expression.

37. $x^2 - 64$ (p. 252)

38. $x^2 - 8x - 48$ (p. 252)

39. $18x^2 - 37x - 20$ (p. 259)

40. $12x^2 - 15x - 18$ (p. 259)

41. $5x^2 + 22x - 30$ (p. 259)

42. $5x^3 + 40$ (p. 353)

43. $x^3 - 4x^2 + 8x - 32$ (p. 353)

44. $x^3 + 2x^2 - 35x$ (p. 353)

45. $x^5 - 9x^3 - 36x$ (p. 353)

Simplify the expression. Tell which properties of exponents you used. (p. 330)

46. $\frac{x^5y}{x^2y^4}$

47. $\frac{48x^{-1}y^4}{6x^2y^3}$

48. $\left(\frac{x^2y^4}{xy^5}\right)^2$

49. $\left(\frac{72x^3y^{-1}}{12x^{-1}y^2}\right)^{-1}$

50. $\frac{6x^{-2}y^2}{36xy^{-3}}$

51. $\left(\frac{x^5y^4}{x^7y^8}\right)^{-2}$

52. $\left(\frac{90x^3y^{-1}}{18x^{-1}y^{-2}}\right)^2$

53. $\left(\frac{xy^6}{x^2y^5}\right)^3$

QUIZ for Lessons 8.1–8.3

The variables x and y vary inversely. Use the given values to write an equation relating x and y . Then find y when $x = -4$. (p. 551)

1. $x = 8, y = 3$

2. $x = 2, y = -9$

3. $x = -5, y = \frac{8}{3}$

4. $x = -\frac{1}{4}, y = -32$

Graph the function.

5. $y = \frac{3}{2x}$ (p. 558)

6. $y = \frac{4}{x-2} + 1$ (p. 558)

7. $f(x) = \frac{-2x}{3x-6}$ (p. 558)

8. $y = \frac{-8}{x^2-1}$ (p. 565)

9. $y = \frac{x^2-6}{x^2+2}$ (p. 565)

10. $g(x) = \frac{x^3-8}{2x^2}$ (p. 565)

11. **SOFTBALL** A pitcher throws 16 strikes in her first 38 pitches. The table shows how the pitcher's strike percentage changes if she throws x consecutive strikes after the first 38 pitches. Write a rational function for the strike percentage in terms of x . Graph the function. How many consecutive strikes must the pitcher throw to reach a strike percentage of 0.60? (p. 558)

x	Total strikes	Total pitches	Strike percentage
0	16	38	0.42
5	21	43	0.49
10	26	48	0.54
x	$x + 16$	$x + 38$?

reached an
n in 2004.

IPLE
PRESENTATIONS

EXAMPLE 7 Divide a rational expression by a polynomial

Divide: $\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$

$$\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x) = \frac{6x^2 + x - 15}{4x^2} \cdot \frac{1}{3x^2 + 5x}$$
 Multiply by reciprocal.

$$= \frac{(3x + 5)(2x - 3)}{4x^2} \cdot \frac{1}{x(3x + 5)}$$
 Factor.

$$= \frac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}}$$
 Divide out common factors.

$$= \frac{2x - 3}{4x^3}$$
 Simplified form

GUIDED PRACTICE for Examples 6 and 7

Divide the expressions. Simplify the result.

11. $\frac{4x}{5x - 20} \div \frac{x^2 - 2x}{x^2 - 6x + 8}$

12. $\frac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$

8.4 EXERCISES**HOMEWORK KEY**○ = WORKED-OUT SOLUTIONS
on p. WS15 for Exs. 7, 25, and 49★ = STANDARDIZED TEST PRACTICE
Exs. 2, 20, 23, 50, and 52**SKILL PRACTICE**1. **VOCABULARY** Copy and complete: To divide one rational expression by another, multiply the first rational expression by the ? of the second rational expression.2. ★ **WRITING** How do you know when a rational expression is simplified?**EXAMPLE 1**on p. 573
for Exs. 3–20**REASONING** Match the rational expression with its simplified form.

3. $\frac{x^2 - 9x + 14}{x^2 - 5x - 14}$

4. $\frac{x^2 - 4}{x^2 + 9x + 14}$

5. $\frac{x^2 + 5x - 14}{x^2 - 4x + 4}$

A. $\frac{x - 2}{x + 7}$

B. $\frac{x - 2}{x + 2}$

C. $\frac{x + 7}{x - 2}$

SIMPLIFYING Simplify the rational expression, if possible.

6. $\frac{4x^2}{20x^2 - 12x}$

7. $\frac{x^2 - x - 20}{x^2 + 2x - 15}$

8. $\frac{x^2 + 2x - 24}{x^2 + 7x + 6}$

9. $\frac{x^2 - 11x + 24}{x^2 - 3x - 40}$

10. $\frac{x^2 + 4x + 4}{x^2 - 5x + 4}$

11. $\frac{2x^2 + 2x - 4}{x^2 - 5x - 14}$

12. $\frac{x - 4}{x^3 - 64}$

13. $\frac{x^2 - 36}{x^2 + 12x + 36}$

14. $\frac{3x^3 + 6x^2 + 12x}{x^3 - 8}$

15. $\frac{8x^2 + 10x - 3}{6x^2 + 13x + 6}$

16. $\frac{5x^2 + 18x - 8}{10x^2 - x - 2}$

17. $\frac{x^3 - 5x^2 - 3x + 15}{x^2 - 8x + 15}$

ERROR ANALYSIS Describe and correct the error in simplifying the rational expression.

18.
$$\frac{x^2 + 16x - 80}{x^2 - 16} = \frac{16x - 80}{-16} = -x + 5$$

✗

19.
$$\frac{x^2 + 16x + 48}{x^2 + 8x + 16} = \frac{x^2 + 2x + 3}{x^2 + x + 1}$$

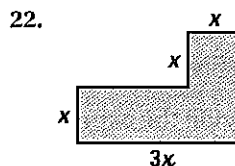
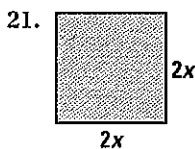
✗

20. ★ **MULTIPLE CHOICE** Which rational expression is in simplified form?

- (A) $\frac{x^2 - x - 6}{x^2 + 3x + 2}$ (B) $\frac{x^2 + 6x + 8}{x^2 + 2x - 3}$ (C) $\frac{x^2 - 6x + 9}{x^2 - 2x - 3}$ (D) $\frac{x^2 + 3x - 4}{x^2 + x - 2}$

EXAMPLE 2
on p. 574
for Exs. 21–23

GEOMETRY A farmer wants to fence in the field shown. Write a simplified rational expression for the ratio of the field's perimeter to its area.



23. ★ **SHORT RESPONSE** Which of the fields in Exercises 21 and 22 has the lower fencing cost per unit of area? Explain.

EXAMPLES 3, 4, and 5
on pp. 575–576
for Exs. 24–33

MULTIPLYING Multiply the expressions. Simplify the result.

24. $\frac{5x^3y}{x^2y^2} \cdot \frac{y^3}{15x^2}$

25. $\frac{48x^5y^3}{y^4} \cdot \frac{x^2y}{6x^3y^2}$

26. $\frac{x(x-3)}{x-2} \cdot \frac{(x+3)(x-2)}{x}$

27. $\frac{4(x+5)}{x^2} \cdot \frac{x(x+1)}{2(x+5)}$

28. $\frac{3x-12}{x+5} \cdot \frac{x+6}{2x-8}$

29. $\frac{x+5}{4x-16} \cdot \frac{2x^2-32}{x^2-25}$

30. $\frac{x^2+3x-4}{x^2+4x+4} \cdot \frac{2x^2+4x}{x^2-4x+3}$

31. $\frac{x^2-3x-10}{x^2-2x-15} \cdot (x^2+10x+21)$

32. $\frac{x^2+5x-36}{x^2-49} \cdot (x^2-11x+28)$

33. $\frac{4x^2+20x}{x^3+4x^2} \cdot (x^2+8x+16)$

EXAMPLES 6 and 7
on pp. 576–577
for Exs. 34–43

DIVIDING Divide the expressions. Simplify the result.

34. $\frac{5x^2y^3}{x^7} \div \frac{30xy^4}{y^3}$

35. $\frac{8x^2y^2z}{xz^3} \div \frac{10xy}{x^4z}$

36. $\frac{(x+3)(x-2)}{x(x+1)} \div \frac{x+3}{x}$

37. $\frac{8x^2}{x+4} \div \frac{x}{2(x-4)}$

38. $\frac{x^2-6x-27}{2x^2+2x} \div \frac{x^2-14x+45}{x^2}$

39. $\frac{x^2-4x-5}{x+5} \div (x^2+6x+5)$

40. $\frac{3x^2+13x+4}{x^2-4} \div \frac{4x+16}{x+2}$

41. $\frac{x^2-x-2}{x^2+4x-5} \div \frac{x-2}{5x+25}$

42. $\frac{x^2-8x+15}{x^2+4x} \div (x^2-x-20)$

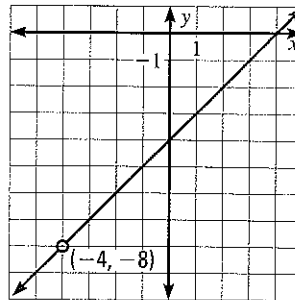
43. $\frac{x^2+12x+32}{6x+42} \div \frac{x^2+4x}{x^2-49}$

POINT DISCONTINUITY In Exercises 44–46, use the following information.

The graph of a rational function can have a hole in it, called a *point discontinuity*, where the function is undefined. An example is shown below.

$$y = \frac{x^2 - 16}{x + 4} = \frac{(x+4)(x-4)}{x+4} = x - 4$$

The graph of $y = \frac{x^2 - 16}{x + 4}$ is the same as the graph of $y = x - 4$ except that there is a hole at $(-4, -8)$ because the rational function is not defined when $x = -4$.



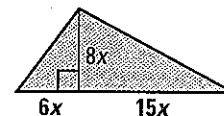
Graph the rational function. Use an open circle for a point discontinuity.

44. $y = \frac{x^2 + 10x + 21}{x + 3}$

45. $y = \frac{x^2 - 36}{x - 6}$

46. $y = \frac{2x^2 - x - 10}{x + 2}$

47. **CHALLENGE** Find the ratio of the perimeter to the area of the triangle shown at the right.

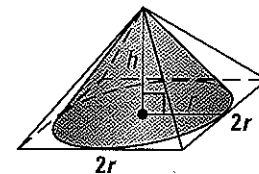


PROBLEM SOLVING

EXAMPLE 2
on p. 574 for
Exs. 48, 50–52

48. **GEOMETRY** Find the ratio of the volume of the square pyramid to the volume of the inscribed cone. Write your answer in simplified form.

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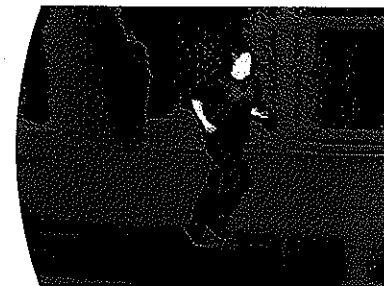
49. **ENTERTAINMENT** From 1992 to 2002, the gross ticket sales S (in millions of dollars) to Broadway shows and the total attendance A (in millions) at the shows can be modeled by

$$S = \frac{-6420t + 292,000}{6.02t^2 - 125t + 1000} \quad \text{and} \quad A = \frac{-407t + 7220}{5.92t^2 - 131t + 1000}$$

where t is the number of years since 1992. Write a model for the *average* dollar amount a person paid per ticket as a function of the year. What was the average amount a person paid per ticket in 1999?

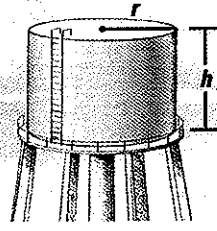
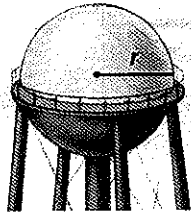
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50. **★ SHORT RESPONSE** Almost all of the energy generated by a long-distance runner is released in the form of heat. For a runner with height H and speed V , the rate h_g of heat generated and the rate h_r of heat released can be modeled by $h_g = k_1 H^3 V^2$ and $h_r = k_2 H^2$ where k_1 and k_2 are constants.
- Write the ratio of heat generated to heat released. Simplify the expression.
 - When the ratio of heat generated to heat released equals 1, how is speed related to height? Does a taller or shorter runner have the advantage? *Explain.*



Thermogram of runner

51. **MULTI-STEP PROBLEM** A manufacturer is comparing two designs for a water tower: a sphere and a cylinder. Both designs have the same volume and the same radius.

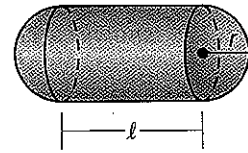


- Show that the height h of the cylindrical tank is $\frac{4}{3}r$.
 - Write an expression for the surface area of each tank in terms of r .
 - Find the ratio of the surface area of the spherical tank to the surface area of the cylindrical tank. *Explain* what the ratio tells you about which water tower would take less material to build.
52. **★ EXTENDED RESPONSE** The surface area S and the volume V of a cylindrical can are given by $S = 2\pi r^2 + 2\pi rh$ and $V = \pi r^2 h$ where r is the radius and h is the height.

- Model** Write and simplify an expression for the efficiency ratio $\frac{S}{V}$.
- Calculate** Find the efficiency ratio for each can listed in the table.

	Soup can	Coffee can	Paint can
Height, h	10.2 cm	15.9 cm	19.4 cm
Radius, r	3.4 cm	7.8 cm	8.4 cm

- Compare** Rank the three cans in part (b) according to efficiency. *Explain* your ranking.
53. **CHALLENGE** A fuel storage container is shaped like a cylinder with a hemisphere on each end, as shown. The length of the cylinder is l and the radius of each hemisphere is r . Show that the ratio of the surface area to the volume of the container is $\frac{6(2r + l)}{r(4r + 3l)}$.



MIXED REVIEW

PREVIEW

Prepare for
Lesson 8.5
in Exs. 54–59.

Find the greatest common factor and the least common multiple of the pair of numbers. (p. 978)

54. 48, 80

55. 120, 155

56. 38, 95

57. 52, 91

58. 66, 154

59. 360, 450

Find the product. (p. 346)

60. $x(x^2 + 4x - 7)$

61. $(x + 9)(x - 5)$

62. $(x + 11)(x - 7)$

63. $(x + 2)(x^2 - 6x + 10)$

64. $(3x - 7)(x^2 - 5x)$

65. $(x + 5)(x^3 + 8x^2)$



8.5 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS
on p. WS15 for Exs. 5, 17, and 43
★ = STANDARDIZED TEST PRACTICE
Exs. 2, 15, 26, 37, and 44

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A fraction that contains a fraction in its numerator or denominator is called a(n) ?.

2. ★ **WRITING** Explain how to add rational expressions with unlike denominators.

EXAMPLE 1

on p. 582
for Exs. 3–8

LIKE DENOMINATORS Perform the indicated operation and simplify.

$$3. \frac{15}{4x} + \frac{5}{4x}$$

$$4. \frac{x}{16x^2} - \frac{4}{16x^2}$$

$$5. \frac{9}{x+1} - \frac{2x}{x+1}$$

$$6. \frac{3x^2}{x-8} + \frac{6x}{x-8}$$

$$7. \frac{5x}{x+3} + \frac{15}{x+3}$$

$$8. \frac{4x^2}{2x-1} - \frac{1}{2x-1}$$

EXAMPLE 2

on p. 583
for Exs. 9–15

FINDING LCMS Find the least common multiple of the polynomials.

$$9. 3x \text{ and } 3(x-2)$$

$$10. 2x^2 \text{ and } 4x+12$$

$$11. 2x \text{ and } 2x(x-5)$$

$$12. 24x^2 \text{ and } 8x^2 - 16x$$

$$13. x^2 - 25, x, \text{ and } x-5$$

$$14. 9x^2 - 16 \text{ and } 3x^2 - 2x - 8$$

15. ★ **MULTIPLE CHOICE** What is the least common multiple of the polynomials $3x^2 - 9x$ and $6x^2$?

(A) $3x(x-3)$

(B) $6x^2$

(C) $6x(x-3)$

(D) $6x^2(x-3)$

EXAMPLES 3 and 4

on pp. 583–584
for Exs. 16–26

UNLIKE DENOMINATORS Perform the indicated operation and simplify.

$$16. \frac{12}{5x} + \frac{7}{6x}$$

$$17. \frac{8}{3x^2} - \frac{5}{4x}$$

$$18. \frac{x-4}{5x} - \frac{12}{5(x-4)}$$

$$19. \frac{12}{x^2+5x-24} + \frac{3}{x-3}$$

$$20. \frac{3}{x+4} - \frac{1}{x+6}$$

$$21. \frac{9}{x-3} + \frac{2x}{x+1}$$

$$22. \frac{x+4}{x^2-4} - \frac{15}{x-2}$$

$$23. \frac{-15x}{x^2-8x+16} + \frac{12}{x-4}$$

$$24. \frac{x^2-5}{x^2+5x-14} - \frac{x+3}{x+7}$$

25. **ERROR ANALYSIS** Describe and correct the error in adding the rational expressions.

$$\frac{x}{x+2} + \frac{4}{x-5} = \frac{x+4}{(x+2)(x-5)} \quad \times$$

26. ★ **MULTIPLE CHOICE** Which expression is equivalent to $\frac{2x}{x+4} - \frac{x^2+4}{x^2-16}$?

(A) $\frac{1}{x+4}$

(B) $\frac{(x+2)(x-2)}{(x+4)(x-4)}$

(C) $\frac{x^2-8x-4}{(x+4)(x-4)}$

(D) $\frac{3x^2-8x+4}{(x+4)(x-4)}$

UNLIKE DENOMINATORS Perform the indicated operation(s) and simplify.

$$27. \frac{x}{x^2-9} + \frac{x+1}{x^2+6x+9}$$

$$28. \frac{x+3}{x^2-2x-8} - \frac{x-5}{x^2-12x+32}$$

$$29. \frac{x+2}{x-4} + \frac{2}{x} + \frac{5x}{3x-1}$$

$$30. \frac{x+3}{x^2-25} - \frac{x-1}{x-5} + \frac{3}{x+3}$$

EXAMPLES
5 and 6
on p. 585
for Exs. 31–36

SIMPLIFYING COMPLEX FRACTIONS Simplify the complex fraction.

31. $\frac{\frac{x-6}{3}}{10 + \frac{4}{x}}$

32. $\frac{15 - \frac{2}{x}}{\frac{x}{5} + 4}$

33. $\frac{\frac{16}{x-2}}{\frac{4}{x+1} + \frac{6}{x}}$

34. $\frac{\frac{1}{2x-5} - \frac{7}{8x-20}}{\frac{x}{2x-5}}$

35. $\frac{\frac{3}{x-2} - \frac{6}{x^2-4}}{\frac{3}{x+2} + \frac{1}{x-2}}$

36. $\frac{\frac{1}{3x^2-3}}{\frac{5}{x+1} - \frac{x+4}{x^2-3x-4}}$

37. ★ **OPEN-ENDED MATH** Write two different complex fractions that each simplify to $\frac{x-3}{x+4}$.

CHALLENGE Simplify the complex fraction.

38. $\frac{\frac{1}{x} - \frac{x}{x^{-1}+1}}{\frac{5}{x}}$

39. $\frac{\frac{3-2x}{x^3}}{\frac{2}{x^2} - \frac{1}{x^3+x^2}}$

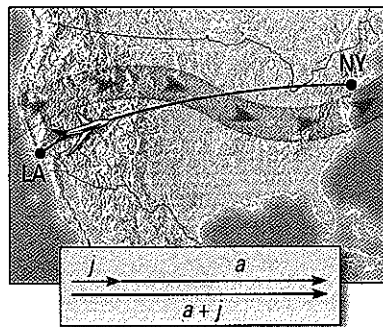
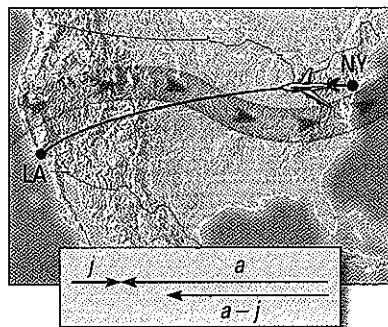
40. $\frac{3x^{-2} + (2x-1)^{-1}}{\frac{6}{x^{-1}+2} + 3x^{-1}}$

PROBLEM SOLVING

EXAMPLE 3
on p. 583
for Ex. 41

41. **JET STREAM** The total time T (in hours) needed to fly from New York to Los Angeles and back (ignoring layovers) can be modeled by the equation in the diagram, where d is the distance each way (in miles), a is the average airplane speed (in miles per hour), and j is the average speed of the jet stream (in miles per hour).

$$T = \frac{d}{a-j} + \frac{d}{a+j}$$

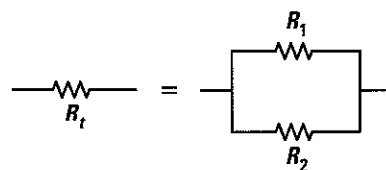


Rewrite the equation so that the right side is simplified. Then find the total time if $d = 2468$ miles, $a = 510$ mi/h, and $j = 115$ mi/h.

Animated Algebra at classzone.com

EXAMPLES
5 and 6
on p. 585
for Exs. 42–43

42. **ELECTRONICS** If two resistors in a parallel circuit have resistances R_1 and R_2 (both in ohms), then the total resistance R_t (in ohms) is given by the equation shown. Simplify the complex fraction. Then find the total resistance if $R_1 = 2000$ ohms and $R_2 = 5600$ ohms.



$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

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43. **CAR LOANS** If you borrow P dollars to buy a car and agree to repay the loan over t years at a monthly interest rate of i (expressed as a decimal), then your monthly payment M is given by either formula below.

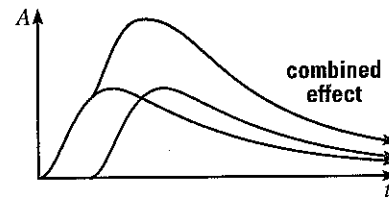
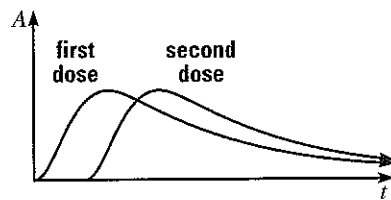
$$\text{Formula 1: } M = \frac{Pi}{1 - \left(\frac{1}{1+i}\right)^{12t}}$$

$$\text{Formula 2: } M = \frac{Pi(1+i)^{12t}}{(1+i)^{12t} - 1}$$

- Show that the formulas are equivalent by simplifying the first formula.
 - Find your monthly payment if you borrow \$15,500 at a monthly interest rate of 0.5% and repay the loan over 4 years.
44. **★ EXTENDED RESPONSE** The amount A (in milligrams) of aspirin in a person's bloodstream can be modeled by

$$A = \frac{391t^2 + 0.112}{0.218t^4 + 0.991t^2 + 1}$$

where t is the time (in hours) after one dose is taken.



- Graph the equation using a graphing calculator.
 - A second dose of the drug is taken 1 hour after the first dose. Write an equation to model the amount of the second dose in the bloodstream.
 - Write and graph a model for the *total* amount of aspirin in the bloodstream after the second dose is taken.
 - About how long after the second dose has been taken is the greatest amount of aspirin in the bloodstream?
45. **CHALLENGE** Find the next two expressions in the pattern shown. Then simplify all five expressions. What value do the expressions approach?

$$1 + \frac{1}{2 + \frac{1}{2}}, 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}, 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}, \dots$$

MIXED REVIEW

PREVIEW
Prepare for
Lesson 8.6
in Exs. 46–54.

Solve the equation.

46. $\frac{1}{3}x + 4 = 15$ (p. 18)

47. $2x = -\frac{5}{8}x - 18$ (p. 18)

48. $12x + 7 = \frac{14}{3}x$ (p. 18)

49. $x^2 + 9x - 36 = 0$ (p. 252)

50. $3x^2 + x - 14 = 0$ (p. 259)

51. $4x(x - 5) = 4x - 35$ (p. 259)

52. $6x^2 - 25 = x^2$ (p. 266)

53. $4(x - 2)^2 = 144$ (p. 266)

54. $3(x + 5)^2 - 10 = 182$ (p. 266)

Graph the function.

55. $y = 4^x$ (p. 478)

56. $y = -2 \cdot 3^x$ (p. 478)

57. $f(x) = \frac{2}{3} \cdot 2^x$ (p. 478)

58. $y = 4\left(\frac{1}{2}\right)^x$ (p. 486)

59. $y = -3\left(\frac{1}{4}\right)^x$ (p. 486)

60. $g(x) = 5\left(\frac{3}{8}\right)^x$ (p. 486)

EXAMPLE 6 Solve a rational equation given a function

VIDEO GAME SALES From 1995 through 2003, the annual sales S (in billions of dollars) of entertainment software can be modeled by

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000}, \quad 0 \leq t \leq 8$$

where t is the number of years since 1995. For which year were the total sales of entertainment software about \$5.3 billion?

**ANOTHER WAY**

For alternative methods for solving the problem in Example 6, turn to page 596 for the **Problem Solving Workshop**.

Solution

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000}$$

Write given function.

$$5.3 = \frac{848t^2 + 3220}{115t^2 + 1000}$$

Substitute 5.3 for $S(t)$.

$$5.3(115t^2 + 1000) = 848t^2 + 3220$$

Multiply each side by $115t^2 + 1000$.

$$609.5t^2 + 5300 = 848t^2 + 3220$$

Simplify.

$$5300 = 238.5t^2 + 3220$$

Subtract $609.5t^2$ from each side.

$$2080 = 238.5t^2$$

Subtract 3220 from each side.

$$8.72 \approx t^2$$

Divide each side by 238.5.

$$\pm 2.95 \approx t$$

Take square roots of each side.

Because -2.95 is not in the domain ($0 \leq t \leq 8$), the only solution is 2.95.

► So, the total sales of entertainment software were about \$5.3 billion about 3 years after 1995, or in 1998.

**GUIDED PRACTICE** for Example 6

11. **WHAT IF?** Use the information in Example 6 to determine in which year the total sales of entertainment software were about \$4.5 billion.

8.6 EXERCISES**HOMEWORK KEY**

- = WORKED-OUT SOLUTIONS on p. WS15 for Exs. 5, 15, and 35
 ★ = STANDARDIZED TEST PRACTICE Exs. 2, 13, 28, 29, 34, and 36

SKILL PRACTICE

- VOCABULARY** Copy and complete: When you write $\frac{x}{3} = \frac{x+2}{5}$ as $5x = 3(x+2)$, you are ? .
- ★ WRITING** A student solved the equation $\frac{5}{x-4} = \frac{x}{x-4}$ and got the solutions 4 and 5. Which, if either, of these is extraneous? *Explain.*
- REASONING** Describe how you can use a graph to determine if an apparent solution of a rational equation is extraneous.

EXAMPLE 1
on p. 589
for Exs. 4–13

CROSS MULTIPLYING Solve the equation by cross multiplying. Check for extraneous solutions.

$$4. \frac{4}{2x} = \frac{5}{x+6}$$

$$5. \frac{9}{3x} = \frac{4}{x+2}$$

$$6. \frac{6}{x-1} = \frac{9}{x+1}$$

$$7. \frac{8}{3x-2} = \frac{2}{x-1}$$

$$8. \frac{x}{x+1} = \frac{3}{x+1}$$

$$9. \frac{x-3}{x+5} = \frac{x}{x+2}$$

$$10. \frac{x}{x^2-2} = \frac{-1}{x}$$

$$11. \frac{4(x-4)}{x^2+2x-8} = \frac{4}{x+4}$$

$$12. \frac{9}{x^2-6x+9} = \frac{3x}{x^2-3x}$$

13. **★ MULTIPLE CHOICE** What is the solution of $\frac{3}{x+2} = \frac{6}{x-1}$?

(A) -5

(B) -4

(C) -1

(D) 4

EXAMPLES 3, 4, and 5
on pp. 590–591
for Exs. 14–27

LEAST COMMON DENOMINATOR Solve the equation by using the LCD. Check for extraneous solutions.

$$14. \frac{4}{x} + x = 5$$

$$15. \frac{2}{3x} + \frac{1}{6} = \frac{4}{3x}$$

$$16. \frac{5}{x} - 2 = \frac{2}{x+3}$$

$$17. \frac{1}{2x} + \frac{3}{x+7} = \frac{-1}{x}$$

$$18. \frac{1}{x-2} + 2 = \frac{3x}{x+2}$$

$$19. \frac{5}{x^2+x-6} = 2 + \frac{x-3}{x-2}$$

$$20. \frac{x+1}{x+6} + \frac{1}{x} = \frac{2x+1}{x+6}$$

$$21. \frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$$

$$22. \frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$$


$$23. \frac{10}{x} + 3 = \frac{x+9}{x-4}$$

$$24. \frac{18}{x^2-3x} - \frac{6}{x-3} = \frac{5}{x}$$

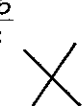
$$25. \frac{x+3}{x-3} + \frac{x}{x-5} = \frac{x+5}{x-5}$$

ERROR ANALYSIS Describe and correct the error in the first step of solving the equation.

26.
$$\frac{3}{2x} + \frac{4}{x^2} = 1$$

$$3x^2 + 8x = 1$$
 

27.
$$\frac{5}{x} + \frac{23}{6} = \frac{45}{x}$$

$$\frac{28}{x+6} = \frac{45}{x}$$
 

28. **★ MULTIPLE CHOICE** What is (are) the solution(s) of $\frac{2}{x-3} = \frac{1}{x^2-2x-3}$?

(A) -3, $-\frac{1}{2}$

(B) $-\frac{1}{2}$, 3

(C) $-\frac{1}{2}$

(D) 3

29. **★ OPEN-ENDED MATH** Give an example of a rational equation that you would solve using cross multiplication. Then give an example of a rational equation that you would solve by multiplying each side by the LCD of the fractions.

CHALLENGE In Exercises 30–32, a is a nonzero real number. Tell whether the algebraic statement is *always true*, *sometimes true*, or *never true*. Explain your answer.

30. For the equation $\frac{1}{x-a} = \frac{x}{x-a}$, $x = a$ is an extraneous solution.


31. The equation $\frac{3}{x-a} = \frac{x}{x-a}$ has exactly one solution.

32. The equation $\frac{1}{x-a} = \frac{2}{x+a} + \frac{2a}{x^2-a^2}$ has no solution.

PROBLEM SOLVING

EXAMPLE 2
on p. 589
for Exs. 33–34

- 33. VOLLEYBALL** So far in your volleyball match, you have put into play 37 of the 44 serves you have attempted. Solve the equation $\frac{90}{100} = \frac{37+x}{44+x}$ to find the number of consecutive serves you need to put into play in order to raise your service percentage to 90%.

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- 34. ★ EXTENDED RESPONSE** A speed skater travels 9 kilometers in the same amount of time that it takes a second skater to travel 8 kilometers. The first skater travels 4.38 kilometers per hour faster than the second skater.

- a. Use the verbal model below to write an equation that relates the skating times of the skaters.

$$\frac{\text{Distance for skater 1}}{\text{Skater 1 speed}} = \frac{\text{Distance for skater 2}}{\text{Skater 2 speed}}$$

- b. Solve the equation in part (a) to find the speeds of both skaters.
c. How long did the skaters skate? *Explain* your answer.

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EXAMPLE 6
on p. 592
for Ex. 35

- 35. MUSIC INDUSTRY** From 1994 through 2003, the number n (in millions) of CDs shipped can be modeled by

$$n = \frac{635t^2 - 7350t + 27,200}{t^2 - 11.5t + 39.4}, \quad 0 \leq t \leq 9$$

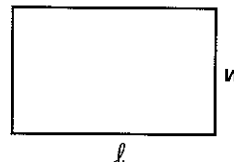
where t is the number of years since 1994. During which year was the total number of CDs shipped about 720 million?

- 36. ★ EXTENDED RESPONSE** You can paint a room in 8 hours. Working together, you and your friend can paint the room in just 5 hours.
- a. Let t be the time (in hours) your friend would take to paint the room when working alone. Copy and complete the table.

	Work Rate	Time	= Work Done
You	$\frac{1 \text{ room}}{8 \text{ hours}}$	5 hours	?
Friend	?	5 hours	?

- b. What is the sum of the expressions in the table's last column? *Explain*.
c. Write and solve an equation to find how long your friend would take to paint the room when working alone. *Explain* your answer.

- 37. GEOMETRY** *Golden rectangles* are rectangles for which the ratio of the width w to the length l is equal to the ratio of l to $l + w$. The ratio of the length to the width for these rectangles is called the *golden ratio*. Find the value of the golden ratio using a rectangle with a width of 1 unit.



- 38. CHALLENGE** Let x be the number of years since 1998, let $g(x)$ be the average monthly bill (in dollars) for mobile phone users in the United States, and let $h(x)$ be the average number of minutes used by U.S. mobile phone users. Then $g(x)$ and $h(x)$ are as given below.

$$g(x) = -0.27x^3 + 1.40x^2 + 1.05x + 39.4$$

$$h(x) = -8.25x^3 + 53.1x^2 - 7.82x + 138$$

- Write a rational function $f(x)$ that gives the average price per minute x years after 1998.
- Find the average price per minute in 1998.
- In what year did the average price per minute fall to 11 cents?

MIXED REVIEW

Graph the function.

39. $y = -2x + 7$ (p. 89)

40. $y = x^2 - 8x + 21$ (p. 236)

41. $f(x) = x^3 - 3$ (p. 337)

42. $y = -\sqrt{x-4} + 1$ (p. 446)

43. $y = \log 4x$ (p. 499)

44. $g(x) = \frac{2}{x+3} + 6$ (p. 558)

Simplify the expression. (p. 266)

45. $\sqrt{52}$

46. $\sqrt{24}$

47. $\sqrt{125}$

48. $\sqrt{252}$

49. $\sqrt{8} \cdot \sqrt{90}$

50. $\sqrt{5} \cdot \sqrt{80}$

51. $\sqrt{\frac{8}{20}}$

52. $\sqrt{\frac{60}{9}}$

PREVIEW

Prepare for
Lesson 9.1
in Exs. 45–52.

QUIZ for Lessons 8.4–8.6

Perform the indicated operation and simplify. (p. 573)

1. $\frac{x^2 - 2x - 24}{x^2 + 3x - 10} \cdot \frac{3x^2 - 6x}{x^3 + 4x^2}$

2. $\frac{x^2 - 10x + 16}{x^2 - 1} \cdot (x - 1)$

3. $\frac{x^2 + 9x + 20}{x^2 - 11x + 28} \div \frac{x^2 + 8x + 15}{x^2 - 3x - 4}$

4. $\frac{x^2 + 12x + 36}{x^2 - 8x + 12} \div (x^2 - 36)$

Perform the indicated operation and simplify. (p. 582)

5. $\frac{1}{x+4} + \frac{1}{x-4}$

6. $\frac{4x+3}{x^2-16} + \frac{2}{x-4}$

7. $\frac{4}{x+5} - \frac{6x-1}{x^2+10x+25}$

Solve the equation. Check for extraneous solutions. (p. 589)

8. $\frac{x-4}{x-1} = \frac{10}{x+7}$

9. $\frac{x-4}{x-2} - \frac{2x-1}{x-2} = 2$

10. $\frac{3x+6}{x^2-4} = \frac{x+1}{x-2}$

11. $\frac{5}{x} + \frac{x+1}{x+2} = \frac{2x+9}{x+2}$

12. $\frac{x-3}{x+2} = \frac{x-1}{3x-1}$

13. $\frac{x-1}{x} + \frac{2x-1}{x+3} = \frac{x+6}{x+3}$

- 14. BATTING AVERAGE** So far this baseball season, you have gotten a hit 12 times out of 60 at-bats. Solve the equation $0.360 = \frac{12+x}{60+x}$ to find the number of consecutive hits you have to get to raise your batting average to 0.360. (p. 589)